The Tale of the Tournament Equilibrium Set

Felix Brandt

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DSS
Decision Sciences & Systems

Rational Choice Theory


Amartya K. Sen

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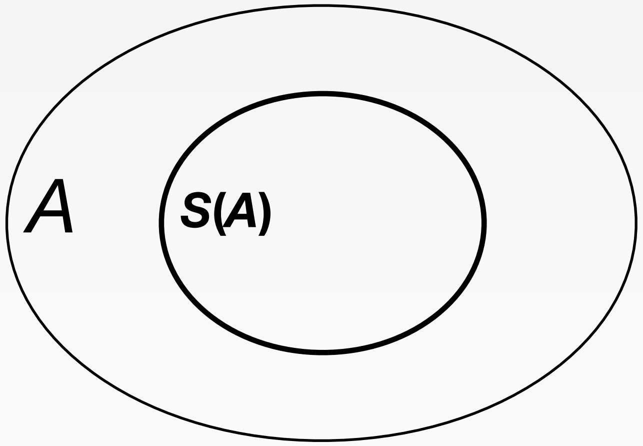

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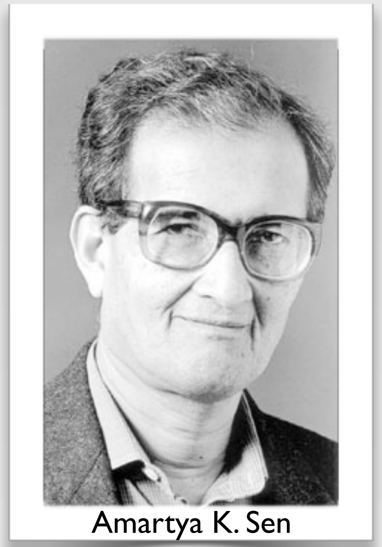

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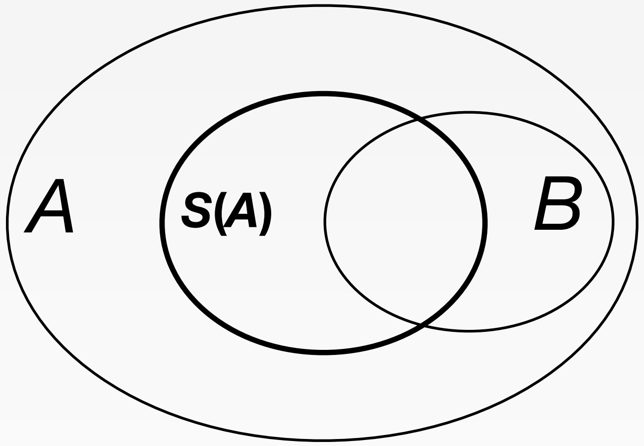

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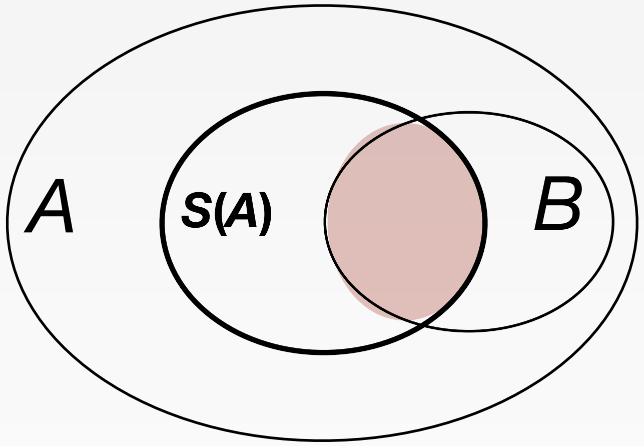

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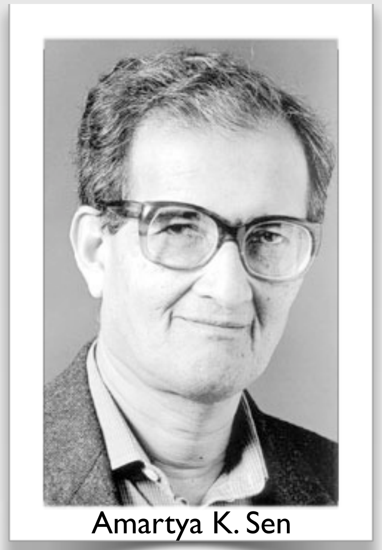

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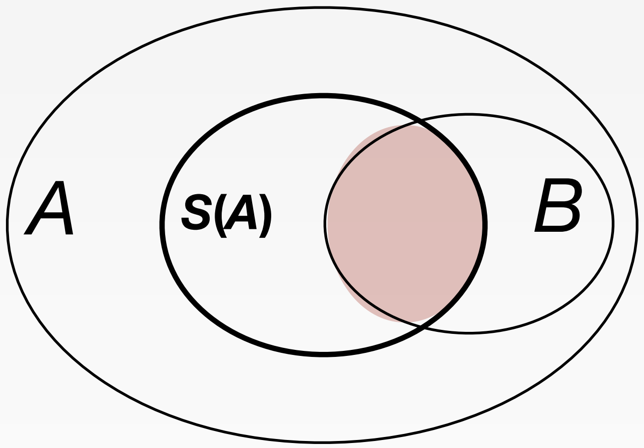

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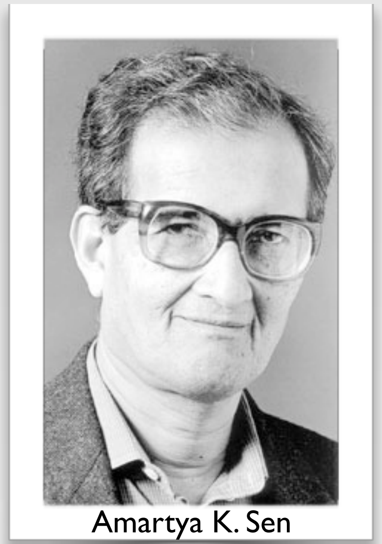

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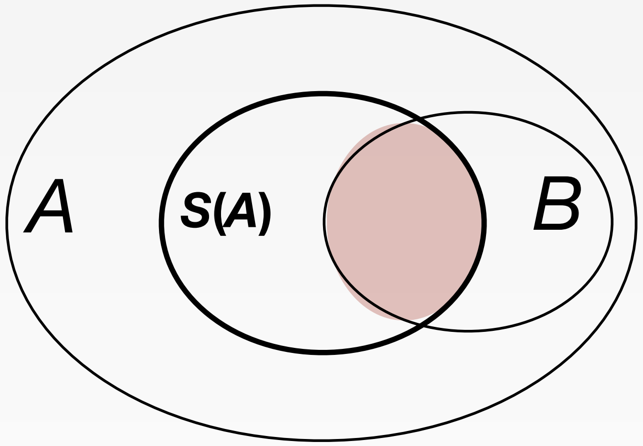

  - The conjunction of both properties is equivalent to Samuelson's *weak axiom of revealed preference* (Sen, 1969; Bordes, 1976).

From Choice to Social Choice

- Let N be a finite set of voters and $R(U)$ the set of all transitive and complete relations over U.
- A social choice function (SCF) is a function $S:R(U)^N \times F(U) \to F(U)$ such that $S(R,A) \subseteq A$.
- Useful conditions on SCFs
  - IIA (Independence of Irrelevant Alternatives): Choice only depends on preferences over alternatives within the feasible set.
  - Pareto-optimality: Alternative y is not chosen if there exists some x that is unanimously strictly preferred to y.
  - Non-dictatorship: There should be no voter whose most preferred alternative is always uniquely chosen.

Arrow’s Impossibility

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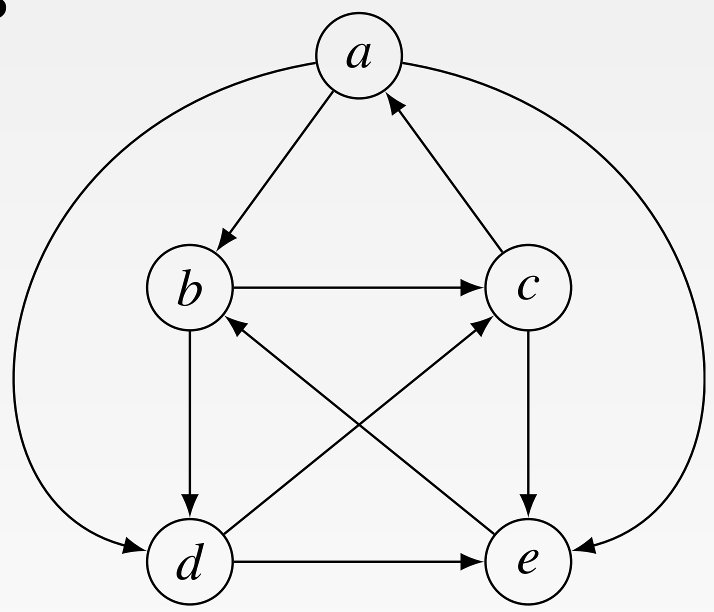
- Dropping α allows for reasonable SCFs!

Majoritarian SCFs

- An SCF is majoritarian if its outcome only depends on the pairwise majority relation $>$ within the feasible set.
    - Majoritarianism implies all Arrovian conditions except α and β^+.
    - We assume for convenience that individual preferences are strict and there is an odd number of voters.
    - Hence, the pairwise majority relation is asymmetric and complete, i.e., it can be represented by a tournament graph.
    - Let $\overline{D}(x)=\{y \mid y>x\}$ denote the dominators of x.



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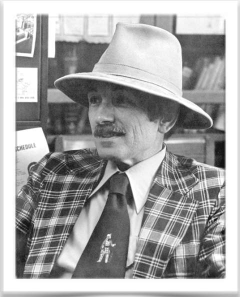
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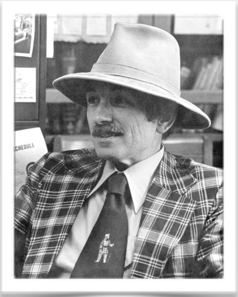

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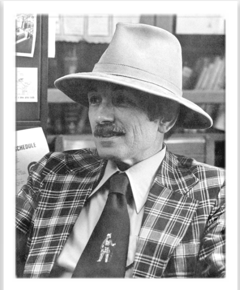
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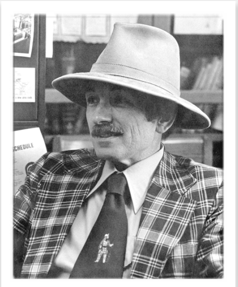
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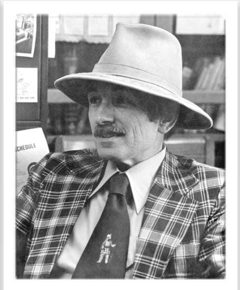
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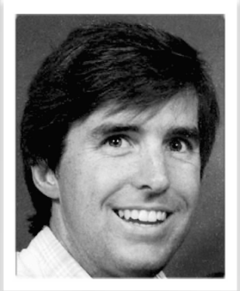
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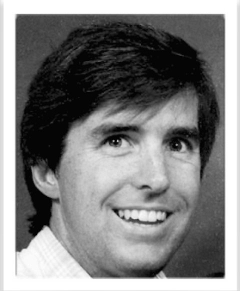

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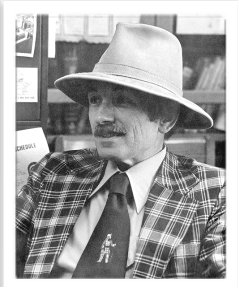
  - ρ^+ can be weakened to ρ: $S(\overline{D}(a)) \subseteq S(A)$ for all $a \in S(A)$.
- Conjecture (Schwartz, 1990): The *tournament equilibrium set* (TEQ) is the smallest majoritarian SCF satisfying ρ.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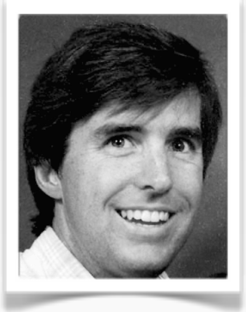
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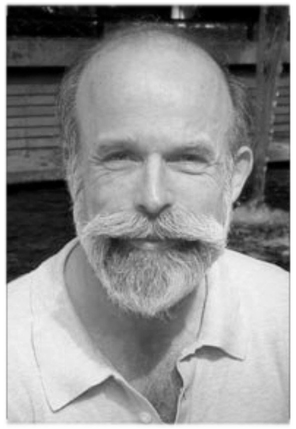
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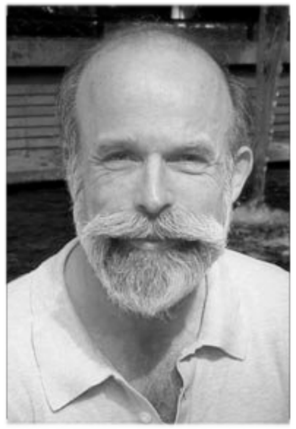
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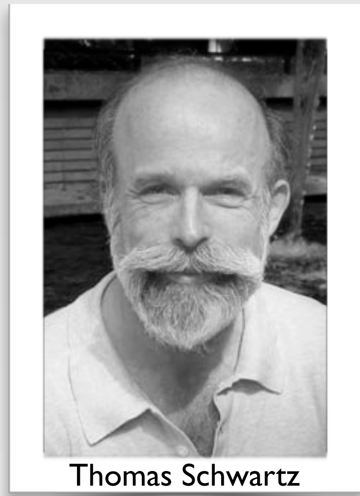

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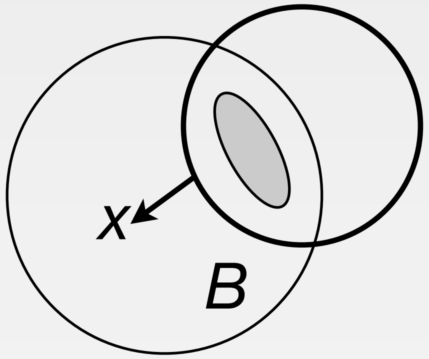

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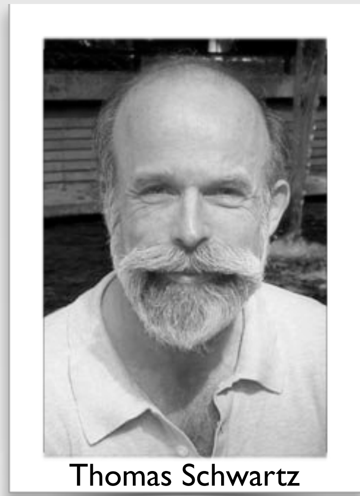

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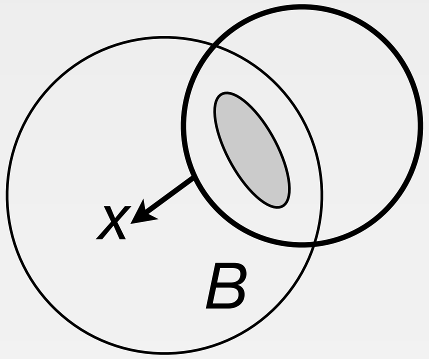
  - A non-empty set of alternatives B is *S-retentive* if $S(\overline{D}(x))\subseteq B$ for all $x\in B$.
  - Idea: No alternative in the set should be "properly" dominated by an outside alternative.



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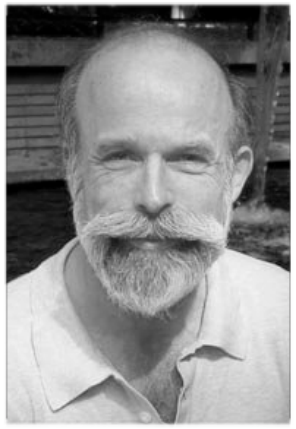
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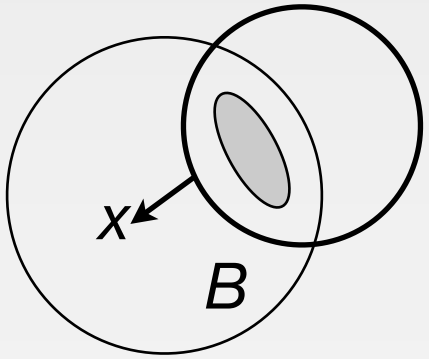

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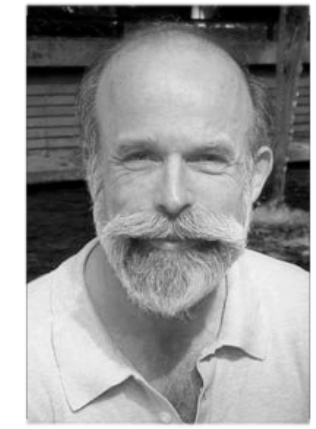

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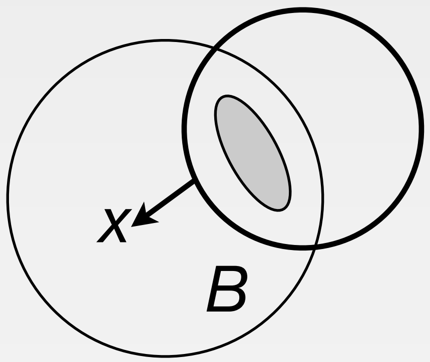

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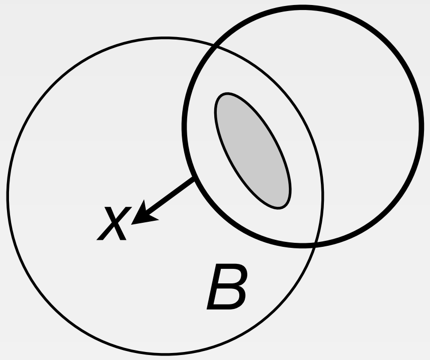

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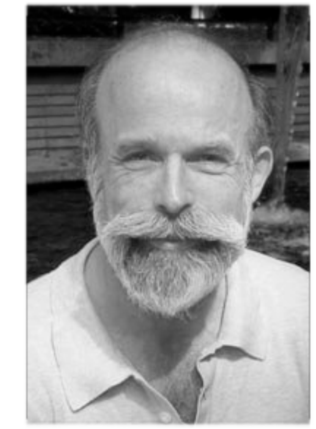
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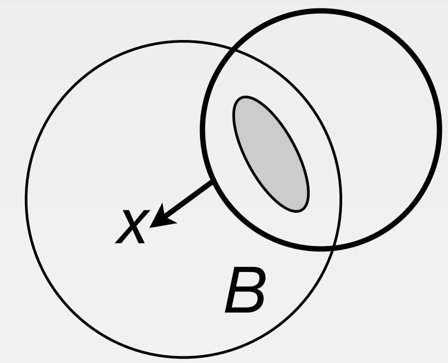

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  - Conjecture (Schwartz, 1990): Every tournament contains a unique inclusion-minimal TEQ-retentive set.

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All or nothing:
Either TEQ is a most appealing SCF or it is severely flawed.

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  - Checking significantly larger tournaments is computationally intractable.
- Over the years, various incorrect proof attempts of Schwartz's conjecture by ourselves and other researchers were discarded.

Weaker Statements

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$TRIV^{(k)}$ Conjecture
Brandt et al. (2010)

ME Conjecture
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MC Conjecture
Dutta (1988)

$\widehat{S}_{\mathcal{M}_k^*}$ Conjecture
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CF Conjecture
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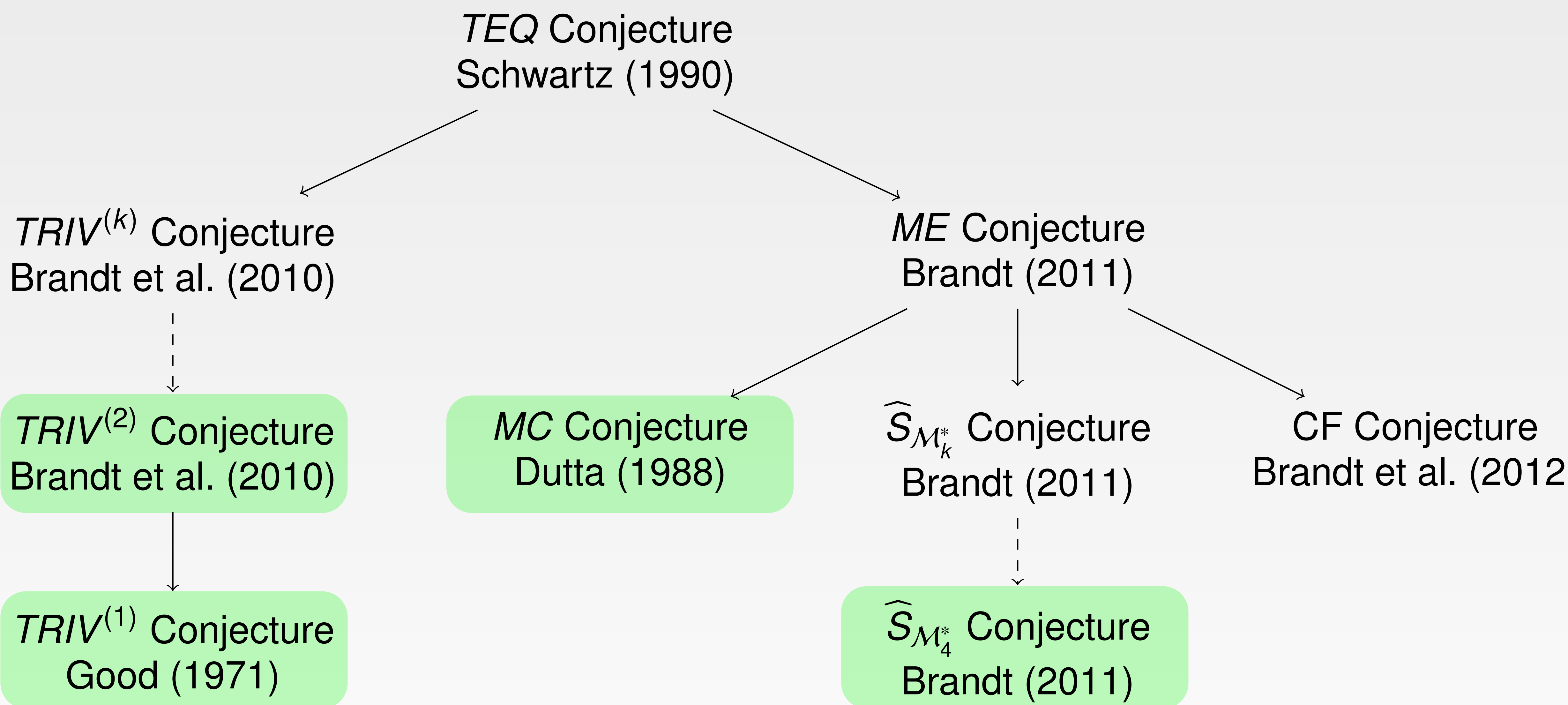

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CF Conjecture: Let (*A*,*B*) be a partition of the vertex set of a tournament *T*. Then *A* or *B* contains a transitive subtournament that is undominated in *T*.

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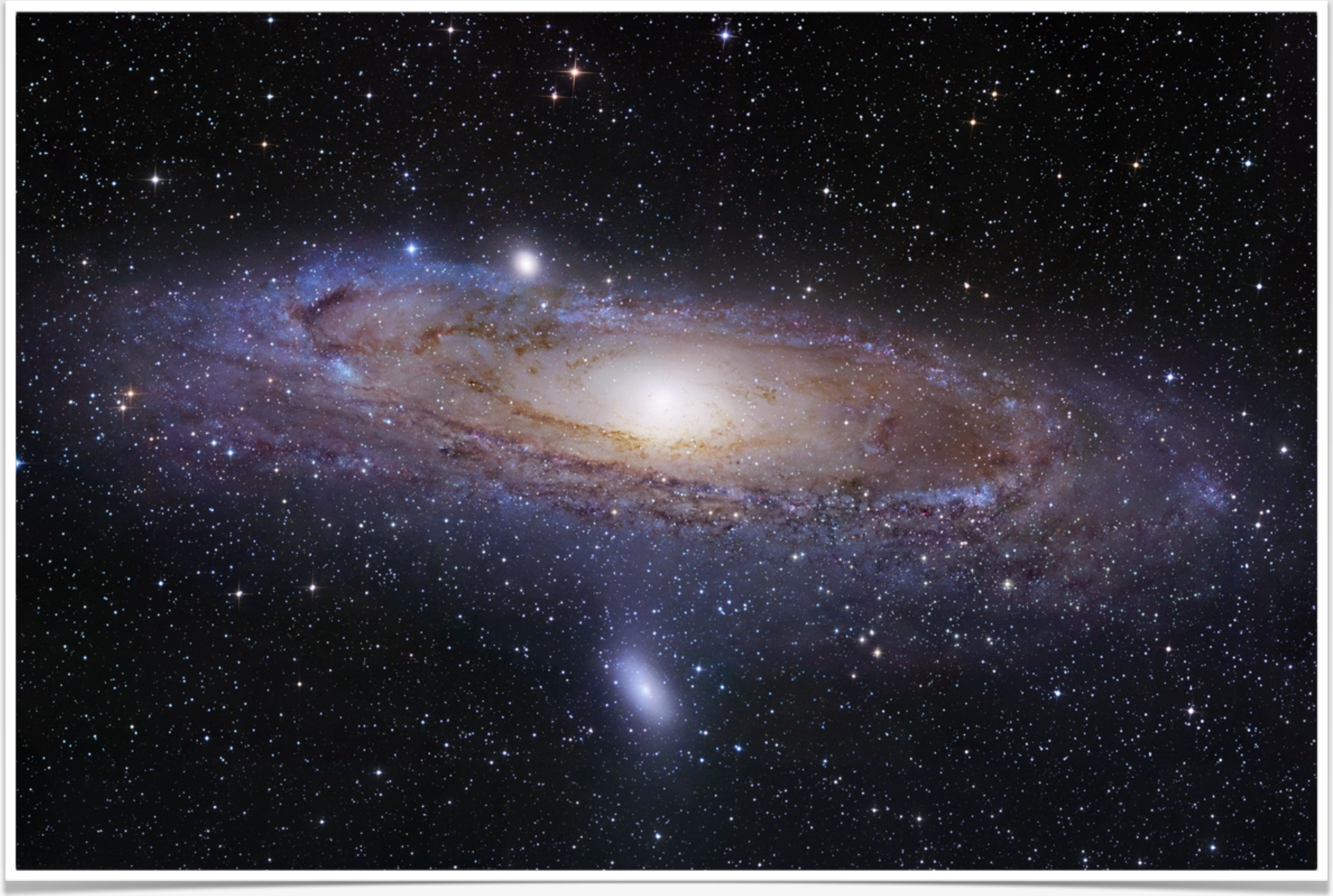

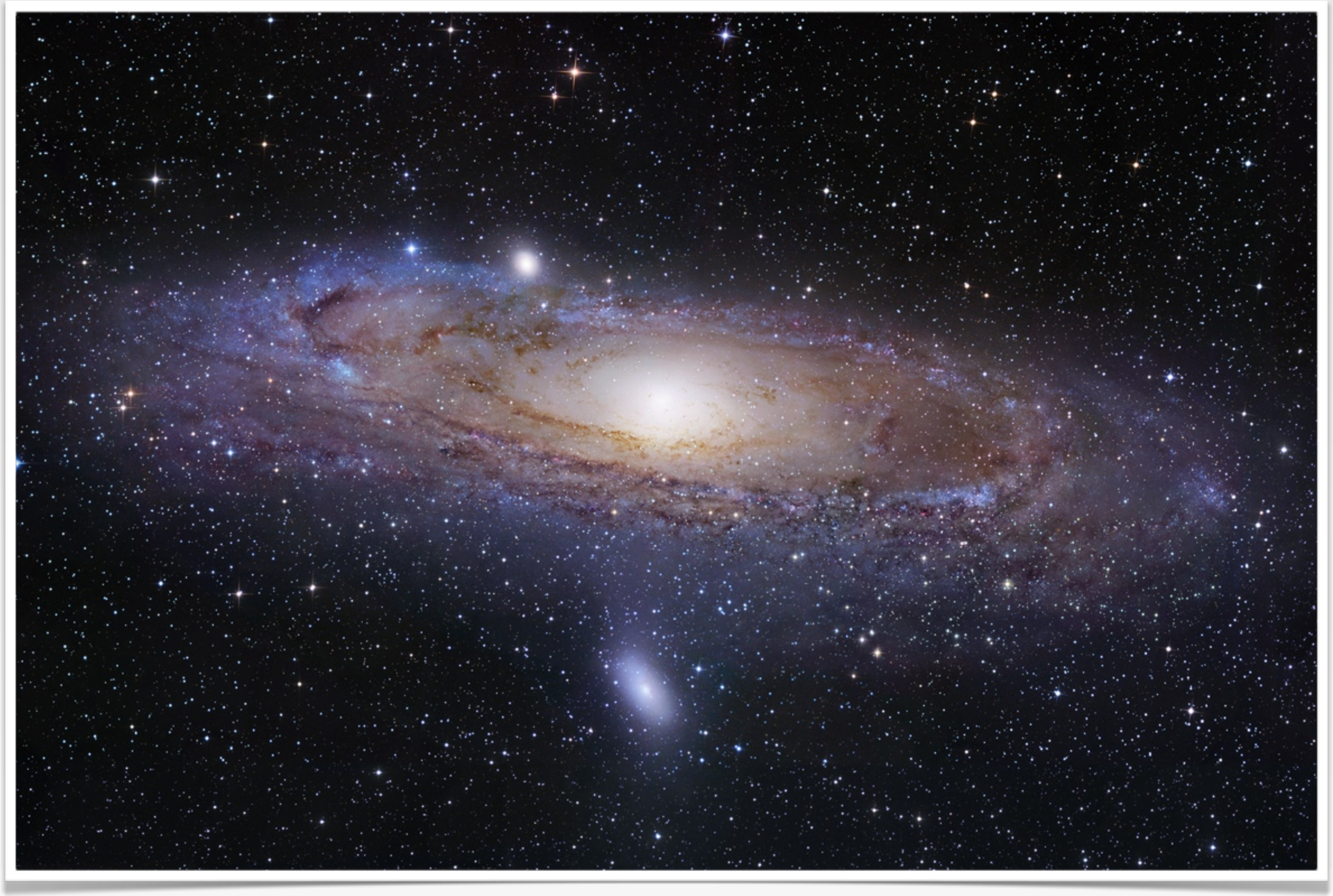

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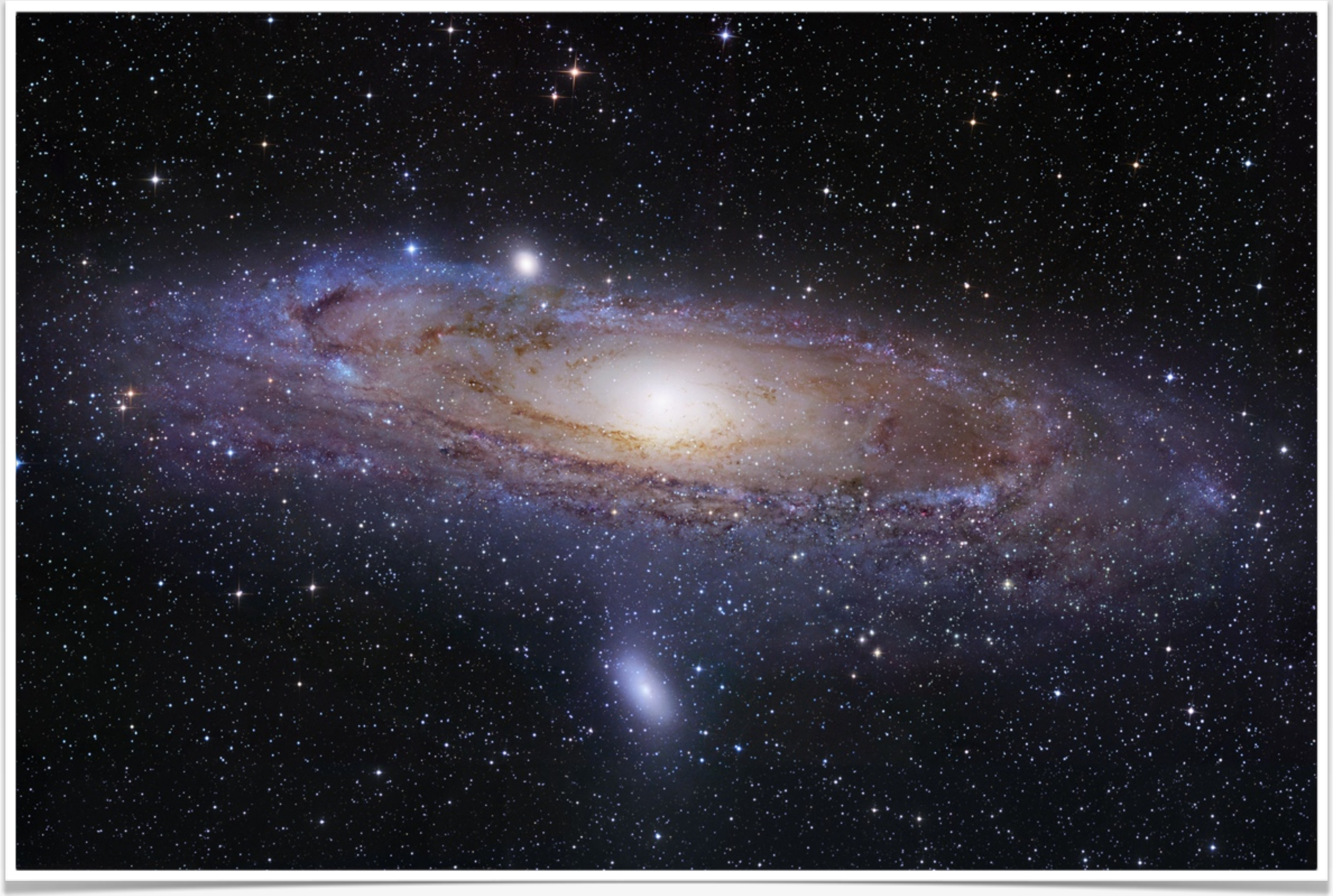

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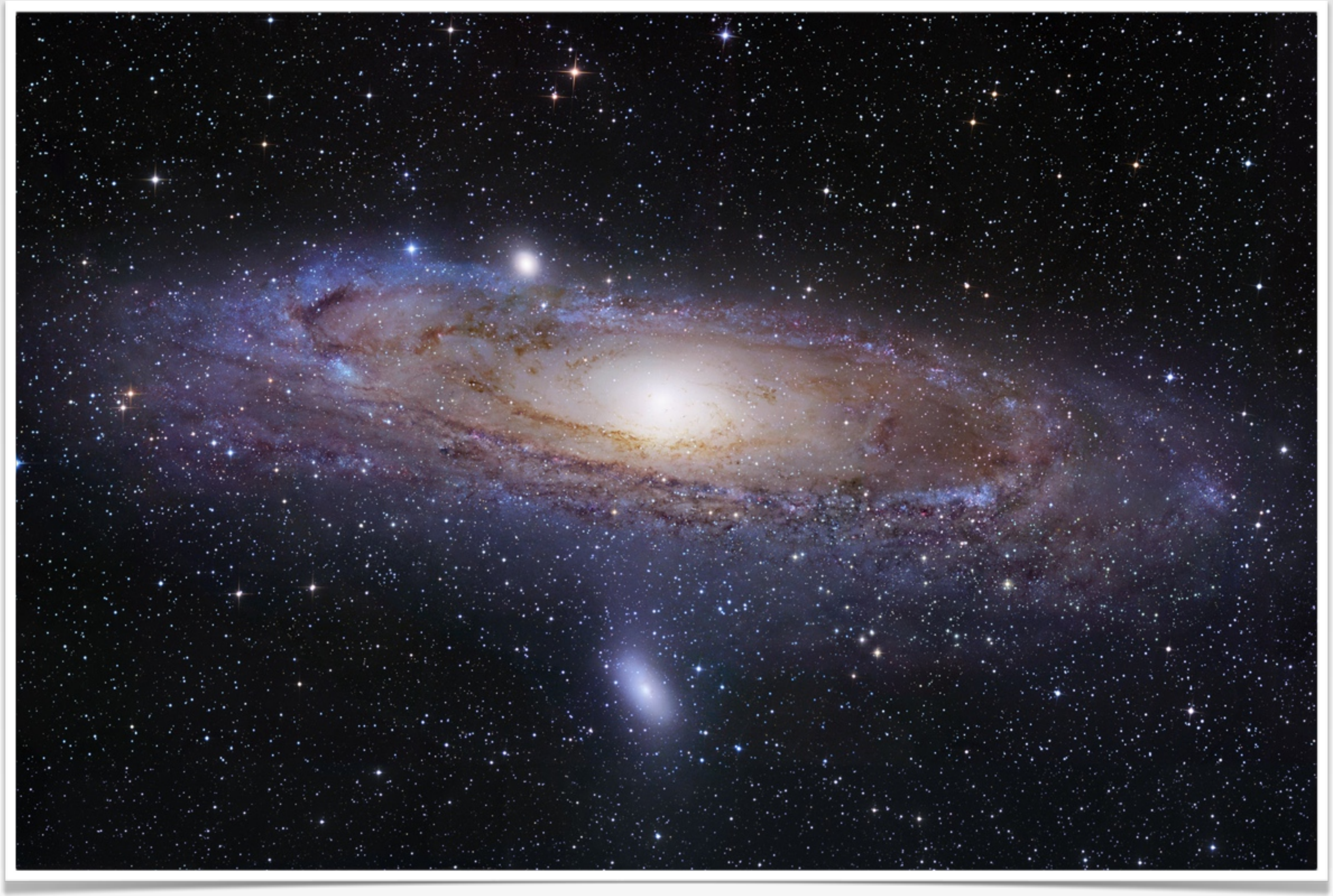

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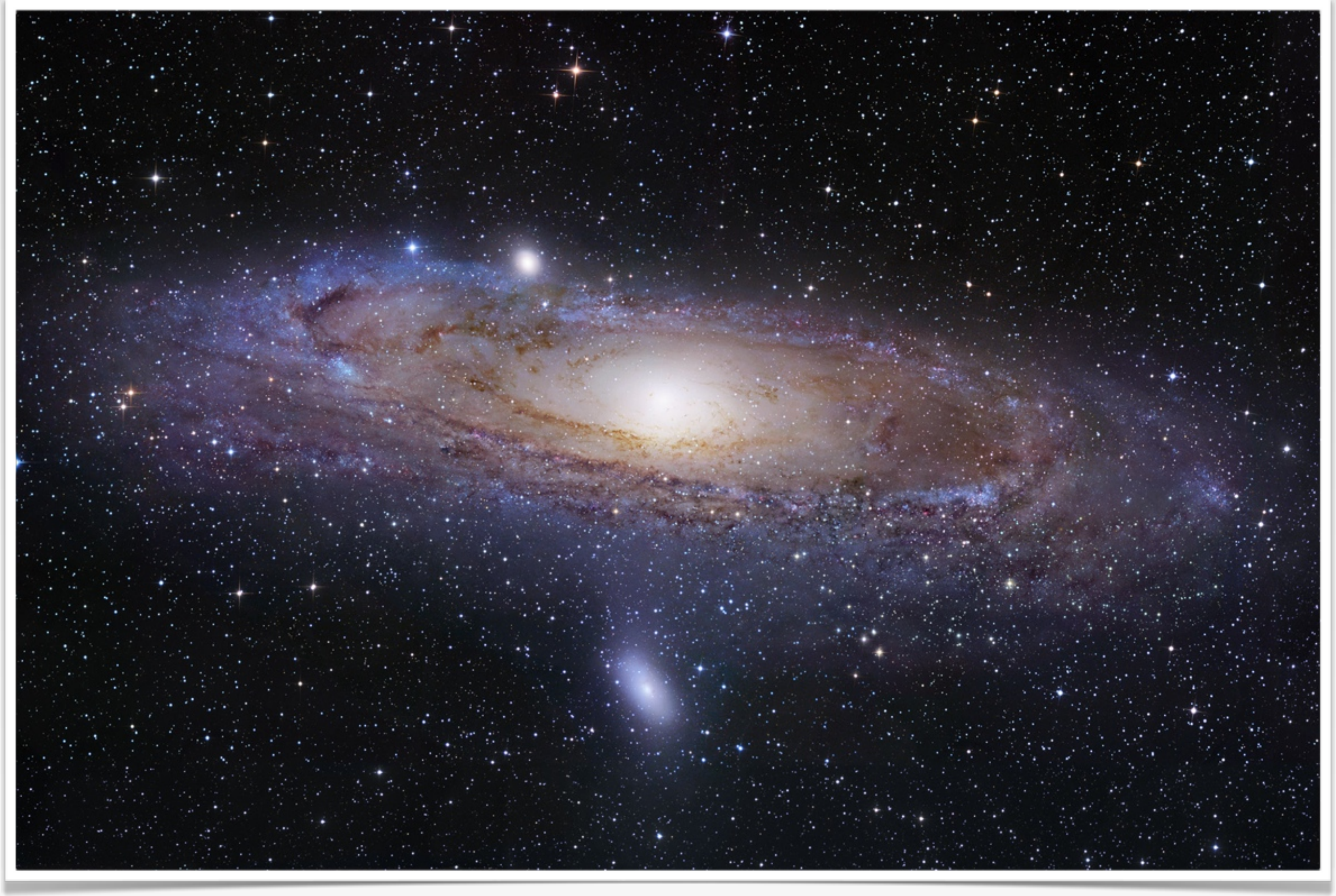

- The proof is non-constructive and relies on a probabilistic argument by Erdös and Moser (1964).
  - Neither the counter-example nor its size can be deduced from proof.
  - Smallest counter-example of this type requires about 10^{136} vertices.
  - The estimated number of atoms in the universe is approx. 10^{80}.

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- In principle, *TEQ* is severely flawed.
- If there does not exist a substantially smaller counter-example, this has no practical consequences.
- The 22-year-old conjecture of a political scientist has been refuted using extremal graph theory.
- "Politics shouldn't be some mind-bending exercise.
  It's about what you feel in your gut"
  (British PM David Cameron, April 2011)