Loyalty in Cardinal Hedonic Games

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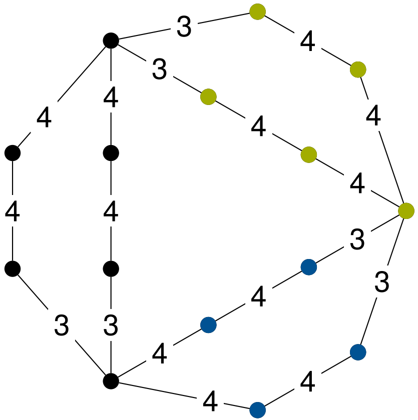
Challenging Selfishness Objectives

- Predominant paradigm: utilitarianism (Bentham, 1789)
- Many delicate situations in non-cooperative game theory (e.g., Prisoner's Dilemma)
- Empirical evidence for aiming at society's well-being (Colman et al., 2008)
- Especially in cooperative settings

Empathy in Coalition Formation

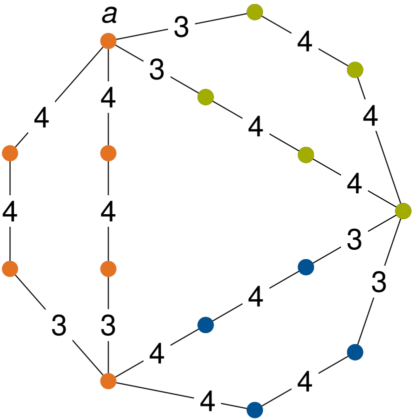
- Specific utility functions designed for empathy aspects
  - Altruistic hedonic games (Nguyen et al., 2016)
  - Aggregating friend-oriented utilities
- Utility functions based on social networks
  - Social distance games (Brânzei and Larson, 2011)
  - Hedonic games with social context (Monaco et al., 2018)
- Goal: integrating empathy into existing benchmark game
- No new game type
- Semantic meaning with respect to original game

Coalition Formation Games



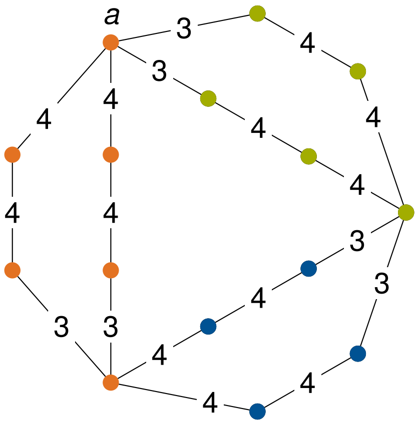
Coalition formation in the framework of hedonic games
(Dréze and Greenberg, 1980; Bogomolnaia and Jackson, 2002)

Coalition Formation Games



- Set of agents
- Individual cardinal utilities
- Aggregated coalitional utilities, e.g. $u_a(C) = 8$ ('addition')
- Output: Partitions of agents, e.g. $u_a(\pi) = u_a(C)$

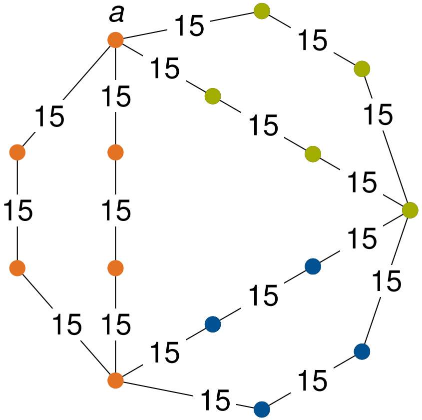
Utility Aggregation



$$u_i(C) = \sum_{j \in C} u_i(j)$$

Additively separable

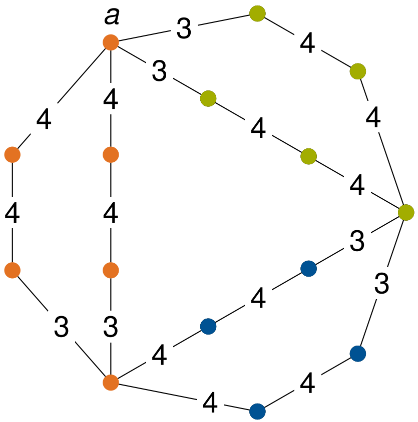
Utility Aggregation



Sum of utilities in $\{n, -1\}$

Friend-oriented

Utility Aggregation

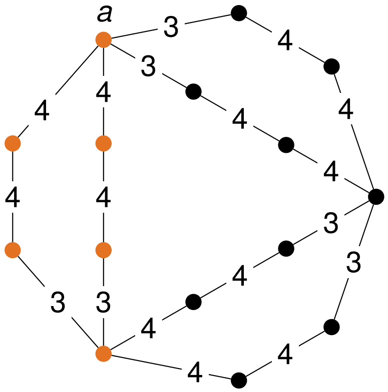


$$u_i(C) = 1/(|C| - 1) \sum_{j \in C} u_i(j)$$

Modified fractional ‘expected utility’

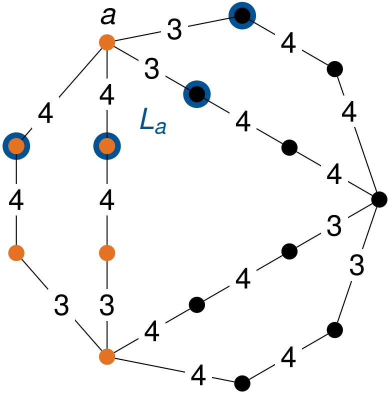
Minimization-Based Loyalty

"A chain is as strong as its weakest link."



Minimization-Based Loyalty

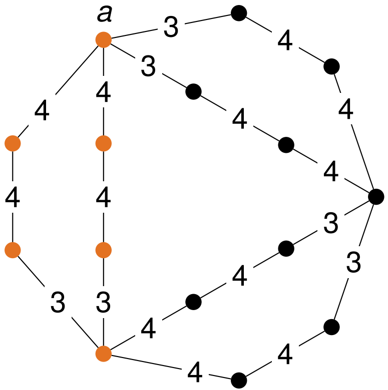
"A chain is as strong as its weakest link."



- Loyalty set $L_i = \{j \in N \setminus \{i\} \colon u_i(\{i,j\}) > 0\}$

Minimization-Based Loyalty

"A chain is as strong as its weakest link."



- Loyalty set $L_i = \{j \in N \setminus \{i\}\colon u_i(\{i,j\}) > 0\}$
- Loyal utilities $u_i^L(C) = \min_{j \in C \cap (L_i \cup \{i\})} u_j(C)$, e.g., $u_a^L(C) = 8$
- Recursively defined loyal variants with utilities u^k

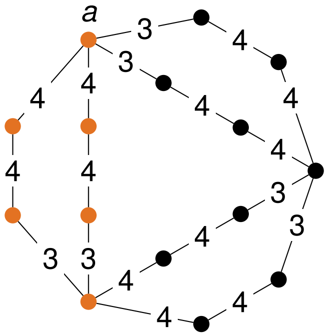
Goals

- Understanding the structure of loyal utilities
  - Loyalty propagation
  - Egalitarianism
  - Best coalitions
- Desirable partitions
  - Group stability: core partitions
  - Efficiency: Pareto optimality

Loyalty Propagation

Proposition

The utilities for the k-fold loyal variant are given as
$u_i^k(C) = \min_{j \in C}\{u_j(C)\colon d_C(i,j) \leq k\}$.

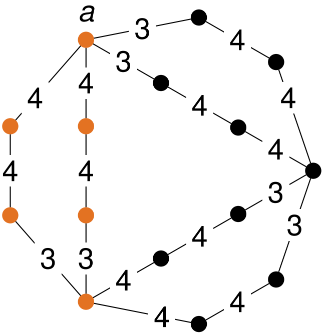


$u_a^2(C) = 7$

Loyalty Propagation

Proposition

The utilities for the k-fold loyal variant are given as
$u_i^k(C) = \min_{j \in C}\{u_j(C) \colon d_C(i,j) \leq k\}$.



$u_a^3(C) = 6$

Loyalty Propagation

Proposition

The utilities for the k-fold loyal variant are given as $u_i^k(C) = \min_{j \in C}\{u_j(C) \colon d_C(i,j) \le k\}$.

Proposition

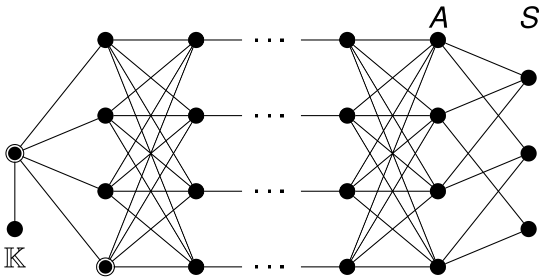
Loyal utilities converge to locally egalitarian utilities, i.e., $u_i^k(C) = \min_{j \in C^*}\{u_j(C)\}$ *for all* $k \ge n$.

Best Coalitions

Theorem

Let $k \geq 1$. For symmetric friend-oriented hedonic games as input, computing best coalitions is NP-hard

- *for the k-fold loyal variant.*

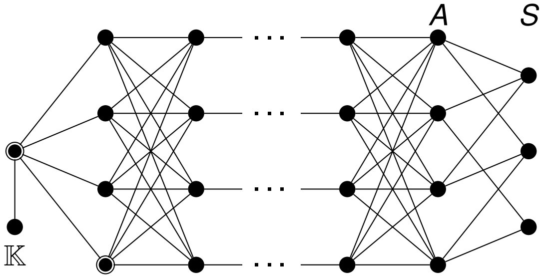


Best Coalitions

Theorem

Let $k \geq 1$. For symmetric friend-oriented hedonic games as input, computing best coalitions is NP-hard

- *for the k-fold loyal variant.*
- *for the locally egalitarian limitgame.*



Finite-diameter variant of reduction for limitgame

Desirable Partitions in the Limitgame

Theorem

Pareto-optimal partitions in the core exist * *for the limitgame.*

Efficient computability?

Theorem

`Computation` *of core partitions*

- *can be done in polynomial time for the limitgame of modified fractional hedonic games.*
- *is NP-hard for the limitgame of friend-oriented hedonic games.*

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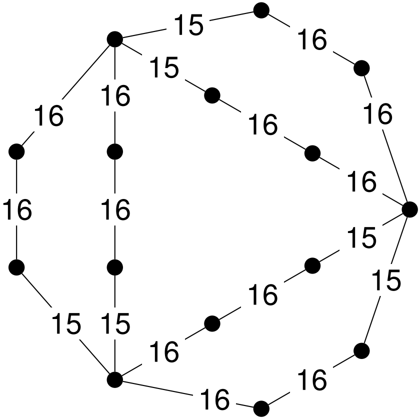
* under a very mild assumption on the utilities.

Group Stability in the Loyal Variants

Proposition

Partitions in the core need not exist for the k-fold loyal variant of additively separable hedonic games.



Group Stability in the Loyal Variants

Proposition

Partitions in the core need not exist for the k-fold loyal variant of additively separable hedonic games.

Theorem

- `Existence` *of core partitions is NP-hard for loyal variants of additively separable hedonic games.*
- `Computation` *of core partitions in the loyal variants of modified fractional hedonic games can be done in polynomial time.*
- `Computation` *of core partitions is NP-hard in the k-fold loyal variant of friend-oriented hedonic games for $k \geq 2$.*

Conclusion

Summary

- Existence of desirable partitions in limitgame
- Tractability of various goals dependent on input game
- Structural similarities of best coalitions and core partitions

Future directions

- Existence of core partitions in loyal variants of FOHGs
- Variant of loyalty (aggregation function)
- Loyalty under ordinal input