Finding and Recognizing Popular Coalition Structures
Joint work with Felix Brandt

Martin Bullinger
Coalition formation games

- Set of agents
- Preferences over coalitions
- Output: partition of agents in coalitions

Alex: Bea ≻ Carl ≻ Don

Bea: Carl ≻ Don ≻ Alex

Carl: Alex ≻ Bea ≻ Don

Don: Alex ≻ Bea ≻ Carl
Popular Partitions

Alex: \[ \text{Bea} \succ \text{Carl} \succ \text{Don} \]

Bea: \[ \text{Carl} \succ \text{Don} \succ \text{Alex} \]

Carl: \[ \text{Alex} \succ \text{Bea} \succ \text{Don} \]

Don: \[ \text{Alex} \succ \text{Bea} \succ \text{Carl} \]

- Pareto-optimal partition
- Overthrow decision by proposing better outcome?
Popular Partitions

- Alex: Bea $\succ$ Carl $\succ$ Don
- Bea: Carl $\succ$ Don $\succ$ Alex
- Carl: Alex $\succ$ Bea $\succ$ Don
- Don: Alex $\succ$ Bea $\succ$ Carl

- Pareto-optimal partition
- Overthrow decision by proposing better outcome?
- More popular partition exists
Popular Partitions

Alex: [Bea] > [Carl] > Don
Bea: [Carl] > [Don] > [Alex]
Carl: [Alex] > [Bea] > [Don]
Don: [Alex] > [Bea] > [Carl]

Popular partitions: weak Condorcet winners
Existence of popular partitions

Alex: Bea ≻ Carl
Bea: Carl ≻ Alex
Carl: Alex ≻ Bea

Popular partitions need not exist.
Classes of games

Flatmate games
  \sqsubseteq
  Roommate games
  \sqsubseteq
  Marriage games
  \sqsubseteq
  Housing games

Cardinal hedonic games
  \sqsubseteq
  ASHGs
  \sqsubseteq
  FHGs
Mixed popularity

- Allow for randomization
- Concept introduced for matchings by Kavitha, Mestre, and Nasre (2011)
- Existence of mixed popular partitions
- Maximin solution to zero-sum game

<table>
<thead>
<tr>
<th></th>
<th>Alex: Bea (\succ) Carl</th>
<th>Bea: Carl (\succ) Alex</th>
<th>Carl: Alex (\succ) Bea</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(AB, C)</td>
<td>(AC, B)</td>
<td>(BC, A)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>–1</td>
<td>1</td>
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<td>1/3</td>
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<tr>
<td>0</td>
<td>–2</td>
<td>–2</td>
<td>–2</td>
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\[
\begin{pmatrix}
AB, C & 0 & 1 & -1 & 2 \\
AC, B & -1 & 0 & 1 & 2 \\
BC, A & 1 & -1 & 0 & 2 \\
A, B, C & -2 & -2 & -2 & 0 \\
\end{pmatrix}
\]
Mixed popularity in roommate games

- Linear feasibility problem in matching polytope
- Computability of mixed popular partitions in roommate games
- Tractable of strongly popular partitions under weak preferences

strong popularity $\implies$ popularity $\implies$ mixed popularity
Popularity under strict preferences

- Popularity is intractable in roommate games (Faenza et al. (2019), Gupta et al. (2019))
- Globally ranked preferences yield existence in roommate games

\[ AE \succ BE \succ CE \succ AF \succ BF \succ CF \succ \ldots \]

Alex: Eve \succ Fred \succ \ldots  
Bea: Eve \succ Fred \succ \ldots  
Carl: Eve \succ Fred \succ \ldots  
Eve: Alex \succ Bea \succ Carl \succ \ldots  
Fred: Alex \succ Bea \succ Carl \succ \ldots  

Martin Bullinger  Popular Coalition Structures
Popularity under strict preferences

- Popularity is intractable in roommate games (Faenza et al. (2019), Gupta et al. (2019))
- Globally ranked preferences yield existence in roommate games
- Allowing larger coalitions of size 3 causes intractability
# Overview of results

<table>
<thead>
<tr>
<th>weak preferences</th>
<th>strict preferences</th>
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<tbody>
<tr>
<td>mPop</td>
<td>sPop</td>
</tr>
<tr>
<td>Flatmates</td>
<td>NP-h. in P</td>
</tr>
<tr>
<td>Roommates</td>
<td>NP-h. in P</td>
</tr>
<tr>
<td>Marriage</td>
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</tr>
<tr>
<td>Housing</td>
<td>NP-h. in P</td>
</tr>
</tbody>
</table>

\( a \): Abraham et al. (2007, Th. 3.9)  
\( b \): Biró, Irving, Manlove (2010, Th. 6)  
\( c \): Gärdernfors (1975, Th. 3)  
\( d \): Gupta et al. (2019, Th. 1.1), Faenza et al. (2019, Th. 4.6)  
\( e \): Kavitha, Mestre, Nasre (2011, Th. 2)

Various hardness results for ASHGs and FHGs
References

Popular matchings.

F. Brandt and M. Bullinger.
Finding and recognizing popular coalition structures.

Popular matchings and limits to tractability.

S. Gupta, P. Misra, S. Saurabh, and M. Zehavi.
Popular matching in roommates setting is np-hard.

T. Kavitha, J. Mestre, and M. Nasre.
Popular mixed matchings.

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