Finding and Recognizing Popular Coalition Structures

Joint work with Felix Brandt

Martin Bullinger

Technical University of Munich

Coalition formation games

Alex: Bea $\succ$ Carl $\succ$ Don

Bea: Carl $\succ$ Don $\succ$ Alex

Carl: Alex $\succ$ Bea $\succ$ Don

Don: Alex $\succ$ Bea $\succ$ Carl

- Set of agents
- Preferences over coalitions
- Output: partition of agents in coalitions

Popular Partitions

Alex: **Bea** $\succ$ Carl $\succ$ Don

Bea: Carl $\succ$ Don $\succ$ **Alex**

Carl: Alex $\succ$ Bea $\succ$ **Don**

Don: Alex $\succ$ Bea $\succ$ **Carl**

- Pareto-optimal partition
- Overthrow decision by proposing better outcome?

Popular Partitions

Alex: Bea $\succ$ Carl $\succ$ Don

Bea: Carl $\succ$ Don $\succ$ Alex

Carl: Alex $\succ$ Bea $\succ$ Don

Don: Alex $\succ$ Bea $\succ$ Carl

- Pareto-optimal partition
- Overthrow decision by proposing better outcome?
- More popular partition exists

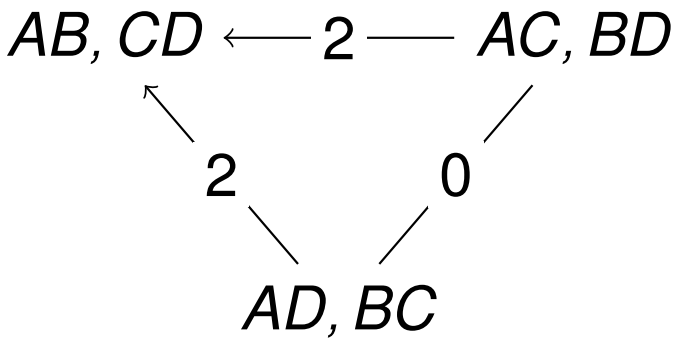
Popular Partitions

Alex: Bea $\succ$ Carl $\succ$ Don

Bea: Carl $\succ$ Don $\succ$ Alex

Carl: Alex $\succ$ Bea $\succ$ Don

Don: Alex $\succ$ Bea $\succ$ Carl



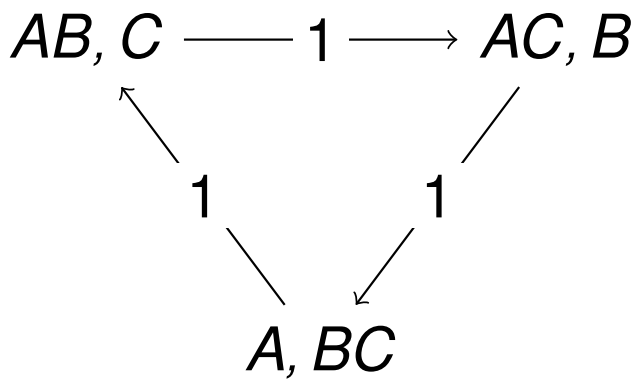
Popular partitions: weak Condorcet winners

Existence of popular partitions

Alex: Bea $\succ$ Carl

Bea: Carl $\succ$ Alex

Carl: Alex $\succ$ Bea



Popular partitions need not exist.

Classes of games

Flatmate games

$\cup|$

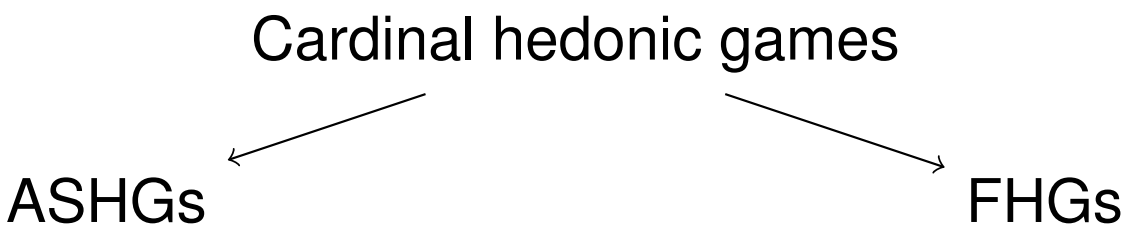
Roommate games

$\cup|$

Marriage games

$\cup|$

Housing games



Mixed popularity

- Allow for randomization
- Concept introduced for matchings by Kavitha, Mestre, and Nasre (2011)
- Existence of mixed popular partitions
- Maximin solution to zero-sum game

Alex: Bea $\succ$ Carl

Bea: Carl $\succ$ Alex

Carl: Alex $\succ$ Bea

		1/3	1/3	1/3	0
		AB, C	AC, B	BC, A	A, B, C
AB, C		0	1	-1	2
AC, B		-1	0	1	2
BC, A		1	-1	0	2
A, B, C		-2	-2	-2	0

Mixed popularity in roommate games

- Linear feasibility problem in matching polytope
- Computability of mixed popular partitions in roommate games
- Tractability of strongly popular partitions under weak preferences

strong popularity $\Longrightarrow$ popularity $\Longrightarrow$ mixed popularity

Popularity under strict preferences

- Popularity is intractable in roommate games (Faenza et al. (2019), Gupta et al. (2019))
- Globally ranked preferences yield existence in roommate games

$$AE \succ BE \succ CE \succ AF \succ BF \succ CF \succ \dots$$

Alex: Eve $\succ$ Fred $\succ \dots$

Bea: Eve $\succ$ Fred $\succ \dots$

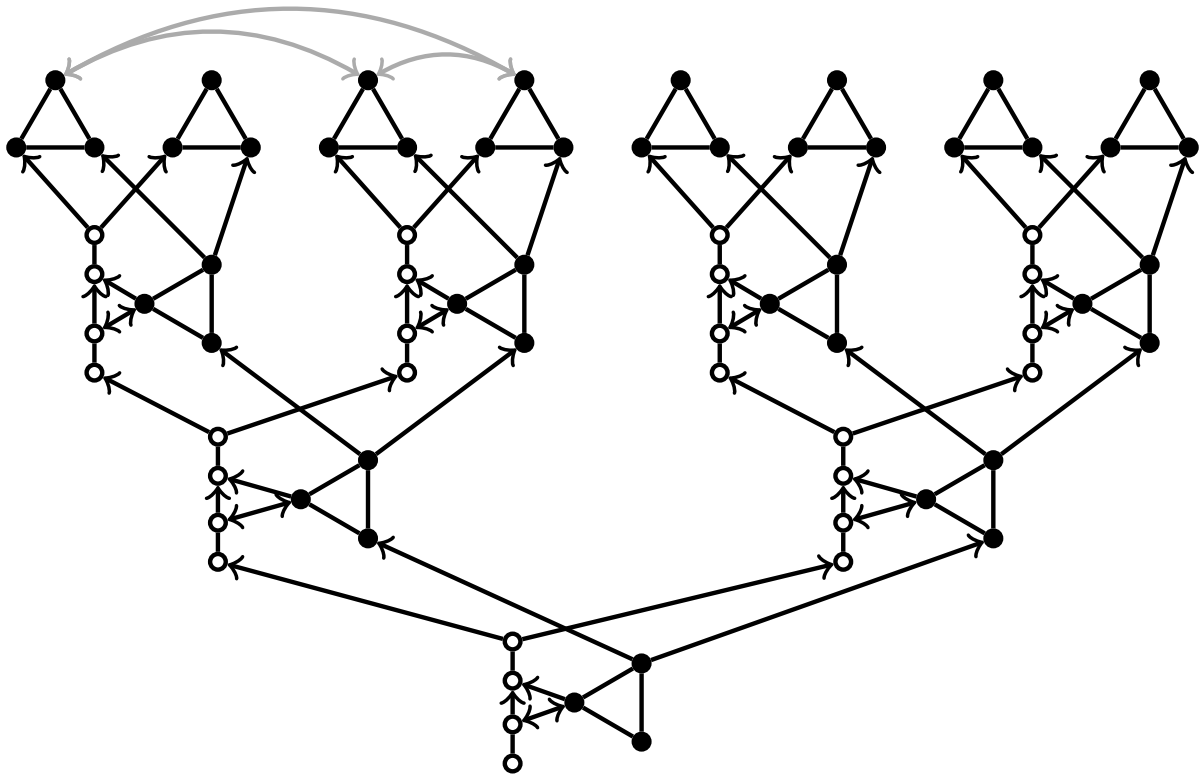
Carl: Eve $\succ$ Fred $\succ \dots$

Eve: Alex $\succ$ Bea $\succ$ Carl $\succ \dots$

Fred: Alex $\succ$ Bea $\succ$ Carl $\succ \dots$

Popularity under strict preferences

- Popularity is intractable in roommate games (Faenza et al. (2019), Gupta et al. (2019))
- Globally ranked preferences yield existence in roommate games
- Allowing larger coalitions of size 3 causes intractability



Overview of results

	weak preferences			*strict preferences*		
	mPop	sPop	Pop	mPop	sPop	Pop
Flatmates	NP-h. in P	NP-h. in P	NP-h.[b] in P[a]	NP-h. in P	NP-h. in P[b]	NP-h.[d] in P[c]
Roommates						
Marriage				in P[e]	in P	in P[a]
Housing						

[a]: Abraham et al. (2007, Th. 3.9)
[b]: Biró, Irving, Manlove (2010, Th. 6)
[c]: Gärdernfors (1975, Th. 3)
[d]: Gupta et al. (2019, Th. 1.1), Faenza et al. (2019, Th. 4.6)
[e]: Kavitha, Mestre, Nasre (2011, Th. 2)

Various hardness results for ASHGs and FHGs

References


D. K. Abraham, R. W. Irving, T. Kavitha, and K. Mehlhorn.
Popular matchings.
SIAM Journal on Computing, 37(4):1030–1034, 2007.


F. Brandt and M. Bullinger.
Finding and recognizing popular coalition structures.
In *Proceedings of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 195–203, 2020.


Y. Faenza, T. Kavitha, V. Power, and X. Zhang.
Popular matchings and limits to tractability.
In *Proceedings of the 30th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pages 2790–2809, 2019.


S. Gupta, P. Misra, S. Saurabh, and M. Zehavi.
Popular matching in roommates setting is np-hard.
In *Proceedings of the 30th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pages 2810–2822, 2019.


T. Kavitha, J. Mestre, and M. Nasre.
Popular mixed matchings.
Theoretical Computer Science, 412(24):2679–2690, 2011.