

On Strategyproofness and Localizedness in Social Choice

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We consider functions that map the preferences of a set of voters to a collective choice and generalize localizedness—an invariance condition proposed by Allan Gibbard—to weak preferences: any minimal change in a voter’s preference relation can only result in different collective outcomes a and b if the voter changed her preference between a and b . We provide new characterizations of (strongly) localized and unanimous social choice functions and social decision schemes. In particular, we prove that serial dictatorships with lexicographic tie-breaking are the only unanimous and strongly localized social choice functions for weak preferences. We also discuss the relationships between localizedness, non-bossiness, and Maskin monotonicity.

1 Introduction

One of the central results in microeconomic theory is the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975). While it is often interpreted as an impossibility result, it essentially represents an axiomatic characterization of dictatorial social choice functions (SCFs). Gibbard (1977) later extended this result to social choice functions that allow for randomization, so-called social decision schemes (SDSs), by characterizing random dictatorships, i.e., dictatorships where the dictator is selected at random. Random dictatorships are much more desirable than dictatorships because impartiality with respect to the agent can be guaranteed by selecting the dictator uniformly at random. Such uniform random dictatorships feature prominently in subdomains of social choice that are concerned with the allocation of private goods, e.g., the *random serial dictatorship*

(aka *random priority*) mechanism in assignment (see, e.g., Abdulkadiroğlu and Sönmez, 1998; Che and Kojima, 2010).

The key axiom in Gibbard’s characterizations is strategyproofness, i.e., no voter can manipulate the outcome in her favor by reporting insincere preferences. Gibbard showed that an SDS is strategyproof if and only if it is localized and monotonic.¹ Under scrutiny, the invariance condition that Gibbard called localizedness is the main ingredient for both of his theorems. For strict preferences, a function is localized if a voter can only change the outcome by swapping two alternatives a and b if the outcome changes from a to b or *vice versa*. Notably, this axiom ignores the “direction” of individual preferences. Gibbard shows that only a rather narrow set of functions is localized. Weak additional axioms suffice to further restrict this class to (mixtures of) dictatorships. In this paper, we show that for strict preferences, both theorems still hold if we replace strategyproofness with localizedness and non-imposition (or Pareto optimality, respectively) with unanimity, which requires that an alternative top-ranked by all voters is selected. In Section 4.1 (Section 5.1), we show that among localized rules, only (mixtures of) dictatorships are unanimous. This does not come as a surprise. In fact, although never explicitly mentioned, these insights are hidden in the proof techniques of existing characterizations (Gibbard, 1977; Tanaka, 2003; Mishra, 2016). When, furthermore, requiring anonymity, each voter gets to be the dictator with the same probability, leading to the uniform random dictatorship theorem.

Another implication of our results is a linear algebra interpretation of Gibbard’s random dictatorship theorem. Localizedness, unanimity, and anonymity are all “equality constraints”. Thus, we can reformulate the question about which rules simultaneously satisfy these axioms as a linear algebra problem where suitable rules correspond to vectors that are solutions to the system of linear equations given by the axioms. For example, since the uniform random dictatorship is the only SDS that satisfies strong localizedness, unanimity, and anonymity, we know that the corresponding system of linear equations has a unique solution and *vice versa*. This insight might help to prove similar results in other domains.

For weak preferences, the Gibbard-Satterthwaite theorem does not completely characterize dictatorships, because not all dictatorial SCFs are strategyproof (see, e.g., Example 2). By considering strategyproofness for weak preferences, Gibbard and Satterthwaite implicitly also define the concept of localizedness for weak preferences. Localized rules are much less restrictive than in the domain of strict preferences. These SCFs can almost arbitrarily change the outcome within an indifference class of a deviating voter. In a sense, this goes against the idea that the outcome should only change due to local changes in the individual preferences.

Following this line of thought, we propose *strong localizedness* as an invariance condition for weak preferences that is stronger than localizedness, but builds upon the same motivation. Both notions coincide for strict preferences. In Section 6, we give additional motivation for strong localizedness via the concept of non-bossiness and discuss the relation between Maskin monotonicity (Maskin, 1999) and our axioms. In particular,

¹Gibbard referred to monotonicity as non-perverseness.

Prefs.	Axioms		Characterized Rules	Source
strict	strategyproofness	non-imposition ²	all dictatorships	Gibbard (1973) and
weak	strategyproofness	non-imposition	some dictatorships	Satterthwaite (1975)
strict	localizedness	unanimity	all dictatorships	
weak	strong localizedness	unanimity	all serial dictatorships	
strict	<i>SD</i> -strategyproofness	Pareto optimality	all random dictatorships	Gibbard (1977)
weak	<i>SD</i> -strategyproofness	non-imposition	some random dictatorships	Gibbard (1978)
strict	localizedness	Pareto optimality	all random dictatorships	Tanaka (2003)
strict	localizedness	unanimity	all random dictatorships	

Table 1: Overview of existing characterizations and our results (highlighted in gray). The first half of the table is concerned with SCFs, the second one with SDSs. For weak preferences, all strategyproof and non-imposing rules are dictatorships and random dictatorships, respectively, but the converse directions do not hold. Hence, only “some” dictatorships and random dictatorships are characterized.

we show that the conjunction of strong localizedness and monotonicity is equivalent to a natural weakening of Maskin monotonicity due to Berga and Moreno (2009). In Section 4.2, we prove that, for weak preferences, a social choice function is strongly localized and unanimous if and only if it is a serial dictatorship with lexicographic tie-breaking. Efforts to obtain characterizations of strategyproof social choice functions (SCFs) for weak preferences have been unsuccessful, because strategyproofness is too weak to significantly narrow down the set of SCFs (see, e.g., Barberà et al., 1998; Barberà, 2007; Lederer, 2024, Open Problem 5.3).

Our characterization of serial dictatorships shows that replacing strategyproofness with strong localizedness enables a statement that is similar in spirit to the original Gibbard-Satterthwaite theorem. Characterizing social decision schemes in a similar fashion remains an intriguing open problem. We propose some ideas in Section 5.2. A comparison of our results for SCFs and SDSs is given in Table 1.

2 Related Work

Barberà (2010) provides an in-depth survey of strategyproofness in social choice theory. Without doubt, the most central result in this stream of research is the Gibbard-Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975).

Proofs of the Gibbard-Satterthwaite theorem and the random dictatorship theorem.

Interestingly, the two original proofs of the theorem differ significantly. While Satterthwaite gives a direct proof, Gibbard reduces the statement to Arrow’s impossibility (Arrow,

²An SDS g is *non-imposing* if for every $a \in A$, there exists $R \in \mathcal{R}^n$ such that $g(R, a) = 1$.

1951). Many other proofs have been given since then (e.g., Beja, 1993; Benoît, 2000; Sen, 2001; Reny, 2001; Cato, 2009; Svensson and Reffgen, 2014).

The random dictatorship theorem by Gibbard (1977) was also the starting point for a thorough investigation of strategyproofness for social decision schemes. Multiple authors gave direct proofs of the theorem (e.g., Duggan, 1996; Nandeibam, 1997; Tanaka, 2003; Sen, 2011; Lederer, 2024), which Gibbard only stated as a corollary of his more general characterization. We would like to highlight that Tanaka (2003) appears to be the first to prove the characterization with localizedness instead of strategyproofness. Unfortunately, his exposition is somewhat imprecise and confusing in this aspect, and the fact that localizedness suffices is easy to miss.

Local variants of localizedness and strategyproofness. Localizedness is known under many names (e.g., set-monotonicity (Börger, 2015) and swap-monotonicity (Mishra, 2016)) and allows for various equivalent definitions. Gibbard (1977) defined it via upper and lower contour sets. By contrast, we use only local constraints between two profiles where *one* voter changes her individual preferences by moving *one* alternative up (or down) by *one* position. Taking such an atomic point of view often helps to get hold of global axioms such as localizedness or, more generally, strategyproofness. Carroll (2012) investigates under which conditions (global) strategyproofness is implied by local strategyproofness for a variety of models. Sato (2013a,b) and Kumar et al. (2021a,b) study notions of strategyproofness where the set of admissible manipulations is restricted, e.g., to pairwise swaps of two alternatives. Note that the definition of localizedness allows for a simultaneous interpretation as a global axiom (via upper and lower contour sets) and a local axiom (as in Definition 5).

Axioms related to localizedness. Muto and Sato (2017) consider a weakening of localizedness called “bounded response” for SCFs and strict preferences. They show that bounded response and unanimity do not imply dictatorships but provide a characterization by strengthening unanimity and weakening bounded response. Our characterization for this setting uses the stronger notion of localizedness but preserves unanimity. In particular, strong localizedness stems from the same motivation as the concept of non-bossiness in public goods (Satterthwaite and Sonnenschein, 1981; Saijo et al., 2007; Berga and Moreno, 2009) — an intriguing connection which is discussed in detail in Section 6.

Subdomains and serial dictatorships. A natural way to escape impossibility theorems is to restrict the domain of admissible preferences. For example, building on an idea by Black (1948), Moulin (1980) characterizes anonymous and strategyproof SCFs on the domain of single-peaked preferences. The assignment and matching domains constitute two specific subdomains of social choice where possible assignments or matchings are interpreted as alternatives. In these settings, voters’ preferences are usually restricted, i.e., voters are indifferent among all assignments in which they receive the same object. Consequently, a dictator has multiple top-ranked alternatives, and other voters further reduce the set of outcomes one after another. Such serial dictatorships have received little

attention in the classical social choice literature (e.g., Aziz et al., 2018), but are among the most prominent rules for private good settings. There are multiple characterizations of serial dictatorships or variations that rely on strategyproofness, or more precisely, mainly again on restrictions of localized functions in assignment (see, e.g., Svensson, 1999; Pápai, 2000). However, as there are large indifference classes, additional invariance conditions are needed.

Maskin monotonicity. A central concept in implementation theory (see e.g., Moore, 1992; Jackson, 2001) is Maskin monotonicity due to its close connection to Nash implementability (Maskin, 1999). For strict preferences, Maskin monotonicity is equivalent to strategyproofness (Muller and Satterthwaite, 1977) which is in turn equivalent to localizedness and monotonicity (Gibbard, 1977). For weak preferences, only constant rules satisfy Maskin monotonicity (Saijo, 1987) while there are nontrivial strategyproof SCFs. Consequently, weakenings of Maskin monotonicity have also been studied (see, e.g., Berga and Moreno, 2009; Kojima and Manea, 2010; Basteck, 2024). We are particularly interested in the variant by Berga and Moreno (2009), which turns out to be equivalent to the conjunction of strong localizedness and monotonicity.

3 Preliminaries

Let $N = \{1, \dots, n\}$ be a set of n voters and A a set of m alternatives. The set of all complete and transitive preference relations over A is denoted by $\mathcal{R}$. The subset of $\mathcal{R}$ that only contains antisymmetric relations is denoted by $\mathcal{L}$. For two alternatives $a, b \in A$, $a \succ_i b$ if and only if $a \succsim_i b$ and not $b \succsim_i a$ and $a \sim_i b$ if and only if $a \succsim_i b$ and $b \succsim_i a$. For $B \subseteq A$ we write $\succsim|_B = \{(a, b) \in \succsim : a, b \in B\}$ for the preference relation $\succsim$ restricted to the set of alternatives B . Furthermore, $\max(\succsim) := \{x \in A : x \succsim y \text{ for all } y \in A\}$ are the maximal elements of $\succsim$ and $[a]_{\succsim} := \{b \in A : b \sim a\}$ corresponds to alternative a 's equivalence class under $\succsim$.

A preference profile $R = (\succsim_1, \dots, \succsim_n) \in \mathcal{R}^n$ consists of n individual preference relations. Moreover, $R_{-i} = R \setminus \{\succsim_i\}$ and we write $(R_{-i}, \succsim'_i)$ for the preference profile in which $\succsim_i$ is replaced with $\succsim'_i$. A lottery p is a probability distribution over the set of alternatives A . It assigns each alternative $x \in A$ a probability $p(x) \geq 0$ such that $\sum_{x \in A} p(x) = 1$. The set of all lotteries over A is denoted by $\Delta(A)$. A social choice function (SCF) $f : \mathcal{R}^n \rightarrow A$ maps each profile R to one alternative $f(R)$, whereas a social decision scheme (SDS) $g : \mathcal{R}^n \rightarrow \Delta(A)$ maps each profile to a probability distribution $g(R)$ over the alternatives. Thus, an SCF admits an interpretation as an SDS that always assigns probability 1 to one alternative. Slightly abusing notation, we denote the sum of probabilities f and g assign to the members of a set of alternatives $A' \subseteq A$ in profile R by $f(R, A')$ and $g(R, A')$, respectively. Note that $f(R, a) \in \{0, 1\}$. The support $\text{supp}(g(R))$ of an outcome $g(R)$ is defined as the set of all alternatives a with $g(R, a) > 0$.

In the following, we define all axioms for weak preferences and SDSs and mention special implications for strict preferences or SCFs explicitly.

Definition 1. An SDS g is *unanimous* if for all $R \in \mathcal{R}^n$, $\bigcap_{i \in N} \max(\succsim_i) \neq \emptyset$ implies that $\text{supp}(g(R)) \subseteq \bigcap_{i \in N} \max(\succsim_i)$.

Unanimity demands that whenever there are alternatives that are top-ranked by all voters, only these alternatives receive positive probability. For strict preferences, this is only possible when all voters have the same top choice.

Definition 2. In a profile $R \in \mathcal{R}^n$, an alternative $a \in A$ is *Pareto optimal* if there does not exist another alternative $b \in A \setminus \{a\}$ with $b \succsim_i a$ for all $i \in N$ and $b \succ_i a$ for some $i \in N$. An alternative $a \in A$ is *weakly Pareto optimal* if there does not exist another alternative $b \in A \setminus \{a\}$ with $b \succ_i a$ for all $i \in N$.

An SDS g satisfies (weak) Pareto optimality if for each profile, only (weakly) Pareto optimal alternatives receive positive probability.

Note that on $\mathcal{L}^n$, Pareto optimality coincides with its weaker version.

It is straightforward to see that

$$\text{Pareto optimality} \implies \text{unanimity} \implies \text{non-imposition}$$

but the reverse implications do not hold.

Gibbard (1977) defined strategyproofness for SDSs by demanding that no voter can increase her expected utility by misrepresenting her preferences for all utility functions that are consistent with her ordinal preferences.

Definition 3. An SDS g is *strategyproof* if for all $a \in A$,

$$\sum_{b: b \succsim_i a} g(R, b) \geq \sum_{b: b \succsim'_i a} g((R_{-i}, \succsim'_i), b) \quad (1)$$

for all $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}^n$.

This strategyproofness notion is sometimes called (strong) *SD*-strategyproofness to distinguish it from other variants of strategyproofness.

For SCFs, (1) reduces to $f(R) \succsim_i f(R_{-i}, \succsim'_i)$. Gibbard then showed that strategyproof SDSs form precisely the intersection of all *localized* and *monotonic* SDSs.

Before introducing localizedness and monotonicity, we define minimal changes of a preference relation. We say that a is *adjacent and above* b for $\succsim$ if $a \succ b$ and no c exists with $a \succ c \succ b$. Similarly, we say that a is *adjacent and below* b for $\succsim$ if $b \succ a$ and no c exists with $b \succ c \succ a$.

Definition 4. Let $\succsim, \succsim' \in \mathcal{R}$. We say $\succsim'$ is a *minimal change* of $\succsim$ if there is an alternative $a \in A$ such that

- $\succsim|_{A \setminus \{a\}} = \succsim'|_{A \setminus \{a\}}$ and
- one of the following conditions holds:
 - $|[a]_{\succsim}| = 1$, and b is adjacent and above a for $\succsim$ and $\succsim' = \succsim \cup \{(a, x) : x \in [b]_{\succsim}\}$.

- $|[a]_{\succsim}| = 1$, and b is adjacent and below a for $\succsim$ and $\succsim' = \succsim \cup \{(x, a) : x \in [b]_{\succsim}\}$.
- $|[a]_{\succsim}| > 1$ and $\succsim' = \succsim \setminus \{(a, x) : x \in [a]_{\succsim} \setminus \{a\}\}$.
- $|[a]_{\succsim}| > 1$ and $\succsim' = \succsim \setminus \{(x, a) : x \in [a]_{\succsim} \setminus \{a\}\}$.

A minimal change can select an alternative that forms an indifference class by itself and put it into the indifference class above or below. Alternatively, it selects an alternative from an indifference class containing more than one element and moves it to a new class above or below. For strict preferences, a minimal change is defined as a pairwise swap of adjacent alternatives to stay in $\mathcal{L}^n$, i.e., (x, a) or (a, x) need to be additionally excluded in $\succsim'$ for the first two cases of Definition 4. Note that such a swap corresponds to *two* minimal changes in $\mathcal{R}^n$ where the two involved alternatives are put into one indifference class in a first step.

We are now ready to define localizedness and monotonicity.

Definition 5. An SDS g is *localized* if for all $R \in \mathcal{R}^n$, $i \in N$, $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ with alternative a , $g(R, [b]_{\succsim_i}) = g((R_{-i}, \succsim'_i), [b]_{\succsim_i})$ for all $b \in A \setminus \{a\}$ with $\succsim_i|_{\{a, b\}} = \succsim'_i|_{\{a, b\}}$.

For strict preferences, localizedness implies that only the probability assigned to the two swapped alternatives can change. Weak preferences offer more freedom as the outcome can arbitrarily move probability not only between $[a]_{\succsim_i}$ and $[a]_{\succsim'_i}$, but also within other indifference classes of voter i .

Localizedness corresponds to equality constraints between profiles and is therefore easier to handle than strategyproofness from a technical point of view, as already shown by Gibbard's original proof (see Lemma 1 below).

Definition 6. An SDS g is *monotonic* if for all $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ in which alternative a is moved *up*, it holds that $g((R_{-i}, \succsim'_i), a) \geq g(R, a)$.

Monotonicity requires that the probability of an alternative cannot decrease when it is strengthened in a voter's preference relation. For SCFs, this means that a chosen alternative is still chosen when a voter moves it upwards.

Next, we propose a refined notion of localizedness for weak preferences and SCFs. In Section 5.2, we discuss possible generalizations for SDSs.

Definition 7. An SCF f is *strongly localized* if for all $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ with alternative a ,

$$f(R_{-i}, \succsim'_i) \begin{cases} \in [a]_{\succsim_i} \cup [a]_{\succsim'_i} & \text{if } f(R) = a, \\ \in \{a, f(R)\} & \text{if } f(R) \in ([a]_{\succsim_i} \cup [a]_{\succsim'_i}) \setminus \{a\}, \\ = f(R) & \text{otherwise.} \end{cases}$$

Strong localizedness ensures that the outcome can only change *from* a or *to* a . In particular, only minimal changes that involve the returned alternative in one of a 's

Localized	$\begin{array}{c} \mathbf{a}, b \\ c \rightarrow c, d \\ d \end{array}$	$\begin{array}{c} \mathbf{a}, \mathbf{b} \\ c \rightarrow \end{array}$	$\begin{array}{c} \mathbf{a}, b \quad \mathbf{a}, b, c \\ c \rightarrow \end{array}$	$\begin{array}{c} \mathbf{a}, b, c \quad \mathbf{c} \\ \rightarrow \mathbf{a}, \mathbf{b} \end{array}$	$\begin{array}{c} \mathbf{a}, b, c \quad \mathbf{a} \\ \rightarrow \mathbf{b}, \mathbf{c} \end{array}$
Strongly localized	$\begin{array}{c} \mathbf{a}, b \\ c \rightarrow c, d \\ d \end{array}$	$\begin{array}{c} \mathbf{a}, b \\ c \rightarrow \end{array}$	$\begin{array}{c} \mathbf{a}, b \quad \mathbf{a}, b, c \\ c \rightarrow \end{array}$	$\begin{array}{c} \mathbf{a}, b, c \quad \mathbf{c} \\ \rightarrow \mathbf{a}, \mathbf{b} \end{array}$	$\begin{array}{c} \mathbf{a}, b, c \quad \mathbf{a} \\ \rightarrow \mathbf{b}, \mathbf{c} \end{array}$

Figure 1: A comparison of localizedness and strong localizedness for four alternatives and SCFs. We assume that alternative a is selected when the voter reports the first preference relation. She then changes her preferences to the second preference relation. The bold alternatives show possible outcomes from localized and strongly localized SCFs.

indifference classes can influence the outcome. Note that for strict preferences, strong localizedness coincides with localizedness (see also Proposition 1) and is therefore implied by strategyproofness. Due to the second case in Definition 7, this relation does not hold for weak preferences as strategyproofness allows for shifts of the outcome in $[a]_{\succsim_i}$ or $[a]_{\succsim'_i}$ that do not involve a .

Strong localizedness can also be seen as an ordinal consistency axiom: the probability of an alternative cannot change when all pairwise comparisons to other alternatives remain the same. This is reminiscent of the concept of non-bossiness for public goods (Saijo et al., 2007; Berga and Moreno, 2009) which states that the outcome cannot change from a to b if agent i is indifferent between them under $\succsim$ (and $\succsim'_i$, respectively). In fact, strong localizedness can be interpreted as a combination of localizedness and a local version of non-bossiness (see Section 6). Berga and Moreno (2009) show relations between (weak) non-bossiness, strategyproofness, and Maskin monotonicity when agents have single-peaked or single-plateaued preferences over a continuum of alternatives. In Section 6, we prove similar results for the entire domain of weak preferences.

The following example provides a comparison of the two notions of localizedness.

Example 1. Consider the four cases illustrated in Figure 1. If a is selected by the SCF, alternatives marked in bold constitute possible outcomes of (strongly) localized SCFs after the voter changed her preferences as indicated.

The first case demonstrates the case where the indifference class that contains a remains unchanged after the minimal change of the voter's preferences. As mentioned previously, the outcome only remains within the indifference class $[a]_{\succsim}$ for localized functions. Strong localizedness, however, requires that alternative a remains the winner. In this particular case, the outcome cannot change to b .

In the second and third cases, an unchosen alternative c joins or leaves the indifference class that contains the winning alternative a . Note that, in contrast to strategyproofness, both strongly localized axioms are ignorant with respect to the direction of changes. Therefore, the resulting constraints are the same, independently of c moving up or down in the voter's preference relation. We observe that localized SCFs allow the outcome to

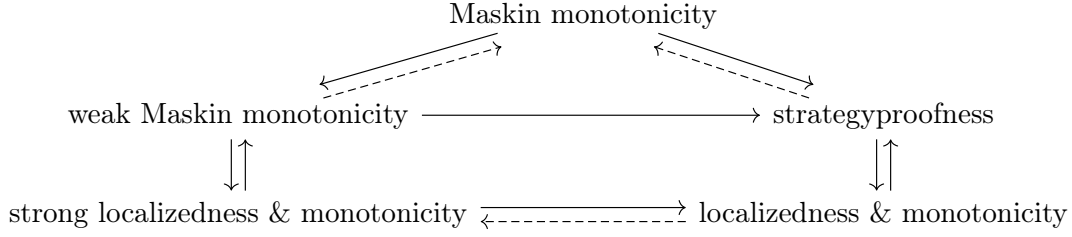


Figure 2: Relationships between strategyproofness, localizedness, and monotonicity for SCFs. Implications depicted by dashed arrows only hold for strict preferences.

change to alternative b while this is not possible for strongly localized SCFs since the voter remains indifferent between a and b .

The fourth case shows that both notions have the same implications when moving the returned alternative a .

Figure 2 illustrates the relations between strategyproofness and variants of localizedness and monotonicity for SCFs (see also Section 6).

Finally, we formally define dictatorships and serial dictatorships.

Definition 8. An SDS g is a *dictatorship* if there exists a voter $i \in N$ such that $g(R) \in \max(\succsim_i)$ for every $R \in \mathcal{R}^n$.

Such a voter i is called a dictator.

In a serial dictatorship, the voters are dictators one after another in a specified order π . The first dictator restricts the set of admissible alternatives from all alternatives A to the set of her top choices. Then, in the order given by π , the other dictators restrict the set of admissible alternatives further by restricting it to their top choices among the remaining admissible alternatives. If, in the end, more than one alternative remains, a tie-breaking rule r is needed to determine the final outcome.

Definition 9. A *tie-breaking rule* $r: 2^A \setminus \{\emptyset\} \rightarrow \Delta(A)$ with $\text{supp}(r(A')) \subseteq A'$ for all $A' \in 2^A \setminus \{\emptyset\}$ maps each set of alternatives A' to a distribution over A' .

For SCFs, r has to put probability 1 on one alternative, i.e., $r: 2^A \setminus \{\emptyset\} \rightarrow A$ with $r(A') \in A'$.

Definition 10. Let π be a permutation of voters and r be a tie-breaking rule. The SDS SD_π^r is the *serial dictatorship* with order π and tie-breaking rule r . We compute $SD_\pi^r(R) = SD_\pi^r(R, A, 1)$ where the later is defined recursively as,

$$SD_\pi^r(R, B, i) = \begin{cases} SD_\pi^r(R, \max(\succsim_{\pi(i)}|_B), i+1) & \text{if } i \leq n, \\ r(B) & \text{otherwise.} \end{cases}$$

Note that every serial dictatorship is a dictatorship, but the converse does not hold.

Example 2. Let $n = 2$, $A = \{a, b\}$ and consider the SCF f , for which Voter 1 is a dictator, i.e., $f(R) \in \max(\succsim_i)$. When $a \sim_1 b$, let

$$f(R) = \begin{cases} a & \text{if } b \succ_2 a, \\ b & \text{otherwise.} \end{cases}$$

By definition, f is a dictatorship. However, f is not strategyproof as Voter 2 can beneficially manipulate from a to b in the profile where $a \sim_1 b$ and $b \succ_2 a$ by reporting $a \sim_2 b$ instead. The same profile also shows that f is no serial dictatorship.

4 Social Choice Functions

In this section, we characterize (serial) dictatorships as the only SCFs that are (strongly) localized and unanimous.

For strict preferences, our characterization and the Gibbard-Satterthwaite theorem imply that strategyproofness together with non-imposition is equivalent to localizedness combined with unanimity. Note that strategyproofness is stronger than localizedness and non-imposition is weaker than unanimity.

For weak preferences, Gibbard (1973) and Satterthwaite (1975) proved that all strategyproof and non-imposing SCFs are dictatorships. However, the converse statement does not hold (see, e.g., Example 2). Using a natural generalization of localizedness to weak preferences results in a characterization of serial dictatorships with lexicographic tie-breaking via strong localizedness and unanimity.

4.1 Strict Preferences

We start by revisiting Lemma 3 in Gibbard (1977) which states that whether or not a voter can change the outcome from a to b is independent of swaps of other pairs of alternatives by other voters. Gibbard called such rules *pairwise isolated* and proved the statement for the more general class of SDSs. We state the lemma in this general form as it is also utilized in Section 5. We include its proof for completeness. Figure 3 illustrates the required profiles.

Lemma 1 (Gibbard, 1977). *Given a profile $R \in \mathcal{L}^n$, two voters $i, j \in N$, two pairs of alternatives $\{a, b\}, \{c, d\} \subset A$ with $\{a, b\} \neq \{c, d\}$, and preference relations $\succ'_i, \succ'_j \in \mathcal{L}$ such that*

- $a \succ_i b$ where a, b are adjacent in $\succ_i$,
- $\succ'_i = \succ_i \setminus \{(a, b)\} \cup \{(b, a)\}$,
- $c \succ_j d$ where c, d are adjacent in $\succ_j$, and
- $\succ'_j = \succ_j \setminus \{(c, d)\} \cup \{(d, c)\}$.

Then, for all localized SDSs g ,

$$\begin{aligned} & g((R_{-\{i,j\}}, \succ_i, \succ_j), a) - g((R_{-\{i,j\}}, \succ'_i, \succ_j), a) \\ &= g((R_{-\{i,j\}}, \succ_i, \succ'_j), a) - g((R_{-\{i,j\}}, \succ'_i, \succ'_j), a), \end{aligned} \quad (2)$$

and

$$\begin{aligned} & g((R_{-\{i,j\}}, \succ_i, \succ_j), b) - g((R_{-\{i,j\}}, \succ'_i, \succ_j), b) \\ &= g((R_{-\{i,j\}}, \succ_i, \succ'_j), b) - g((R_{-\{i,j\}}, \succ'_i, \succ'_j), b). \end{aligned} \quad (3)$$

Proof. Without loss of generality, assume $a \notin \{c, d\}$, otherwise we can rename a, b and consider $\succ'_i = \succ_i \setminus \{(b, a)\} \cup \{(a, b)\}$. When voter j swaps c and d , i.e. moves from $\succ_j$ to $\succ'_j$, we have

$$g((R_{-\{i,j\}}, \succ_i, \succ_j), a) = g((R_{-\{i,j\}}, \succ_i, \succ'_j), a), \quad (4)$$

and

$$g((R_{-\{i,j\}}, \succ'_i, \succ_j), a) = g((R_{-\{i,j\}}, \succ'_i, \succ'_j), a) \quad (5)$$

since g is localized. Subtracting (5) from (4) gives (2).

Furthermore, again by localizedness, the sum of probabilities on a and b does not change when voter i swaps a and b , i.e.,

$$\begin{aligned} & g((R_{-\{i,j\}}, \succ_i, \succ_j), a) + g((R_{-\{i,j\}}, \succ_i, \succ_j), b) \\ &= g((R_{-\{i,j\}}, \succ'_i, \succ_j), a) + g((R_{-\{i,j\}}, \succ'_i, \succ_j), b), \end{aligned} \quad (6)$$

and

$$\begin{aligned} & g((R_{-\{i,j\}}, \succ_i, \succ'_j), a) + g((R_{-\{i,j\}}, \succ_i, \succ'_j), b) \\ &= g((R_{-\{i,j\}}, \succ'_i, \succ'_j), a) + g((R_{-\{i,j\}}, \succ'_i, \succ'_j), b). \end{aligned} \quad (7)$$

Next, we subtract the right from the left hand side for both equations and equate them, i.e.,

$$\begin{aligned} & g((R_{-\{i,j\}}, \succ_i, \succ_j), a) - g((R_{-\{i,j\}}, \succ'_i, \succ_j), a) \\ &+ g((R_{-\{i,j\}}, \succ_i, \succ_j), b) - g((R_{-\{i,j\}}, \succ'_i, \succ_j), b) \\ &= g((R_{-\{i,j\}}, \succ_i, \succ'_j), a) - g((R_{-\{i,j\}}, \succ'_i, \succ'_j), a) \\ &+ g((R_{-\{i,j\}}, \succ_i, \succ'_j), b) - g((R_{-\{i,j\}}, \succ'_i, \succ'_j), b). \end{aligned} \quad (8)$$

Finally, we subtract (2) from (8) and get (3). □

For localized SCFs, repeatedly applying Lemma 1 implies that, if the outcome changes from a to b when voter i swaps adjacent alternatives a and b in her preferences, the same happens for every other profile in which voter i has the same preferences and all other voters have the same binary relations over a and b . This insight is essential for proving the following theorem.

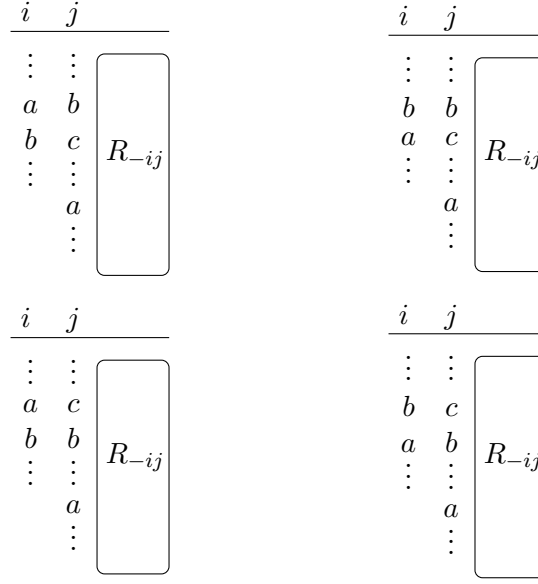


Figure 3: Illustration of the four preference profiles needed for Lemma 1. The lemma implies that the change in outcome between the upper and lower pair of profiles is the same.

Theorem 1. *Let $m \geq 3$. An SCF f is localized and unanimous on $\mathcal{L}^n$ if and only if f is dictatorial.*

Proof. It is straightforward to see that all dictatorships are unanimous and localized. In particular, localizedness is weaker than strategyproofness.

It remains to show that all localized and unanimous SCFs are dictatorships. Let $R^0, \dots, R^n$ be a sequence of profiles such that in R^0 , all voters have the same top-ranked alternative (without loss of generality a) and rank the same alternative (without loss of generality b) second. Furthermore, $R^k = (R_{-k}^{k-1}, \succ_k^{k-1} \setminus \{(a, b)\} \cup \{(b, a)\})$ for $k \in \{1, \dots, n\}$, i.e., when moving from R^{k-1} to R^k , voter k swaps a and b . In R^n , b is top-ranked by all voters.

By unanimity, $f(R^0) = a$ and $f(R^n) = b$. Thus, there exist $i \in \{1, \dots, n\}$ with $f(R^{i-1}) = a$ and $f(R^i) = b$. We show that voter i is the dictator.

To see this, first consider profiles $\tilde{R}^{i-1}$ and $\tilde{R}^i$ where, compared to R^{i-1} and R^i , voter i moves c up to her second rank instead of swapping a and b and all other voters rank a third alternative c at their tops, but preferences are the same otherwise, i.e., $\succ_j|_{A \setminus c} = \tilde{\succ}_j|_{A \setminus \{c\}}$ for all $j \neq i$ in R^{i-1} , R^i , and $\tilde{R}^{i-1}$, $\tilde{R}^i$. By localizedness and Lemma 1, $f(\tilde{R}^{i-1}) = f(R^{i-1}) = a$ and $f(\tilde{R}^i) = f(R^i) = b$.

Next, consider profiles $\bar{R}^{i-1}$ and $\bar{R}^i$ where, compared to $\tilde{R}^{i-1}$ and $\tilde{R}^i$, an arbitrary subset of voters $N' \subset N \setminus \{i\}$ swapped a and b . Again, $f(\bar{R}^{i-1}), f(\bar{R}^i) \in \{a, b\}$ due to localizedness as a and b are adjacent in all preferences of voters from N' . If $f(\bar{R}^{i-1}) = b$, then voter i could swap c from second to first rank without changing the outcome.

However, this would contradict unanimity as in the resulting profile, c is top-ranked by all voters. Thus, $f(\bar{R}^{i-1}) = a$. Analogously, $f(\bar{R}^i) = b$.

As N' was chosen arbitrarily, the outcome in these profiles does not even depend on how voters $j \neq i$ rank a versus b . Together with Lemma 1, $f(R) = \max(\succ_i)$ whenever $\max(\succ_i) \in \{a, b\}$.

As the final step, consider the same sequence of profiles $R^0, \dots, R^n$ where b is replaced by c and the outcome flips from a to c at voter i' . By the same arguments, $f(R) = \max(\succ_{i'})$ whenever $\max(\succ_{i'}) \in \{a, c\}$. Consequently, $i' = i$ as otherwise, profiles with $\max(\succ_i) = a$ and $\max(\succ_{i'}) = c$ would lead to a contradiction. As c was chosen arbitrarily, it follows that $f(R) = \max(\succ_i)$ whenever $\max(\succ_i) \in A$, so voter i is the dictator. $\square$

4.2 Weak Preferences

This section is dedicated to proving that an SCF is strongly localized and unanimous if and only if it is a serial dictatorship with lexicographic tie-breaking. We first show that every SCF that is strongly localized is also localized, and subsequently that all strongly localized and unanimous SCFs are dictatorships.

Proposition 1. *Every strongly localized SCF on $\mathcal{R}^n$ is also localized.*

Proof. Let f be a strongly localized SCF and $R \in \mathcal{R}^n$ an arbitrary profile. For arbitrary $i \in N$, let $\tilde{\succ}'_i \in \mathcal{R}$ be a minimal change of $\tilde{\succ}_i$ with alternative a (see Definition 4). For arbitrary $b \in A \setminus \{a\}$, $\tilde{\succ}_i|_{\{a,b\}} = \tilde{\succ}'_i|_{\{a,b\}}$ is equivalent to $b \notin ([a]_{\tilde{\succ}_i} \cup [a]_{\tilde{\succ}'_i})$. By strong localizedness, $f(R) \neq b \neq f(R_{-i}, \tilde{\succ}'_i)$ if $f(R) \in ([a]_{\tilde{\succ}_i} \cup [a]_{\tilde{\succ}'_i})$, and $f(R) = f(R_{-i}, \tilde{\succ}'_i)$, otherwise. Thus, f satisfies localizedness. $\square$

In the following theorem, we prove that strongly localized and unanimous SCFs are dictatorships. This theorem is a variant of the Gibbard-Satterthwaite theorem using localizedness and unanimity instead of strategyproofness and non-imposition.

Theorem 2. *Let $m \geq 3$ and f be a strongly localized SCF on $\mathcal{R}^n$ that satisfies unanimity. Then, f is a dictatorship.*

Proof. Let f be a strongly localized SCF that satisfies unanimity. Without loss of generality, let Voter 1 be the dictator on the subdomain $\mathcal{L}^n$ whose existence is guaranteed by Theorem 1. For all $k \in \{0, \dots, n\}$, let $\mathcal{R}_k^n = \mathcal{L}^{n-k} \times \mathcal{R}^k$ be the domain in which the last k voters have weak preferences and all others have strict preferences. Clearly, $\mathcal{L}^n = \mathcal{R}_0^n \subset \mathcal{R}_1^n \subset \dots \subset \mathcal{R}_n^n = \mathcal{R}^n$.

Furthermore, if Voter 1 is a dictator on $\mathcal{R}_{n-1}^n$, then also on $\mathcal{R}_n^n = \mathcal{R}^n$. To see this, consider an arbitrary $R = (\succ_i, R_{-1}) \in \mathcal{R}_{n-1}^n$ where Voter 1 is a dictator by assumption. Hence, $f(R) = \max(\succ_1)$. Let Voter 1 now change to any weak order $\tilde{\succ}'_1$ with $\max(\succ_1) \in \max(\tilde{\succ}'_1)$. By localizedness, $f((\tilde{\succ}_1, R_{-1})) \in \max(\tilde{\succ}_1)$. Since R was chosen arbitrarily, Voter 1 is a dictator in $\mathcal{R}^n$.

For $0 < k < n$, we show by induction that Voter 1 is a dictator on $\mathcal{R}_k^n$ if he is a dictator on $\mathcal{R}_{k-1}^n$. The statement for $k = n$ then follows from above. By assumption, Voter 1 is a

dictator on $\mathcal{L}^n = \mathcal{R}_0^n$ ($k = 0$). Assuming that Voter 1 is a dictator on $\mathcal{R}_{k-1}^n$ for $0 < k < n$, we now prove that Voter 1 is also a dictator on $\mathcal{R}_k^n$.

Consider an arbitrary profile $R \in \mathcal{R}_k^n$. Note that Voter 1 has strict preferences as $k < n$. Without loss of generality, $\max(\succ_1) = a$ is Voter 1's most preferred alternative. Our goal is to show that Voter 1 remains dictator when voter $\ell = n - k + 1$ moves from strict to weak preferences. If $f(R) = a$, we are done, so assume $f(R) \neq a$. Let $\succ_\ell \in \mathcal{L}$ such that $\succ_\ell \subset \widetilde{\succ}_\ell$, i.e., the relation $\succ_\ell$ breaks ties in $\widetilde{\succ}_\ell$ arbitrarily.

First, assume $[a]_{\widetilde{\succ}_\ell} \neq A$. Then, there exists an alternative $b \in A \setminus [a]_{\widetilde{\succ}_\ell}$. Let Voter 1 change her preferences to $\succ'_1: a \succ'_1 b \succ'_1 \dots \succ'_1 [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$, where the alternatives from $[a]_{\widetilde{\succ}_\ell} \setminus \{a\}$ are ranked in an arbitrary strict order. Since $f(R) \neq a$, and f is localized, $f(\succ'_1, R_{-1}) \neq a$. Hence, either $f((\succ'_1, R_{-1})) \in [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$ or $f((\succ'_1, R_{-1})) \notin [a]_{\widetilde{\succ}_\ell}$.

For $f((\succ'_1, R_{-1})) \notin [a]_{\widetilde{\succ}_\ell}$, assume that voter ℓ changes her preferences to $\succ_\ell$. Then, $f((\succ'_1, \succ_\ell, R_{-\{1,\ell\}})) \notin [a]_{\widetilde{\succ}_\ell}$ since f is localized. But $(\succ'_1, \succ_\ell, R_{-\{1,\ell\}}) \in \mathcal{R}_{k-1}^n$ and thus, $f((\succ'_1, \succ_\ell, R_{-\{1,\ell\}})) = a$ by assumption, resulting in a contradiction.

For $f((\succ'_1, R_{-1})) \in [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$, consider Voter 1 swapping a and b , i.e., $\succ_1^* = \succ'_1 \setminus \{(a, b)\} \cup \{(b, a)\}$. Then, $f((\succ_1^*, R_{-1})) = f((\succ'_1, R_{-1})) \in [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$ since f is localized. In particular, $b \notin [a]_{\widetilde{\succ}_\ell}$ is not the winner. Next, let voter ℓ change her preferences to $\succ_\ell$. We have $f((\succ_1^*, \succ_\ell, R_{-\{1,\ell\}})) \in [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$ since $f((\succ_1^*, R_{-1})) \in [a]_{\widetilde{\succ}_\ell} \setminus \{a\}$. But $(\succ_1^*, \succ_\ell, R_{-\{1,\ell\}}) \in \mathcal{R}_{k-1}^n$ and thus, $f((\succ_1^*, \succ_\ell, R_{-\{1,\ell\}})) = b$ by assumption, a contradiction. We conclude that $f(R) = a$ when $[a]_{\widetilde{\succ}_\ell} \neq A$.

Otherwise, $[a]_{\widetilde{\succ}_\ell} = A$ and, without loss of generality, $f(R) = b$. As $m \geq 3$, there exists an alternative $c \neq a, b$. Let voter ℓ change her preferences to $\widetilde{\succ}'_\ell$ for which $x \succ'_\ell c$ and $x \sim'_\ell y$ for all $x, y \in A \setminus \{c\}$. By strong localizedness, $f(R_{-\ell}, \widetilde{\succ}'_\ell) \in \{c, f(R)\}$. But if $f(R_{-\ell}, \widetilde{\succ}'_\ell) = c$, then voter ℓ could move to a strict ranking without changing the outcome by localizedness. This would contradict our assumption and thus, $f(R_{-\ell}, \widetilde{\succ}'_\ell) = f(R)$. We are now back in the first case, therefore $f((R_{-\ell}, \widetilde{\succ}'_\ell)) = a$ and thus, $f(R) = a$. This completes the induction showing that Voter 1 is a dictator on $\mathcal{R}^n$. $\square$

Large parts of the proof of Theorem 2 require only localizedness. In fact, strong localizedness is only needed in the last case where a non-dictator is indifferent between all alternatives. This insight allows us to define a localized and unanimous SCF for $m = 3$ and $n = 3$, which is not a dictatorship.

Example 3. Let $n = 3$, $A = \{a, b, c\}$, and consider the SCF f defined as the serial dictatorship with order $(1, 2, 3)$ and lexicographic tie-breaking unless Voter 3 is indifferent between all three alternatives. In that case, the serial dictatorship order is $(2, 1, 3)$.

This SCF is not a dictatorship as the dictator depends on the preferences of Voter 3.

We show later that all serial dictatorships are strongly localized and unanimous. Therefore, localizedness is potentially violated only when switching orders, i.e., when Voter 3 changes her preferences from or to complete indifference. However, localizedness does not impose any constraints when applying such a minimal change (see Definition 5).

Hence, f satisfies localizedness and unanimity without being a dictatorship.

We are now ready to prove the main theorem of this section.

Theorem 3. *Let $m \geq 3$. Then, f is a unanimous and strongly localized SCF on $\mathcal{R}^n$ if and only if it is a serial dictatorship with lexicographic tie-breaking.*

Proof. First, we show that all serial dictatorships with lexicographic tie-breaking $SD_\pi^>$ satisfy strong localizedness and unanimity. For unanimity, note that in profiles R for which $\bigcap_{i \in N} \max(\succsim_i) \neq \emptyset$, applying a serial dictatorship eliminates all alternatives not in $\bigcap_{i \in N} \max(\succsim_i)$. Consequently, tie-breaking chooses an alternative from $\bigcap_{i \in N} \max(\succsim_i)$ and $SD_\pi^>(R) \in \bigcap_{i \in N} \max(\succsim_i)$.

Next, we show that serial dictatorships satisfy strong localizedness. Remember that by the recursive definition of serial dictatorships, agents (and in the end the lexicographic tiebreaking) restrict the set of possible outcomes B until a single alternative is returned. Consider an arbitrary $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ with alternative a . Let $R' := (R_{-i}, \succsim'_i)$. This change has no influence on the elimination of alternatives by voters ranked higher than voter i , therefore i selects from the same set of alternatives B in both R and R' . We need to consider three cases.

First, if $SD_\pi^>(R) = a$ then strong localizedness requires that $SD_\pi^>(R') \in [a]_{\succsim_i} \cup [a]_{\succsim'_i}$. If the minimal change moved a up, then $SD_\pi^>(R) = SD_\pi^>(R')$ since serial dictatorships are monotonic. Otherwise, the minimal change of voter i moved a down. Since $a \in \max(\succsim_i | B)$, it follows that $\max(\succsim'_i | B) \subseteq [a]_{\succsim_i} \cup [a]_{\succsim'_i}$ and therefore $SD_\pi^>(R') \in [a]_{\succsim_i} \cup [a]_{\succsim'_i}$.

Second, if $SD_\pi^>(R) \in ([a]_{\succsim_i} \cup [a]_{\succsim'_i}) \setminus \{a\}$, strong localizedness requires that $SD_\pi^>(R') \in \{a, SD_\pi^>(R)\}$. If the minimal change moved a down then $\max(\succsim'_i | B) \subseteq \max(\succsim_i | B)$ since $a \neq SD_\pi^>(R)$ and $SD_\pi^>(R) \in \max(\succsim_i | B)$. Thus, $SD_\pi^>(R) = SD_\pi^>(R')$. If the minimal change moved a up then $SD_\pi^>(R) = SD_\pi^>(R')$ if $a \notin B$. If $a \in B$, either $\max(\succsim'_i | B) = \{a\}$ and $SD_\pi^>(R') = a$ or $\max(\succsim'_i | B) = \max(\succsim_i | B) \cup \{a\}$. In the latter case, either $SD_\pi^>(R') = a$ or a gets eliminated later in the process resulting in $SD_\pi^>(R) = SD_\pi^>(R')$.

Finally, if $SD_\pi^>(R) \notin [a]_{\succsim_i} \cup [a]_{\succsim'_i}$, strong localizedness requires that $SD_\pi^>(R) = SD_\pi^>(R')$. This follows directly from the fact that $\max(\succsim_i | B) = \max(\succsim'_i | B)$. Thus, serial dictatorships satisfy strong localizedness.

For the reverse direction, we find an order of the voters π and a linear order $>$ on the alternatives such that $f = SD_\pi^>$. Let $m \geq 3$ be fixed. We prove the statement by induction on the number of voters.

Let $N = \{1\}$. Unanimity implies that $f(R) \in \max(\succsim_1)$ for all profiles R and thus, Voter 1 is the dictator. It remains to show that a linear order $>$ on A is used for tie-breaking.

First, note that it is sufficient to define f on all subsets of (top-ranked) alternatives $T \subseteq A$ as alternatives ranked below the top indifference class of the voter do not matter since f is unanimous. Second, for an arbitrary pair of top-ranked alternatives (a, b) , assume without loss of generality, $f(\{a, b\}) = a$. Then, for every $T \subseteq A$ with $a, b \in T$, $f(T) \neq b$ as otherwise, Voter 1 could move all alternatives $c \in T \setminus \{a, b\}$ down one class which would not affect the outcome due to strong localizedness, resulting in $f(\{a, b\}) = b$, a contradiction.

For each $T \subseteq A$, alternative $f(T)$ thus needs to also win when being top-ranked with only one other alternative from T . Choosing $T = A$, we deduce that $f(A)$ has to be

chosen whenever $f(A) \in T$. If $f(A) \notin T$, the same argument applies for $f(A \setminus f(A))$. Iterating this procedure, we get the linear order $f(A) > f(A \setminus f(A)) > \dots$, which proves the statement.

We now assume that when $|N| = n - 1 \geq 1$, the only SCF that satisfies all axioms are serial dictatorships $SD_{\pi'}^>$ with an order over $n - 1$ voters π' and a lexicographic tie-breaking order on the alternatives $>$ when there are $m \geq 3$ alternatives.

Let $N = \{1, \dots, n\}$. Theorem 2 implies that f is a dictatorship when $m \geq 3$. Let voter $d \in N$ be the dictator.

We proceed in two steps. First, we show that f is a serial dictatorship on a subdomain. Then, we reduce the choices f makes on all profiles to the choices made in the subdomain. As a result, we conclude that f is a serial dictatorship.

Let $\mathcal{D} \subset \mathcal{R}^n$ be the subdomain in which the dictator d is indifferent between all alternatives. We now consider f restricted to $\mathcal{D}$ and name this function f' , i.e., $f'(R) = f(R)$ if $R \in \mathcal{D}$, otherwise f' is undefined. We note that f' is strongly localized and unanimous on this subdomain. If the former property is violated, a violation is passed on to f on the entire domain, contradicting our assumption that f is strongly localized. For unanimity, note that the dictator is indifferent between all alternatives and will thus not influence the intersection of top-ranked alternatives and f' inherits unanimity from f . As a result, f' is a strongly localized and unanimous SCF for $n - 1$ voters and $m \geq 3$ alternatives. By induction hypothesis, it is a serial dictatorship with lexicographic tie-breaking. Therefore, there exist orders over the voters π' and over the alternatives $>$ such that $f' = SD_{\pi'}^>$. Let π be the order of voters where dictator d is ranked first, and the remaining voters are ordered according to π' . We have shown that f is a serial dictatorship on $\mathcal{D}$ with the order π and lexicographic tie-breaking according to $>$.

Let $R \in \mathcal{R}^n$. We show that $f(R) = SD_{\pi}^>$. We write $\sim_d$ for the preference relation where the dictator d expresses complete indifference between all alternatives. Also, remember that d is the dictator, and therefore it holds that $f(R) \in \max(\succsim_d)$.

If $R \in \mathcal{D}$, we are done. Otherwise, we consider two cases. For the simpler case, assume $f(R) = f(\sim_d, R_{-d})$. We note that $f(\sim_d, R_{-d}) = SD_{\pi}^>(\sim_d, R_{-d}) = SD_{\pi}^>(R)$ where the last equality follows from $f(R) \in \max(\succsim_d)$. Thus, $f(R) = SD_{\pi}^>(R)$.

For the other case, assume $f(R) \neq f(\sim_d, R_{-d})$. Let $B = A \setminus \max(\succsim_d)$ be the set of alternatives not top-ranked by the dictator d . In particular, $f(R) \notin B$. We now let all voters $N \setminus \{d\}$ change their preferences one after another via minimal changes to the profile where they all bottom rank all alternatives in B , and we call the resulting profile R' . Each time a voter i pushes an alternative $b \in B$ down in her preference relation, the winner can only change to b or remain the same due to strong localizedness. But $b \notin \max(\succsim_d)$ by definition and a change to b would contradict the assumption that d is a dictator. Therefore, $f(R) = f(R')$.

Now, let d change her preferences to $\sim_d$. Since f is strongly localized, $f(\sim_d, R'_{-d}) \in \{f(R)\} \cup B$. Moreover, note that all voters $i \in N \setminus \{d\}$ have $f(R) \succ_i b$ for all $b \in B$. If $f(\sim_d, R'_{-d}) \in B$, all voters can push $f(R)$ to the top of their preferences but due to strong localizedness, the outcome cannot change, leading to a contradiction with unanimity as $f(R)$ is uniquely top-ranked by all agents apart from d . Thus, $f(R) = f(\sim_d, R'_{-d})$.

Finally, $f(\sim_d, R'_{-d}) = SD_{\pi}^>(\sim_d, R'_{-d}) = SD_{\pi}^>(R') = SD_{\pi}^>(R)$. The second to last

equality follows from $SD_\pi^>(\sim_d, R'_{-d}) \in \max(\succsim_d)$ and the last equality holds due to the fact that the outcome of $SD_\pi^>$ does not depend on how voters other than the dictator rank alternatives in B as they are excluded immediately by the dictator in the first step of the serial dictatorship.

We have shown that $f = SD_\pi^>$ for all profiles, which concludes the proof. $\square$

Remark 1. It is straightforward to see that serial dictatorships satisfy Pareto optimality and are also strategyproof for weak preferences (in contrast to dictatorships). In light of Theorem 3 and the original motivation for localizedness, serial dictatorships appear to be the “correct” generalization of dictatorships to weak preferences.

5 Social Decision Schemes

Gibbard (1977, 1978) showed that for both strict and weak preferences, strategyproof and weakly Pareto optimal SDSs are random dictatorships where each voter is chosen as the dictator with a fixed probability.³ Additionally requiring anonymity leads to a characterization of uniform random dictatorships.

In Section 5.1, we show that the random dictatorship theorem still holds when weakening strategyproofness to localizedness and (weak) Pareto optimality to unanimity.

In Section 5.2, we discuss possible generalizations of strong localizedness for SDSs and illustrate that none of them allows for statements in the spirit of Theorem 3.

5.1 Strict preferences

Tanaka (2003) gave an alternative proof of the random dictatorship theorem for strict preferences, which does not use monotonicity. We extend this generalization by showing that Pareto optimality can be weakened to unanimity.

Lemma 2. *Every localized and unanimous SDS g on $\mathcal{L}^n$ is Pareto optimal.*

Proof. Assume for contradiction that the g is a localized and unanimous SDS that fails Pareto optimality. Then, there exists a profile $R \in \mathcal{L}^n$ and alternatives $a, b \in A$ such that $a \succ_i b$ for all $i \in N$ but $g(R, b) > 0$. We now let one voter after the other change preferences by pushing a to the top of her preferences. Let R' be the profile where all voters moved a to the top. By localizedness, the probability of b does not change whenever a voter moves a upwards since $a \succ_i b$. Thus, $0 < g(R, b) = g(R', b)$. However, unanimity implies $g(R', a) = 1$, a contradiction. Hence, g has to be Pareto optimal. $\square$

Applying Lemma 2 and the theorem by Tanaka (2003), we obtain the following theorem, which constitutes a generalization of Theorem 1 to SDSs.

Theorem 4. *Let $m \geq 3$. An SDS g is localized and unanimous on $\mathcal{L}^n$ if and only if g is a mixture of dictatorships.*

³For strict preferences, random dictatorships are completely characterized by these properties, see Figure 1.

Alternatively, the statement can be proven directly as follows. Consider a pair of alternatives a, b in some profile that are adjacent but not top-ranked by voter i . By Lemma 1, we can reorder the preferences of all other voters to be equal to voter i 's preferences except the relative ranking of a and b . Since a and b are not top-ranked, some other alternative c is top-ranked by all voters and selected with probability 1 by unanimity. In particular, this argument shows that the resulting lottery can only change when the top alternative of some voter changes. By a similar argument as in Theorem 1, one can then show that each voter is a dictator with some constant probability on all profiles.

As a consequence, the only mixture of dictatorships, where permuting the voters' identities does not change the outcome (a property known as anonymity), is the uniform random dictatorship.

Corollary 1. *Let $m \geq 3$. The uniform random dictatorship is the only SDS on $\mathcal{L}^n$ that satisfies anonymity, unanimity, and localizedness.*

5.2 Weak preferences

For weak preferences, localizedness and unanimity are insufficient to characterize mixtures of dictatorships as in Theorem 4. This already follows from Example 3. Moreover, it is unclear how to define strong localizedness for SDSs. One way to translate the idea behind strong localizedness for SCFs (Definition 7) to probabilistic outcomes is the following.

Definition 11. An SDS g is strongly localized if for all $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ with alternative a ,

1. $g((R_{-i}, \succsim'_i), b) = g(R, b)$ for all $b \notin [a]_{\succsim_i} \cup [a]_{\succsim'_i}$.
2. $\sum_{b \in ([a]_{\succsim_i} \cup [a]_{\succsim'_i}) \setminus \{a\}} |g((R_{-i}, \succsim'_i), b) - g(R, b)| = |g((R_{-i}, \succsim'_i), a) - g(R, a)|$

The first condition guarantees that the probability assigned to alternatives b with $\succsim_i|_{\{a,b\}} = \succsim'_i|_{\{a,b\}}$ does not change. The second condition ensures that a minimal change with a does not cause a “probability flow” between two alternatives $b, c \neq a$. More precisely, alternatives whose probability increases need to receive this probability mass from a . Analogously, alternatives whose probability decreases need to transfer this probability mass exclusively to a . As a consequence of these two conditions, it is not possible to have two alternatives $b, c \neq a$ where the probability of b increases while the probability of c decreases.

It is straightforward to see that a strongly localized SDS also satisfies localizedness. Furthermore, this definition is consistent with Definition 7 for SCFs. In particular, if $g(R, a) = 0$, only the probability on alternative a might increase.

Unfortunately, Theorem 3 does not generalize to SDSs in the same way as the Gibbard-Satterthwaite theorem generalizes to the random dictatorship theorem.

Theorem 5. *Not every strongly localized and unanimous SDS on $\mathcal{R}^n$ is a mixture of serial dictatorships with lexicographic tie-breaking.*

Proof. We give a counterexample with a single voter and $m = 4$ alternatives. Unanimity implies that the voter is a dictator. Strong localizedness imposes constraints on how probability can flow when the voter changes her top indifference class. Consider the following SDS, where the outcome only depends on the top indifference class of the voter.

$$\begin{array}{cccccc}
\{a, b\} & \{a, c\} & \{a, d\} & \{b, c\} & \{b, d\} & \{c, d\} \\
a & \frac{1}{2}a + \frac{1}{2}c & \frac{1}{2}a + \frac{1}{2}d & \frac{1}{2}b + \frac{1}{2}c & \frac{1}{2}b + \frac{1}{2}d & \frac{1}{2}c + \frac{1}{2}d \\
\\
\{a, b, c\} & \{a, b, d\} & \{a, c, d\} & \{b, c, d\} & \{a, b, c, d\} & \\
\frac{1}{2}a + \frac{1}{2}c & \frac{1}{2}a + \frac{1}{2}d & \frac{1}{2}a + \frac{1}{2}d & \frac{1}{2}c + \frac{1}{2}d & \frac{1}{2}a + \frac{1}{2}d &
\end{array}$$

If there is a single alternative in the voter's top indifference class, this alternative receives probability 1. This SDS satisfies unanimity because it always chooses a mixture of top-ranked alternatives. It is also strongly localized, which can be seen by checking all minimal changes that influence the voter's top indifference class.

We now show that this SDS cannot be represented as a convex combination of linear tie-breaking rules restricted to the voter's top choices. All tie-breaking orders have to rank a above b as a is always selected among $\{a, b\}$. We select $\frac{1}{2}a + \frac{1}{2}c$ among $\{a, b, c\}$. Therefore, exactly half of all tie-breaking orders have a above c . At the same time, the choice from $\{b, c\}$ implies that exactly half of the time, b is ranked above c . If c is ranked above a it is also before b . Thus, the two possible orders for $\{a, b, c\}$ are abc and cab , each occurring with the same probability.

We select $\frac{1}{2}a + \frac{1}{2}d$ among $\{a, b, c, d\}$. Therefore, half of the orders rank a and the other half rank d in the first place. In all rankings with cab , alternative a cannot be ranked first, and the ranking has to be $dcab$. In all other rankings (abc), we need to have $abcd$ as $\frac{1}{2}c + \frac{1}{2}d$ is returned for $\{c, d\}$. Thus, we found that the mixture of tie-breaking rules has to be $\frac{1}{2}abcd + \frac{1}{2}dcab$.

This implies that $\frac{1}{2}b + \frac{1}{2}d$ needs to be selected on the set $\{b, c, d\}$ but our SDS returns $\frac{1}{2}c + \frac{1}{2}d$. All in all, this specific SDS cannot be represented as a mixture of (serial) dictatorships with lexicographic tie-breaking. $\square$

Defining alternate versions of strong localizedness for SDSs and obtaining a similar result as Theorem 3 are interesting directions for future research.

6 Connections to Non-Bossiness and Maskin monotonicity

In this section, we discuss connections to notions of non-bossiness and Maskin monotonicity for SCFs.

6.1 Non-Bossiness

As already mentioned in Section 3, the additional requirements of strong localizedness (compared to localizedness) can also be interpreted as a non-bossiness condition. The

concept of non-bossiness was introduced by Satterthwaite and Sonnenschein (1981) and has been mainly applied to private goods (e.g., in assignment) since then. In particular, their definition is trivially satisfied when considering public goods, e.g., in voting where all voters obtain the same outcome (see Satterthwaite and Sonnenschein, 1981, p. 590).

Saijo et al. (2007) proposed a version for public goods which states that the outcome cannot change if voter i who changed her preferences to $\succsim'_i$, remains indifferent between the old and the new outcome according to her original preferences $\succsim$.⁴ Notably, this notion clashes with unanimity for weak preferences. To see this, consider the case of two alternatives $\{a, b\}$ and a single voter who is indifferent. Without loss of generality, f returns a . If the voter changes to $b \succ a$, then f still needs to return a by non-bossiness, which violates unanimity.

Berga and Moreno (2009) consider a weakened notion of non-bossiness due to Ritz (1985), which additionally requires that the voter is also indifferent under $\succsim'$.⁵ It turns out that strong localizedness is equivalent to localizedness and a local version of weak non-bossiness.

Definition 12. An SCF f satisfies *local weak non-bossiness* if for all $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$, $f(R) \sim_i f(R_{-i}, \succsim'_i)$ and $f(R) \sim'_i f(R_{-i}, \succsim'_i)$ implies $f(R) = f(R_{-i}, \succsim'_i)$.

Proposition 2. *Strong localizedness is equivalent to localizedness in conjunction with local weak non-bossiness for SCFs on $\mathcal{R}^n$.*

Proof. $\Rightarrow$: Assume that f is strongly localized. Proposition 1 already shows that strong localizedness implies localizedness. Furthermore, assume that the prerequisites of Definition 12 are given (minimal change with a). If $f(R) = a$, then $[a]_{\succsim_i} \cap [a]_{\succsim'_i} = a$ and thus, $f(R_{-i}, \succsim'_i) = a$ by strong localizedness and the fact that voter i is indifferent between a and $f(R_{-i}, \succsim'_i)$ under both $\succsim_i$ and $\succsim'_i$. If $f(R) \in ([a]_{\succsim_i} \cup [a]_{\succsim'_i}) \setminus \{a\}$, $f(R_{-i}, \succsim'_i) \neq a$ as voter i cannot be indifferent between $f(R)$ and a for both $\succsim_i$ and $\succsim'_i$. Thus, $f(R) = f(R_{-i}, \succsim'_i)$ by strong localizedness. Otherwise, $f(R) = f(R_{-i}, \succsim'_i)$ holds directly by definition of strong localizedness. All in all, f is locally weakly non-bossy.

$\Leftarrow$: Assume that f is localized, locally weakly non-bossy, and the prerequisites of Definition 7 are given. First, assume that $f(R) \in ([a]_{\succsim_i} \cup [a]_{\succsim'_i}) \setminus \{a\}$. By localizedness, $f(R_{-i}, \succsim'_i) \in [a]_{\succsim_i} \cup [a]_{\succsim'_i}$. If $f(R_{-i}, \succsim'_i) \neq a$, voter i is indifferent between $f(R)$ and $f(R_{-i}, \succsim'_i)$ under both $\succsim_i$ and $\succsim'_i$. By weak non-bossiness, $f(R) = f(R_{-i}, \succsim'_i)$. The other two cases from Definition 7 are directly implied by localizedness. All in all, f is strongly localized. $\square$

Note that the statement also holds for strict preferences, because local non-bossiness is always satisfied in that case.

Weak non-bossiness is too strong to be implied by strong localizedness, as shown by the following example.

⁴An SCF f satisfies *non-bossiness* if $f(R) \sim_i f(R_{-i}, \succsim'_i)$ implies $f(R) = f(R_{-i}, \succsim'_i)$.

⁵An SCF f satisfies *weak non-bossiness* if $f(R) \sim_i f(R_{-i}, \succsim'_i)$ and $f(R) \sim'_i f(R_{-i}, \succsim'_i)$ implies $f(R) = f(R_{-i}, \succsim'_i)$.

Example 4. Consider an instance with one voter, three alternatives $\{a, b, c\}$, and an SCF defined as follows.

$$\begin{array}{cccccc}
 \{a, b, c\} & \{b, c\} \succ a & a \succ \{b, c\} & \{a, c\} \succ b & b \succ \{a, c\} & \{a, b\} \succ c & c \succ \{a, b\} \\
 a & b & c & a & b & a & c \\
 \\
 a \succ b \succ c & a \succ c \succ b & b \succ a \succ c & b \succ c \succ a & c \succ a \succ b & c \succ b \succ a & \\
 b & c & b & b & c & c &
 \end{array}$$

It can be checked that this SCF is strongly localized. However, weak non-bossiness is violated due to the outcomes for the second and third profiles.

Still, every serial dictatorship with lexicographic tie-breaking satisfies weak non-bossiness. If $f(R) \sim_i f(R_{-i}, \succ'_i)$ and $f(R) \sim'_i f(R_{-i}, \succ'_i)$, this means that all alternatives that voter i strictly prefers to $f(R)$, $f(R_{-i}, \succ'_i)$ are excluded by higher-ranked voters whereas $f(R)$ and $f(R_{-i}, \succ'_i)$ are always among their top choices. As a consequence, voter i herself excludes neither $f(R)$ nor $f(R_{-i}, \succ'_i)$ from the set of admissible outcomes. Thus, $f(R) = f(R_{-i}, \succ'_i)$. Consequently, we obtain the following corollary of Theorem 3 and Proposition 2.

Corollary 2. *Let $m \geq 3$. Then, f is a unanimous, localized, and weakly non-bossy SCF on $\mathcal{R}^n$ if and only if it is a serial dictatorship with lexicographic tie-breaking.*

6.2 Maskin Monotonicity

We conclude the paper by discussing localizedness in light of Maskin monotonicity (Maskin, 1999). Maskin monotonicity is a central concept in implementation theory. It requires that, if we consider two preference profiles R, R' and an alternative a such that $a \succ b$ implies $a \succ' b$ for all $b \in A$, then a has to be the outcome in R' if a is the outcome in R . In this section, we discuss the connections between strong localizedness, Maskin monotonicity, and related axioms.

Definition 13 (Maskin, 1999). An SCF f satisfies *Maskin monotonicity* if for all $R, R' \in \mathcal{R}^n$ and $a \in A$, if $f(R) = a$ and $a \succsim_i b \Rightarrow a \succ'_i b$ for all $i \in N$ and all $b \in A$, then $f(R') = a$.

Maskin monotonicity is equivalent to strategyproofness for strict preferences (Muller and Satterthwaite, 1977). When allowing for weak preferences, however, it can be seen that strengthening an alternative by moving it upwards next to the winning alternative cannot change the outcome. As a consequence, serial dictatorships violate Maskin monotonicity. More generally, Saijo (1987) showed that only constant rules satisfy Maskin monotonicity for weak preferences. Subsequently, weak versions of Maskin monotonicity have been proposed for weak preferences. We consider weak Maskin monotonicity as proposed by Berga and Moreno (2009), which augments Maskin monotonicity with an additional condition demanding that alternatives that are *strictly* preferred remain *strictly* preferred.

Definition 14. An SCF f satisfies weak Maskin monotonicity if for all $R, R' \in \mathcal{R}^n$ and $a \in A$, if $f(R) = a$ and for all $i \in N$ and all $b \in A$ with $a \succsim_i b \Rightarrow a \succsim'_i b$ and $a \succ_i b \Rightarrow a \succ'_i b$, then $f(R') = a$.

Serial dictatorships satisfy weak Maskin monotonicity. We establish this fact by showing that weak Maskin monotonicity is equivalent to strong localizedness and monotonicity. For strict preferences, this statement reduces to Maskin monotonicity being equivalent to strategyproofness (Muller and Satterthwaite, 1977). Therefore, we only consider minimal swaps with respect to weak preferences.

Theorem 6. *An SCF f on $\mathcal{R}^n$ satisfies strong localizedness and monotonicity if and only if it satisfies weak Maskin monotonicity.*

Proof. $\Leftarrow$: Let f be an SCF that satisfies weak Maskin monotonicity.

We first show that f is monotonic. Consider an arbitrary $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ in which alternative a is moved up. Let $R' := (R_{-i}, \succsim'_i)$. We need to show that if $f(R) = a$, then $f(R') = a$. This follows directly from weak Maskin monotonicity as for all $b \in A$, $a \succsim_i b \Rightarrow a \succsim'_i b$ and $a \succ_i b \Rightarrow a \succ'_i b$ since we made no change to the profile except strengthening a in voter i 's preference relation. Therefore, f satisfies monotonicity.

We now show f satisfies strong localizedness. Consider an arbitrary $R \in \mathcal{R}^n$, $i \in N$, and $\succsim'_i \in \mathcal{R}$ such that $\succsim'_i$ is a minimal change of $\succsim_i$ with alternative a . Let $R' := (R_{-i}, \succsim'_i)$. We need to consider three cases.

First, if $f(R) = a$ then strong localizedness requires that $f(R') \in [a]_{\succsim_i} \cup [a]_{\succ'_i}$. Assume for contradiction that $f(R') \notin [a]_{\succsim_i} \cup [a]_{\succ'_i}$. Then, for all $i \in N$ and for all $b \in A$ $f(R') \succ'_i b \Rightarrow f(R') \succsim_i b$ and $f(R') \succ'_i b \Rightarrow f(R') \succ_i b$. Consequently, weak Maskin monotonicity implies that $f(R') = f(R) \neq a$ contradicting our assumption $f(R) = a$.

Next, if $f(R) \in ([a]_{\succsim_i} \cup [a]_{\succ'_i}) \setminus \{a\}$ then strong localizedness requires that $f(R') \in \{a, f(R)\}$. If the minimal change moved a down in voter i 's preferences then for all $i \in N$ and for all $b \in A$, $f(R) \succsim_i b \Rightarrow f(R) \succ'_i b$ and $f(R) \succ_i b \Rightarrow f(R) \succ'_i b$. Consequently, weak Maskin monotonicity implies that $f(R) = f(R')$. Otherwise, a was moved up by the minimal change. In this case, assume for contradiction $f(R') \notin \{a, f(R)\}$. Then, for all $i \in N$ and for all $b \in A$, $f(R') \succ'_i b \Rightarrow f(R') \succsim_i b$ and $f(R') \succ'_i b \Rightarrow f(R') \succ_i b$. Therefore, weak Maskin monotonicity implies that $f(R') = f(R)$ contradicting our assumption $f(R') \notin \{a, f(R)\}$.

Finally, if $f(R) \notin [a]_{\succsim_i} \cup [a]_{\succ'_i}$ then strong localizedness requires that $f(R) = f(R')$. This follows directly from weak Maskin monotonicity since the minimal change of a did not alter the relative preferences of $f(R)$ with other alternatives. Thus, weak Maskin monotonicity implies strong localizedness.

$\Rightarrow$: Let f be an SCF that satisfies monotonicity and strong localizedness. We show that f satisfies weak Maskin monotonicity. Consider two profiles $R, R' \in \mathcal{R}^n$ such that $a = f(R)$ and for all $b \in A$ $a \succsim_i b \Rightarrow a \succsim'_i b$ and $a \succ_i b \Rightarrow a \succ'_i b$. We show $a = f(R')$. To this end, we express the change from R to R' as a sequence of minimal changes $R = R_0, \dots, R_K = R'$ such that $a = f(R_k)$ for all $k \in \{0, 1, \dots, K\}$. By assumption, this can be achieved without reinforcing any alternative b against a in the process.

We show by induction that the outcome does not change. By assumption, $f(R_0) = f(R) = a$. For $k \in \{0, \dots, K-1\}$, assume $f(R_k) = a$. We want to prove that $f(R_{k+1}) = a$ where R_{k+1} results from a minimal change of an alternative $b \neq a$ for voter i in R_k . If the relation between b and a does not change, strong localizedness directly implies $f(R_{k+1}) = a$. Otherwise, $b \in [a]_{\succsim_i^k} \cup [a]_{\succsim_i^{k+1}}$ and b is moved down in the minimal change. By strong localizedness, $f(R_{k+1}) \in \{b, f(R^k)\}$. If $f(R_{k+1}) = b$, monotonicity would imply $f(R^k) = b$, a contradiction. Thus, $f(R_{k+1}) = f(R^k) = a$. All in all, $f(R^k) = a$ for all $k \in \{0, \dots, K\}$ and in particular $f(R') = a$ which proves weak Maskin monotonicity of f . $\square$

Theorem 3 and Theorem 6 can be combined to prove that serial dictatorships are the only SCFs that satisfy weak Maskin monotonicity and unanimity. Furthermore, since non-imposition and monotonicity imply unanimity, we can weaken unanimity to non-imposition.

Corollary 3. *Let $m \geq 3$. Then, serial dictatorships with lexicographic tie-breaking are the only SCFs that satisfy weak Maskin monotonicity and non-imposition on $\mathcal{R}^n$.*

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