Distribution Rules Under Dichotomous Preferences

Felix Brandt
(with F. Brandl, D. Peters, and C. Stricker)

22nd ACM Conference on Economics and Computation, July 2021

DSS
Decision Sciences & Systems

Donor Coordination

- Customers can select **one** of over 1 million charitable organizations.

amazonsmile
You Shop. **Amazon Gives.**

- Amazon donates 0.5% of the price of a customer's purchase to his selected charity.
- Imagine customers could approve **more than one** charity.
- Amazon could cleverly distribute the contribution of each customer among his approved charities.
  - Customer 1 approves a and b. Customer 2 approves b and c. An efficient distribution rule would donate both contributions to b.
  - Both customers are happier than without coordination because their approved charities receive more money.

Donor Coordination

amazonsmile You Shop. **Amazon Gives.**

- Which distribution rule should be used?
- Minimal requirement to incentivize customers to participate: Amazon donates money to at least one approved charity of each customer.
- **Main result**: No such rule can simultaneously satisfy efficiency and strategyproofness!
  - Confirms a conjecture by Bogomolnaia, Moulin, and Stong (BMS 2005)

The Model

- Allocate a divisible and homogeneous resource among a set of public projects A.
  - The resource could for example be money, time, or probability.
- Each agent $i \in N$ contributes amount C_i to a common pool.
  - $C = \sum_{i \in N} C_i$ is called the endowment.
- Each agent i approves a non-empty set of projects $A_i \subseteq A$.
- Distribution rule f returns $\delta \in [0,C]^A$ with $\sum_{x \in A} \delta(x) = C$.
- Each agent i receives utility $u_i(\delta) = \sum_{x \in A_i} \delta(x)$.

Three Axioms

- Efficiency
  - There is no $\delta' \in \Delta(C)$ with $u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and
    $u_i(\delta') > u_i(\delta)$ for some $i \in N$.
- Strategyproofness
  - $u_i(f(A_1, \ldots, A_n)) \geq u_i(f(A_1, \ldots, A_i', \ldots, A_n))$ for all $i \in N$ and A_i'.
- Positive Share (BMS 2005)
  - $u_i(\delta) > 0$ for all $i \in N$.

Three Rules

UTIL

	a	b	c	d	u_i
1	1				5
2	1				5
3	1				0
4	1				0
5	1				5
δ	5				

CUT

	a	b	c	d	u_i
1	1				3.5
2	1				3.5
3		.5	.5		1.5
4		.5		.5	1.5
5	1				3
δ	3	1	.5	.5	

NASH

	a	b	c	d	u_i
1	1				3
2	1				3
3		1			2
4		1			2
5	1				3
δ	3	2			

- Utilitarian rule (*UTIL*) **violates positive share!**
  - distributes endowment uniformly over most approved projects
- Conditional utilitarian rule (*CUT*) **violates efficiency!**
  - distributes each C_i uniformly over most approved projects in A_i
- Nash product rule (*NASH*) **violates strategyproofness!**
  - returns distribution that maximizes the product of agents' utilities

Two Out of Three Ain't Bad

	UTIL	*CUT*	*NASH*
Efficiency	✓	–	✓
Strategyproofness	✓	✓	–
Positive share	–	✓	✓

- **Theorem**: No distribution rule satisfies efficiency, strategyproofness, and positive share when $|A| \geq 4$ and $|N| \geq 6$.
- Strengthens three existing theorems by BMS 2005 and Duddy 2015.
- Significantly weaker notion of strategyproofness suffices.
  - Manipulator has to enforce that the *entire endowment* is distributed on his approved projects.

A FULL PROOF OF THEOREM

A.1 Assuming $f(\mathcal{A}_1)$ has support *bc* or *abc* leads to contradiction.

	A_1	A_2	A_3	A_4	A_5	A_6	possible supports	dominated supports
Profile 1	*b*	*c*	*ab*	*ac*	*bd*	*cd*	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrowtail bc$
Profile 2	*b*	*c*	*abc*	*ac*	*bd*	*cd*	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 3	*b*	*c*	*bc*	*ac*	*bd*	*cd*	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 4	*bc*	*c*	*bc*	*ac*	*bd*	*cd*	$cd, \underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 5	*bc*	*c*	*bc*	*ac*	*bd*	*acd*	$cd, \underline{bc}, \underline{bcd}$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail cd$
Profile 6	*bc*	*c*	*bc*	*ac*	*bd*	*ad*	$cd, acd, \underline{bcd}$	$ab \leftarrowtail cd$
Profile 7	*bc*	*c*	*bc*	*ac*	*bcd*	*ad*	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail cd$
Profile 8	*bc*	*c*	*bc*	*ac*	*cd*	*ad*	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$
Profile 9	*bc*	*c*	*cd*	*ac*	*cd*	*ad*	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$
Profile 10	*bc*	*c*	*cd*	*abc*	*cd*	*ad*	$ac, \underline{cd}, \underline{acd}$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$

Profile 369	b	abd	bd	bd	bc	ac	$\underline{ab}, bc, abc$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 370	b	ab	bd	bd	bc	ac	$\underline{ab}, bc, abc$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 371	b	ab	bd	bd	bcd	ac	$\underline{ab}, bc, \underline{abc}$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail bc$
Profile 372	b	ab	bd	bd	cd	ac	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrowtail bc$
Profile 373	b	abc	bd	bd	cd	ac	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 374	b	bc	bd	bd	cd	ac	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 375	b	bc	bd	bd	cd	bc	$\underline{bc}, bd, bcd$	$a \leftarrowtail b,\ ab \leftarrowtail bc,\ ac \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 376	b	bc	ad	bd	cd	bc	$bd, \underline{bcd}$	$a \leftarrowtail d,\ ab \leftarrowtail bd,\ ac \leftarrowtail bd$
Profile 377	b	bc	ad	abd	cd	bc	$bd, \underline{bcd}$	$a \leftarrowtail d,\ ab \leftarrowtail bd,\ ac \leftarrowtail bd$
Profile 378	b	bc	ad	ab	cd	bc	$bd, abd, \underline{bcd}$	$ac \leftarrowtail bd$
Profile 379	b	bc	ad	ab	bcd	bc	$ab, \underline{bd}, abd$	$c \leftarrowtail b,\ ac \leftarrowtail ab,\ cd \leftarrowtail bd$
Profile 380	b	bc	ad	ab	bd	bc	$ab, \underline{bd}, abd$	$c \leftarrowtail b,\ ac \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 381	b	bc	ad	abc	bd	bc	$ab, \underline{bd}, \underline{abd}$	$c \leftarrowtail b,\ ac \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 382	b	bc	ad	ac	bd	bc	$ab, \underline{abd}, abc$	$cd \leftarrowtail ab$
Profile 383	b	bc	abd	ac	bd	bc	$\underline{ab}, bc, abc$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 384	b	bc	ab	ac	bd	bc	$\underline{ab}, bc, abc$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail ab$
Profile 385	b	bc	ab	ac	bd	bcd	$\underline{ab}, bc, \underline{abc}$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail bc$
Profile 386	b	c	ab	ac	bd	bcd	$bc, \underline{abc}$	$d \leftarrowtail b,\ ad \leftarrowtail ab,\ cd \leftarrowtail bc$
Profile 1	b	c	ab	ac	bd	cd	$bc, \underline{abc}, bcd$	$ad \leftarrowtail bc$

Decomposability & Fairness

- Decomposability (Brandl et al. 2021)
  - The distribution can be decomposed into individual distributions of each agent's contribution on his approved projects.
  - $\delta = \sum_{i\in N} \delta_i$ with $\sum_{x\in A} \delta_i(x) = C_i$ and $u_i(\delta_i) = C_i$
- Group Fair Share (BMS 2005)
  - The cumulative contribution of each coalition is distributed on projects approved by at least one member of the coalition.
  - $\sum_{x\in\bigcup_{i\in S} A_i} \delta(x) \geq \sum_{i\in S} C_i$ for all $S \subseteq N$
- Both axioms obviously imply positive share.
- **Theorem**:
  A distribution is decomposable iff it satisfies group fair share.

Participation Incentives

- Participation
  - Agents prefer participating to abstaining.
  - $u_i(f((A_j)_{j \in N})) > u_i(f((A_j)_{j \in N \setminus \{i\}}))$
- Contribution Incentive-Compatibility
  - Agents weakly prefer participating to abstaining *and* spending their contribution on approved projects themselves.
  - $u_i(f((A_j)_{j \in N})) \geq u_i(f((A_j)_{j \in N \setminus \{i\}})) + C_i$
- Both axioms obviously imply positive share.

Summary

	UTIL	*CUT*	*NASH*	*SUT*	No Rule!
Efficiency	✓	–	✓	✓	↯
↳ Decomposable Efficiency	✓	✓	✓	✓	
Decomposability (Group Fair Share)	–	✓	✓	✓	
↳ Positive Share	–	✓	✓	✓	↯
Strategyproofness	✓	✓	–	–	↯
↳ Monotonicity	✓	✓	–	✓	
Contribution Incentive-Compatibility	–	✓	✓	–	
↳ Participation	✓	✓	✓	–	

- Unknown whether there is a rule that satisfies efficiency, monotonicity, and contribution incentive-compatibility.