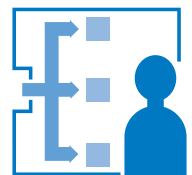


Distribution Rules Under Dichotomous Preferences

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DSS
Decision Sciences & Systems



Donor Coordination

- ▶ Customers can select **one** of over 1 million charitable organizations.
- ▶ Amazon donates 0.5% of the price of a customer's purchase to his selected charity.
- ▶ Imagine customers could approve **more than one** charity.
- ▶ Amazon could cleverly distribute the contribution of each customer among his approved charities.
 - ▶ Customer 1 approves *a and b*. Customer 2 approves *b and c*. An efficient distribution rule would donate both contributions to *b*.
 - ▶ Both customers are happier than without coordination because their approved charities receive more money.



Donor Coordination



- Which distribution rule should be used?
- Minimal requirement to incentivize customers to participate: Amazon donates money to at least one approved charity of each customer.
- **Main result:** No such rule can simultaneously satisfy *efficiency* and *strategyproofness*!
 - Confirms a conjecture by Bogomolnaia, Moulin, and Stong (BMS 2005)



The Model

- Allocate a **divisible** and **homogeneous** resource among a set of **public** projects A .
 - The resource could for example be money, time, or probability.
- Each agent $i \in N$ **contributes** amount C_i to a common pool.
 - $C = \sum_{i \in N} C_i$ is called the **endowment**.
- Each agent i **approves** a non-empty set of projects $A_i \subseteq A$.
- **Distribution rule** f returns $\delta \in [0, C]^A$ with $\sum_{x \in A} \delta(x) = C$.
- Each agent i receives **utility** $u_i(\delta) = \sum_{x \in A_i} \delta(x)$.



Three Axioms

- ▶ **Efficiency**

- ▶ There is no $\delta' \in \Delta(C)$ with $u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and $u_i(\delta') > u_i(\delta)$ for some $i \in N$.

- ▶ **Strategyproofness**

- ▶ $u_i(f(A_1, \dots, A_n)) \geq u_i(f(A_1, \dots, A'_i, \dots, A_n))$ for all $i \in N$ and A'_i .

- ▶ **Positive Share** (BMS 2005)

- ▶ $u_i(\delta) > 0$ for all $i \in N$.



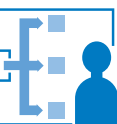
Three Rules

<i>UTIL</i>					
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	u_i
1	1				5
2	1				5
3	1				0
4	1				0
5	1				5
δ	5				

<i>CUT</i>					
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	u_i
1	1				3.5
2	1				3.5
3		.5	.5		1.5
4		.5		.5	1.5
5	1				3
δ	3	1	.5	.5	

<i>NASH</i>					
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	u_i
1	1				3
2	1				3
3		1			2
4		1			2
5	1				3
δ	3	2			

- ▶ Utilitarian rule (*UTIL*) **violates positive share!**
 - distributes endowment uniformly over most approved projects
- ▶ Conditional utilitarian rule (*CUT*) **violates efficiency!**
 - distributes each C_i uniformly over most approved projects in A_i
- ▶ Nash product rule (*NASH*) **violates strategyproofness!**
 - returns distribution that maximizes the product of agents' utilities



Two Out of Three Ain't Bad

	<i>UTIL</i>	<i>CUT</i>	<i>NASH</i>
Efficiency	✓	–	✓
Strategyproofness	✓	✓	–
Positive share	–	✓	✓

- ▶ **Theorem:** No distribution rule satisfies efficiency, strategyproofness, and positive share when $|A| \geq 4$ and $|N| \geq 6$.
- ▶ Strengthens three existing theorems by BMS 2005 and Duddy 2015.
- ▶ Significantly weaker notion of strategyproofness suffices.
 - ▶ Manipulator has to enforce that the *entire endowment* is distributed on his approved projects.



A FULL PROOF OF THEOREM

A.1 Assuming $f(\mathcal{A}_1)$ has support bc or abc leads to contradiction.

	A_1	A_2	A_3	A_4	A_5	A_6	possible supports	dominated supports
Profile 1	b	c	ab	ac	bd	cd	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrow bc$
Profile 2	b	c	abc	ac	bd	cd	\underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$
Profile 3	b	c	bc	ac	bd	cd	\underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$
Profile 4	bc	c	bc	ac	bd	cd	cd, \underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$
Profile 5	bc	c	bc	ac	bd	acd	$cd, \underline{bc}, \underline{bcd}$	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow cd$
Profile 6	bc	c	bc	ac	bd	ad	cd, acd, \underline{bcd}	$ab \leftarrow cd$
Profile 7	bc	c	bc	ac	bcd	ad	ac, \underline{cd}, acd	$b \leftarrow c, ab \leftarrow ac, bd \leftarrow cd$
Profile 8	bc	c	bc	ac	cd	ad	ac, \underline{cd}, acd	$b \leftarrow c, ab \leftarrow ac, bd \leftarrow ac$
Profile 9	bc	c	cd	ac	cd	ad	ac, \underline{cd}, acd	$b \leftarrow c, ab \leftarrow ac, bd \leftarrow ac$
Profile 10	bc	c	cd	abc	cd	ad	$ac, \underline{cd}, \underline{acd}$	$b \leftarrow c, ab \leftarrow ac, bd \leftarrow ac$

Profile 369	b	aba	ba	ba	bc	ac	\underline{ab}, bc, abc	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow ab$
Profile 370	b	ab	bd	bd	bc	ac	\underline{ab}, bc, abc	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow ab$
Profile 371	b	ab	bd	bd	bcd	ac	$\underline{ab}, bc, \underline{abc}$	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow bc$
Profile 372	b	ab	bd	bd	cd	ac	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrow bc$
Profile 373	b	abc	bd	bd	cd	ac	\underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$
Profile 374	b	bc	bd	bd	cd	ac	\underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$
Profile 375	b	bc	bd	bd	cd	bc	\underline{bc}, bd, bcd	$a \leftarrow b, ab \leftarrow bc, ac \leftarrow bc, ad \leftarrow bc$
Profile 376	b	bc	ad	bd	cd	bc	bd, \underline{bcd}	$a \leftarrow d, ab \leftarrow bd, ac \leftarrow bd$
Profile 377	b	bc	ad	abd	cd	bc	bd, \underline{bcd}	$a \leftarrow d, ab \leftarrow bd, ac \leftarrow bd$
Profile 378	b	bc	ad	ab	cd	bc	bd, abd, \underline{bcd}	$ac \leftarrow bd$
Profile 379	b	bc	ad	ab	bcd	bc	ab, \underline{bd}, abd	$c \leftarrow b, ac \leftarrow ab, cd \leftarrow bd$
Profile 380	b	bc	ad	ab	bd	bc	ab, \underline{bd}, abd	$c \leftarrow b, ac \leftarrow ab, cd \leftarrow ab$
Profile 381	b	bc	ad	abc	bd	bc	$ab, \underline{bd}, \underline{abd}$	$c \leftarrow b, ac \leftarrow ab, cd \leftarrow ab$
Profile 382	b	bc	ad	ac	bd	bc	ab, \underline{abd}, abc	$cd \leftarrow ab$
Profile 383	b	bc	abd	ac	bd	bc	\underline{ab}, bc, abc	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow ab$
Profile 384	b	bc	ab	ac	bd	bc	\underline{ab}, bc, abc	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow ab$
Profile 385	b	bc	ab	ac	bd	bcd	$\underline{ab}, bc, \underline{abc}$	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow bc$
Profile 386	b	c	ab	ac	bd	bcd	bc, \underline{abc}	$d \leftarrow b, ad \leftarrow ab, cd \leftarrow bc$
Profile 1	b	c	ab	ac	bd	cd	bc, \underline{abc}, bcd	$ad \leftarrow bc$

Decomposability & Fairness

- **Decomposability** (Brandl et al. 2021)
 - The distribution can be decomposed into individual distributions of each agent's contribution on his approved projects.
 - $\delta = \sum_{i \in N} \delta_i$ with $\sum_{x \in A} \delta_i(x) = C_i$ and $u_i(\delta_i) = C_i$
- **Group Fair Share** (BMS 2005)
 - The cumulative contribution of each coalition is distributed on projects approved by at least one member of the coalition.
 - $\sum_{x \in \bigcup_{i \in S} A_i} \delta(x) \geq \sum_{i \in S} C_i$ for all $S \subseteq N$
- Both axioms obviously imply positive share.
- **Theorem:**
A distribution is decomposable iff it satisfies group fair share.



Participation Incentives

- **Participation**
 - Agents prefer participating to abstaining.
 - $u_i(f((A_j)_{j \in N})) > u_i(f((A_j)_{j \in N \setminus \{i\}}))$
- **Contribution Incentive-Compatibility**
 - Agents weakly prefer participating to abstaining *and* spending their contribution on approved projects themselves.
 - $u_i(f((A_j)_{j \in N})) \geq u_i(f((A_j)_{j \in N \setminus \{i\}})) + C_i$
- Both axioms obviously imply positive share.



Summary

	<i>UTIL</i>	<i>CUT</i>	<i>NASH</i>	<i>^{NEW}SUT</i>	No Rule!
Efficiency	✓	–	✓	✓	⚡
↳ Decomposable Efficiency	✓	✓	✓	✓	
Decomposability (Group Fair Share)	–	✓	✓	✓	
↳ Positive Share	–	✓	✓	✓	⚡
Strategyproofness	✓	✓	–	–	⚡
↳ Monotonicity	✓	✓	–	✓	
Contribution Incentive-Compatibility	–	✓	✓	–	
↳ Participation	✓	✓	✓	–	

- Unknown whether there is a rule that satisfies efficiency, monotonicity, and contribution incentive-compatibility.

