Computer Science*

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In some sense the disciplines of computer science and game theory can be considered siblings because they share one of their founding fathers. John von Neumann initiated the formal study of games together with Oscar Morgenstern and their joint book *Games and Economic Behavior*, which appeared in 1944 (von Neumann and Morgenstern, 1944). Together with others such as Alan Turing and Charles Babbage, von Neumann is also considered one of the pioneers of computer science. Shortly after the publication of *Games and Economic Behavior*, von Neumann laid out the details of the first electronic and binary computer (von Neumann, 1945) and foresaw the importance of polynomial-time algorithms (von Neumann, 1953). To this date, students of computer science learn about the von Neumann computer architecture.

The purpose of this book is to celebrate one hundred years of game theory. *Algorithmic game theory* has been an active subarea for at least twenty of them (Nisan et al., 2007). While it is impossible to pinpoint the exact origin of computational analyses of game-theoretic problems, there are notable examples of pioneering papers that brought techniques and insights from computer science to game theory. One of the earliest such examples is a paper by Donald Knuth, Christos Papadimitriou, and John Tsitsiklis on the complexity of iterated dominance (Knuth et al., 1988), but there have been even earlier precursors with a computational flavor such as the stable marriage algorithm, proposed by David Gale and Lloyd Shapley (Gale and Shapley, 1962). The strong connection between zero-sum games and linear programming, which is almost as old as game theory itself, constitutes another example (e.g., Vajda, 1956).

Nowadays, the computational perspective on game-theoretic problems is ubiquitous. Next to algorithmic game theory, related interdisciplinary areas such as *algorithmic mechanism design* or *computational social choice* are flourishing and there are numerous regular international conferences such as the *ACM Conference on Economics and Computation* (since 1999), the *Conference on Web and Internet Economics* (since 2005), and the *Symposium on Algorithmic Game Theory* (since 2008), as well as a dedicated journal, the *ACM Transactions on Economics and Computation* (since 2013). But algorithmic game theory is not all about algorithms and complexity. The algorithmic game

*To appear in M. Voorneveld, J. W. Weibull, T. Andersson, R. B. Myerson, J.-F. Laslier, R. Laraki, and Y. Koriyama, editors (Eds.), One Hundred Years of Game Theory: A Nobel Symposium, Econometric Society Monographs, Cambridge University Press, 2026.

theory community works to apply concepts from game theory to many areas of computer science from traffic routing through designing cryptocurrency systems and many others. The community also considers the applications of computer science notions in game theory, such as modeling users as learners, or quantifying the efficiency of game theoretic outcomes via approximation ratios. On top of that, computer scientists have embraced the axiomatic approach of economic theory. For example, none of the papers in this chapter is exclusively concerned with computational questions.

This chapter consists of seven papers that were presented in two sessions and demonstrate the breadth and maturity of research at the intersection of computer science and game theory.

The paper by **Costis Daskalakis** challenges one of the most basic assumptions of game theory, namely that each player's utility function is linear in her strategy. While there have been generalizations to concave utility functions, very little progress has been made for non-concave utility functions because concavity is essential for the existence of Nash equilibria. Motivated by applications in machine learning, Daskalakis introduces the class of differentiable games and proposes the notion of local Nash equilibria (which are guaranteed to exist in differentiable games) and discusses their computational intractability even in two-player zero-sum games.

Edith Elkind's paper is concerned with multi-winner elections, an emerging topic in the area of computational social choice. Much of the classic literature in social choice theory deals with functions that map ordinal preferences over candidates to a winning candidate or perhaps a ranking of the candidates. The goal of multi-winner elections is to select a fixed-size set of candidates: a committee. This gives rise to new rules as well as new axioms. Elkind's piece focuses on the case of approval-based preferences and axioms capturing the idea of proportional representation.

The contribution by **Felix Brandt** shows how the theory of symmetric two-player zero-sum games, which was initiated by Borel in 1921, can be used for randomly selecting an alternative based on quantified pairwise comparisons between alternatives. He points out desirable properties satisfied by the equilibrium distribution and gives examples where these distributions arise as the limit of simple dynamic processes that have been studied across various disciplines such as population biology, quantum physics, and machine learning.

Paul Milgrom describes the successful application of advances in practical truthful mechanisms design to a large-scale computationally hard problem: the FCC's 2016-17 incentive auction, which reallocated tens of billions of dollars of radio spectrum resources from use in television broadcasting to higher value uses in mobile broadband. The mechanism used was an impressive combination of advances in efficiently solving NP-hard resource allocation problems (in most cases) and in new mechanism design that is simple to implement and that adapts well to limited computation capacity. The auction resulted in repurposing 84 megahertz of spectrum, and yielded $19.8 billion in revenue.

Nash equilibria of games have been classically used as the default solution concept: the expected outcome of strategic interaction. In the last 20 years, it was realized there are no efficient algorithms that find Nash equilibria in general games, yet there was some remaining hope that repeated interaction of the players will help them eventually find an equilibrium (even if not efficiently). To date no learning dynamics have been identified that provide such guarantees. **Christos Papadimitriou** reports on impossibility results showing that there is a positive measure of games where no dynamics is guaranteed to converge to Nash equilibria or even to an approximate version of equilibria. The result is shattering a common defense of using Nash equilibria as the default solution concept for all games.

Tuomas Sandholm gives an extensive overview of techniques and algorithms for representing and solving large imperfect-information extensive-form games and reports on recent breakthroughs that have been achieved for the game of poker. These breakthroughs were made possible by advances in three key areas: *(i)* game abstraction, i.e., the systematic construction of significantly smaller extensive-form games that are strategically similar to the original game, *(ii)* equilibrium-finding algorithms, and *(iii)* solving subgames during gameplay in much finer abstractions than would be possible in advance. A new proposal put forward in Sandholm's paper is to reason about games whose rules are modeled via a programming language.

The talk by **Tim Roughgarden** on the axiomatic study of transaction fee mechanisms for cryptocurrencies is, unfortunately, not included in this book.

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