Exploring auction-based leveled-commitment contracting

Part I: English-type auctioning

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Abstract

A key problem addressed in the area of multiagent systems is the automated assignment of multiple tasks to executing agents. The automation of multiagent task assignment requires that the individual agents *(i)* use a common protocol that prescribes how they have to interact in order to come to an agreement and *(ii)* fix their final agreement in a contract that specifies the commitments resulting from the assignment on which they agreed. This report describes a novel approach to automated task assignment in multiagent systems that is based on an auction-based protocol and on leveled commitment contracting. This approach is applicable in a broad range of realistic scenarios in which knowledge-intensive negotiation among agents is not feasible and in which future environmental changes may require agents to breach their contracts.

1 Introduction

The area of multiagent systems (e.g., [5, 8, 16]), which is concerned with systems composed of technical entities called agents that interact and in some sense can be said to be intelligent and autonomous, has achieved steadily growing interest in the past decade for two major reasons. First, it provides innovative methods and concepts for designing, realizing, and handling modern—distributed, large-scale, dynamic, open, and heterogeneous—information processing systems. The Internet is just the most prominent example of such systems; others are multi-database systems and in-house information systems. Second, it offers useful technology for developing and analyzing models and theories of interactivity among humans. Humans, like other intelligent natural beings, do not function in isolation, but interact in various ways and at various levels; however, the relationships between intelligence and interactivity are still poorly understood.

A key problem addressed in this area is the automated assignment of multiple tasks to executing agents under criteria such as efficiency and reliability. The automation of task assignment requires that the agents *(i)* use a common

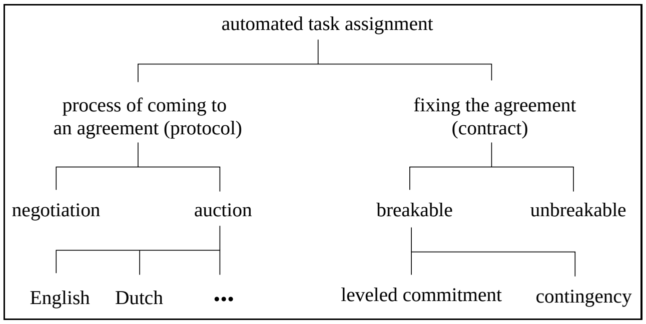

Figure 1: Automated task assignment.

protocol that prescribes how they have to interact in order to come to an agreement on "who does what" and *(ii)* are willing to fix their final agreement in a formal or "legally valid" contract. The protocol concerns the act or process of finding an appropriate task assignment, while the contract concerns the consequences and commitments resulting from the assignment on which the agents agreed. Two standard types of task assignment protocols are negotiation-based protocols (e.g., [3, 7, 15]) and auction-based protocols (e.g., [2]). Examples of widely applied auction protocols are the English auction, the Dutch auction, and the Vickrey auction (e.g., [10]). Compared to negotiation-based protocols, auction-based protocols show several distinct and advantageous features: they are easily implementable, they enforce an efficient (low-cost and/or low-time) assignment process, and they guarantee an agreement even in scenarios in which the agents possess only very little domain- or task-specific knowledge. Two standard types of task assignment contracts are unbreakable contracts (e.g., [6, 11, 12]) and breakable contracts, where common forms of breakable contracts are contingency contracts (e.g., [9]) and leveled commitment contracts (e.g., [1, 4, 13, 14]). Compared to unbreakable contracts, breakable contracts offer a significant advantage: they allow agents acting in dynamic environments to flexibly react upon future environmental changes that make existing contracts unfavorable. Figure 1 summarizes this rough overview of available approaches to automated task assignment.

This report describes work that aims at investigating automated task assignment in multiagent systems that combines auction-based protocols and breakable contracts. More specifically, an approach to multiagent task assignment is introduced that is based on an English-type auction protocol *and* leveled commitment contracting. The advantage of such a combination is that it is applicable in a very broad range of realistic scenarios in which knowledge-intensive negotation among agents is not feasible and in which future environmental changes may require agents to breach their contracts. To our knowledge such a combination has not been investigated so far. This work is structured as follows. Section 2 describes the contracting framework, and Section 3 explains our variation of the English auction. Section 4 presents experimental results that indicate the benefits of this approach. Finally, Section 5 concludes the report with a brief overview of potential research directions evoked by the idea of combining auctioning and leveled commitment contracting.

2 Auction-Based Contracting (ABC)

We consider a group of agents that contains two different types of business partners. *Contractors* CR_i $(i = 1 \dots m)$ who offer a unique task i and *Contractees* CE_j $(j = 1 \dots n)$ who are willing to execute tasks. A contractor CR_i is capable of executing task i by himself for his prime costs[1] $C[CR_i]$. A contractee CE_j is able to do each task i for $C[CE_j, i]$.

We assume that contractees can accomplish tasks cheaper than contractors by defining two intervals.

$$\begin{aligned} \forall i : C[CR_i] &\in [cr_{min}, cr_{max}] \\ \forall j, i : C[CE_j, i] &\in [ce_{min}, ce_{max}] \\ ce_{max} &\leq cr_{min} \end{aligned}$$

This ensures that both, contractors and contractees, are interested in signing contracts with each other.

Pursuing conflicting goals, both types of agents are "true capitalists": contractors intend to pay the lowest feasible price for a task, while contractees try to earn as much money as possible.

During an auction round, each contractor offers his task, where the contractor sequence randomly varies from round to round. Applying an auction-based protocol the agents then come to an agreement which contractee will execute the task. A contractee is only able to accept *one* task per round. For this reason, we consider two basic types of contract obligation: *full commitment* (a contractee has to stay with the first deal he made) and *leveled commitment* (contractors can breach contracts by paying a fine $Penalty_j$ to the concerning contractor CR_i).

We investigate two types of penalty. The first is defined as a fraction of the contract value $P[i]$, the second is a fraction of the contractor's task costs.

Price penalty: $Penalty_j = ppr \cdot P[i]$

Cost penalty: $Penalty_j = cpr \cdot C[CR_i]$

ppr and *cpr* are constants called price penalty rate and cost penalty rate, respectively. After buyer CE_j and seller CR_i negotiated a price $P[i]$ for task i, profits of both agent types are defined as follows.

$$\begin{aligned} CR_i : Profit_i &= C[CR_i] - P[i] \\ CE_j : Profit_j &= P[i] - C[CE_j, i] - PenaltySum_j \end{aligned}$$

$PenaltySum_j$ is the sum of penalties CE_j paid during one round.

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[1] All prices and bids are integers.

3 English-type auction

The ABC system described in the previous section works with any (auction-based) protocol that defines how the agents have to interact to come to an agreement. The protocol investigated in this work is a variation of the English auction.

Whenever a contractor announces a task, interested contractees calculate bids and inform the announcing contractor. The contractees then continuously lower their bids, and the contractee who eventually offers the lowest bid is declared the auction winner. The lowest bid is the *price* of the announced task. This kind of auctioning can be viewed as an "inverse variant" of the standard *English (first-price open-cry) auction* in which the contractees successively *raise* the prices they are willing to pay. The contractees are allowed to decommit from a contract whenever there is a more profitable task announcement by simply paying a decommitment penalty to the corresponding contractor. The penalties are assumed to be specified in the contracts. In particular, they are assumed to be variable and not conditioned on future events. These kinds of breakable contracts are known as *leveled commitment contracts*, in contrast to contingency contracts.

Figure 2 shows the basic framework of the auction. We would like to mention two important notes concerning the line marked with an asterik. First, the set of contractees must be passed through in random order because there might be two or more equal winning bids. In this case, the winner has to be picked randomly. Secondly, `CE` (the leading contractee) has to be prevented from underbidding his own bid by removing him temporarily from the set of bidders.

We assume that the number of contractees is higher than the number of contractors ($n > m$) to ensure that at least two bidders participate in each auction.

3.1 Bidding Details

There is a whole spectrum of possible bidding strategies. The realization described in the following has been chosen because it is intuitively clear, easily extensible, and efficiently realizable. Whenever a contractor CR_i initiates a new auction by announcing his task, each potential contractee CE_j calculates his *opening bid*. This calculation is done as follows. If CE_j is not already involved in another contract in the current auction round, then his opening bid is given by

$$Bid_j = (1 + dp_{ji}) \cdot C[CE_j, i] \tag{1}$$

where dp_{ji} is a variable factor called desired profit (of contractee CE_j w.r.t. the tasks announced by contractor CR_i). Whenever a contractee CE_j wins an auction for a task announced by a contractor CR_i with his opening bid, he raises the factor dp_{ji} according to

$$dp_{ji} = (1 + IncreaseInit_j) \cdot dp_{ji} \tag{2}$$

where $IncreaseInit_j$ is a contractee-specific constant. This ensures that contractees who are successful with their opening bids will start with higher bids

```
FOR Round = 1 TO Max_Rounds
  CR_Set := {CR_i: i = 1 ... m};
  CE_Set := {CE_j: j = 1 ... n};
  WHILE CR_Set ≠ {} AND CE_Set ≠ {}
    CR := Choose_random(CR_Set);
    CR_Set := CR_Set \ CR;
    Min_Bid := Min_Bid_temp := maxcr;
    FOR Bid_Cycle = 1 TO Max_Bid_Cycle
      FOREACH c ∈ (CE_Set \ CE)                    // *
        bid := c.Bid(CR.Task,Min_Bid,Bid_Cycle);
        IF bid<Min_Bid_Temp
          Min_Bid_Temp := bid;
          CE := c;
        ENDIF
      ENDFOREACH
      EXIT IF Min_Bid = Min_Bid_Temp;
      Min_Bid := Min_Bid_Temp;
    ENDFOR
    Contract(CR,CE,Min_Bid);
    IF Commitment = full THEN CE_Set = CE_Set \ CE;
  ENDWHILE
ENDFOR
```

Figure 2: Conception of the English-type auction and contracting framework.

in future auctions and thus try to further increase their profits. The situation is somewhat more sophisticated if CE_j is already involved in a contract signed with another contractor CR_k. In this case CE_j additionally takes into consideration the difference $P[k] - C[CE_j, k]$ (i.e., his potential gain from the already existing contract) and the penalty $Penalty_j$ (i.e., the penalty he would have to pay for decommitting from this contract). Formally, under the assumption that CE_j is already committed to CR_k in the current auction round, CE_j calculates his opening bid for a task announced by CR_i as follows:

$$\begin{aligned} Bid_j &= \max\{(1 + dp_{ji}) \cdot C[CE_j, i], \\ &\quad C[CE_j, i] + P[k] - C[CE_j, k] + Penalty_j\} \end{aligned} \tag{3}$$

where dp_{ji} is defined as above.

Based on the lowest bid $MinBid_i$ currently available, each contractee CE_j calculates and submits his new bid (called a *regular bid* in contrast to an opening bid) as follows. If CE_j is not already committed to another contractor, the difference between the current minimal bid and CE_j's prime costs for tasks offered by contractor CR_i is calculated.

$$Gap_j = MinBid_i - C[CE_j, i] \tag{4}$$

If Gap_j is positive, CE_j's new bid is given by

$$Bid_j = MinBid_i - rr_j \cdot Gap_j \quad . \tag{5}$$

where $rr_j \in [0, 1]$ is a constant called reduction rate. If $Gap_j \leq 0$, CE_j quits the current auction.

If CE_j is already involved in another contract with a contractor CR_k, then Gap_j is given by

$$\begin{aligned} Gap_j &= MinBid_i - (C[CE_j, i] \\ &\quad + P[k] - C[CE_j, k] + Penalty_j) \quad . \end{aligned} \tag{6}$$

The calculation and submission of regular bids is iterated until the winner of the current auction is determined. (Note that according to the above definitions a contractee decommits from a contract only if the new contract would result in a higher profit.)

In the full commitment setting, a contractee can sign at most one contract per auction round. After a contractee signed a contract, he cannot join any auctions in the same round. In this variant the bids are calculated according to the formulas 1, 5 and 4 (formulas 3 and 6 are not applicable in this variant).

4 Experimental Results

All results presented in this section are based on the following parameter setting (for all i and j): $dp_{ji} = 1.0$ (i.e., all contractees intend to make 100% profit), $IncreaseInit_j = 0.1$, $rr_j = 0.2$, and `Max_Bid_Cycle` $= 32$ (this value has never been reached in our experiments). At the beginning of each round none of the potential contractees is involved in a contract and all penalties $Penalty_j$ are set to zero. Other parameters are chosen as described below. In the following several scenarios are investigated, differing in the number of contractors and contractees.

4.1 1 Contractor and 2 Contractees ("1+2 Scenario")

Figure 3 shows a simple scenario consisting of just one contractor and two contractees how the price evolves in 20 consecutive auctions. Because the prime costs of contractee CE_2 are considerably lower than those of contractee CE_1, CE_2 wins the first eleven auctions with his opening bid. As a consequence, CE_2 increases his opening bid in each new auction in order to maximize his profit. In round 12 CE_1 is able to undercut the opening bid of CE_2, resulting in a second bid cycle in which both contractees submit regular bids. The price remains constant in the subsequent auction rounds. This illustrates that the bidding mechanism described in 3.1 realizes what is intuitively expected from an (English-type) electronic auction procedure.

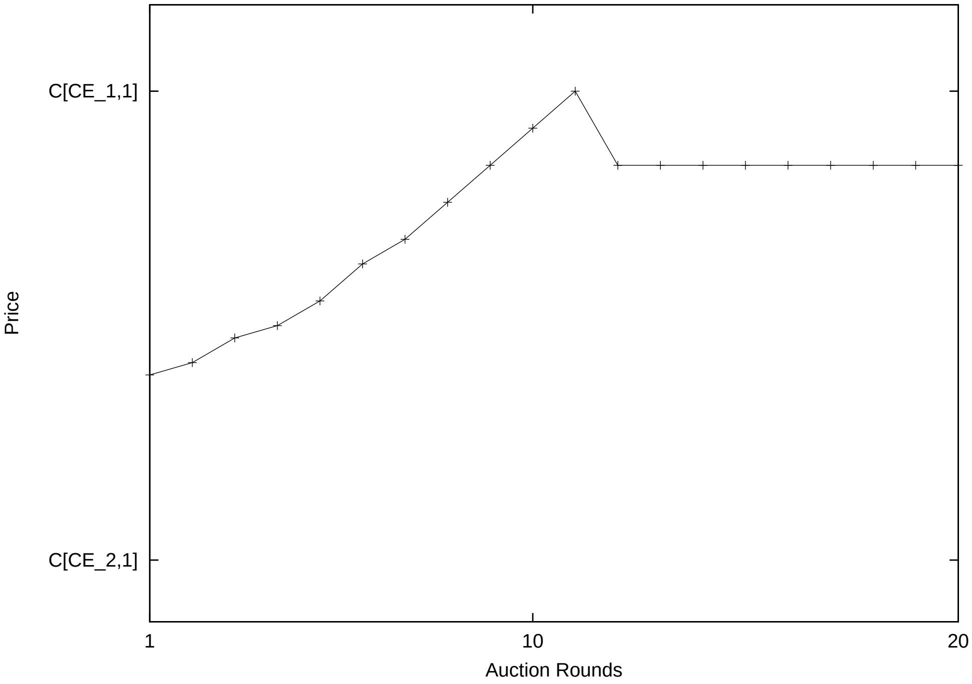

Figure 3: Price development in a simple 1+2 scenario.

4.2 3 Contractors and 4 Contractees ("3+4 Scenario")

Table 1 shows a cost scenario for three contractors and four contractees. The table entries (i.e., the agents' prime costs $C[CR_i]$ and $C[CE_j, i]$) are chosen from the intervals defined by the parameters $ce_{min} = 10$, $ce_{max} = 99$, $cr_{min} = 100$, and $cr_{max} = 200$.

	Task 1	Task 2	Task 3
CR_1	196	–	–
CR_2	–	193	–
CR_3	–	–	115
CE_1	42	68	53
CE_2	22	46	46
CE_3	24	27	59
CE_4	12	11	19

Table 1: Cost table for the 3+4 scenario.

Bid cycle	Bid CE_1	CE_2	CE_3	CE_4
1	84	44	48	44
2	43	–	40	–
3	–	36	–	–
4	–	–	33	–
5	–	30	–	–
6	–	–	28	–
7	–	26	–	–
8	–	–	25	–
9	–	24	–	–
10	–	–	–	–

Table 2: Typical bid development within an auction in the 3+4 scenario.

Table 2 is intended to give an idea how the bids typically evolve in an auction in this scenario. In the concrete auction shown in this table CR_1 is offering his task. CE_4's lowest possible bid is 44, because he is already committed to another contractor and thus would have to pay a penalty for decommitting. After ten bid cycles, CE_2 turns out to be the winner; CR_1 pays a price of 24, which means that CE_2's profit is 2 (i.e., final price minus prime costs).

Figures 4, 5, 6, 7, 8, 9, and 10 show how much profit the contractees accumulated in 100 auctions for different commitment levels (full, three different price penalty rates, and three different cost penalty rates). Note that contractees that made no profit at all are not visible in the "accumulated profit" figures. A key observation with these data is that leveled commitment contracting and full commitment contracting significantly differ in the resulting profit profiles. In particular, leveled commitment contracting turns out to be much fairer than full commitment contracting in that contractees having lower prime costs can effectively make more profit than contractees having higher prime costs. This can be best seen by taking a closer look on the profit profiles of CE_4 and CE_1. As Figure 4 shows, under the full commitment condition CE_4 makes less profit than some other contractee (CE_3) although he is the "best" among all contractees (he can accomplish each task for the cheapest price), and CE_1 makes some profit although he is the "worst" among all contractees. The situation is completely different under leveled commitment conditions; for instance, for a price penalty rate of 0.25 CE_4 makes the highest profit compared to other contractees, while CE_1 makes no profit at all (Figure 7). A second key observation is that these fairness effects are correlated with the level of commitment; in particular, the lower the penalty rates are, the more flexible and the fairer is the contracting (Figures 4 to 10). A third key observation is that there is *no* remarkable difference between price- and cost-oriented penalty (fairness effects can be achieved with both); this indicates that the choice of the penalty mode is not crucial, as long as the penalty mode chosen allows to flexibly decommit from contracts (Figures 5 and 7 versus Figures 9 and 10).

Figures 11, 12, 13, 14, 15, 16, and 17 show how the prices develop under different commitment levels. (Prices for tasks not sold in an auction round are assumed to be zero in these figures; this ensures that only prices paid by the contractors are taken into consideration.) Given these data, a fourth key observation is that leveled commitment contracting, compared to full commitment contracting, results in a significant price pressure and thus typically to lower prices. The reason for this is that even contractees already involved in other contracts contribute to the decrease of task prices whenever they participate in auctions. For instance, the prices for the tasks 2 and 3 are much lower compared to full commitment contracting because now contractee CE_4 (the "best" contractee) participates in auctions even after having signed a contract (provided that CE_4 has the chance to make more profit despite the penalty he would have to pay for breaching an already existing contract).

4.3 3 Contractors and 6 Contractees ("3+6 Scenario")

In order to investigate what happens if the competition increases, two additional contractees were added to the 3+4 scenario, resulting in the scenario shown in Table 3. Again the prime costs were chosen from the intervals defined by the parameters $ce_{min} = 10$, $ce_{max} = 99$, $cr_{min} = 100$, and $cr_{max} = 200$. The

	Task 1	Task 2	Task 3
CR_1	196	–	–
CR_2	–	193	–
CR_3	–	–	115
CE_1	42	68	53
CE_2	22	46	46
CE_3	24	27	59
CE_4	12	11	19
CE_5	31	64	37
CE_6	65	24	55

Table 3: Cost table for the 3+6 scenario.

results for the 3+6 scenario are summarized in the Tables 4 and 5. (Figures showing the detailed price and profit curves for this scenario are not included for reasons of limited space.) These results show, in particular, that increased competition results in lower prices and therefore lower profits of the contractees and higher profits of the contractors. This observation further indicates that the computational approach described in the preceding section in fact realizes what is intuitively expected by "English-type leveled commitment contracting."

It is interesting to discover, that CE_4 is capable of reducing the profit of most of the competitors in both leveled commitment environments, sometimes even without breaking a single contract. Due to the fact, that CE_4 still joins auctions although he already signed a contract, CE_4 pushes down the prices of other deals.

Scenario	Commitment			Accumulated Profit			
				CR_1	CR_2	CR_3	$\sum_i CR_i$
3+4	full	(no penalty)		16,872	15,097	6,537	38,506
	leveled	price penalty	ppr=1.00	16,913	15,493	6,829	39,235
			ppr=0.50	12,357	9,829	6,972	29,158
			ppr=0.25	11,679	6,879	6,996	25,554
		cost penalty	cpr=0.15	16,750	15,621	6,540	38,911
			cpr=0.10	16,920	14,340	6,905	38,165
			cpr=0.05	11,934	11,560	7,087	30,581
3+6	full	(no penalty)		17,368	16,810	7,179	41,357
	leveled	price penalty	ppr=1.00	17,352	16,768	7,378	41,498
			ppr=0.50	17,333	16,786	7,909	42,028
			ppr=0.25	11,397	11,502	7,989	30,888
		cost penalty	cpr=0.15	17,360	16,750	7,348	41,458
			cpr=0.10	17,348	16,792	7,424	41,564
			cpr=0.05	12,017	16,843	7,958	36,818

Table 4: Contractors' profits accumulated in 100 rounds in the 3+4 and 3+6 scenarios for different commitment levels.

4.4 32 Contractors and 40 Contractees ("32+40 Scenario")

A number of experiments with different contractor/contractee scenarios had been conducted. As an example of a larger scenario, here some of the data obtained for a market consisting of 32 contractors and 40 contractees are shown in Table 6. (The prime cost intervals for this scenario were defined by $ce_{min} = 100$, $ce_{max} = 999$, $cr_{min} = 1000$ and $cr_{max} = 2000$.) The experiments showed that the key observations mentioned hold for small as well as large-scale scenarios. In particular, they showed that the auctioning mechanism described in this paper works robust over a very broad range of parameter settings.

5 Conclusions

Automated task assignment that combines auction-based protocols and leveled commitment contracting defines a new field of research in the area of multiagent

Scenario	Commitment			Broken	Accumulated Profit						
					CE_1	CE_2	CE_3	CE_4	CE_5	CE_6	$\sum_j CE_j$
3+4	full	(no penalty)		–	54	622	2,005	1,497	–	–	4,178
	leveled	price penalty	ppr=1.00	1	0	498	1,441	1,436	–	–	3,375
			ppr=0.50	66	0	451	885	1,926	–	–	3,262
			ppr=0.25	72	0	348	742	2,103	–	–	3,193
		cost penalty	cpr=0.15	0	82	680	1,387	1,428	–	–	3,577
			cpr=0.10	9	0	576	1,077	1,452	–	–	3,105
			cpr=0.05	65	0	354	548	1,984	–	–	2,886
3+6	full	(no penalty)		–	0	130	0	1,052	668	192	2,042
	leveled	price penalty	ppr=1.00	0	0	144	0	1,066	592	171	1,973
			ppr=0.50	0	0	147	0	1,110	44	185	1,486
			ppr=0.25	60	0	106	0	1,279	0	132	1,517
		cost penalty	cpr=0.15	0	0	120	0	1,062	588	222	1,992
			cpr=0.10	0	0	128	0	1,062	567	172	1,929
			cpr=0.05	35	0	95	0	1,076	6	163	1,340

Table 5: Number of broken contracts and contractees' profits accumulated in 100 rounds in the 3+4 and 3+6 scenarios for different commitment levels.

Commitment			Broken	Accumulated Profit	
				$\sum_j CR_i$	$\sum_j CE_j$
full	(no penalty)		–	21,790,126	578,606
leveled	price penalty	ppr=0.25	1519	20,118,475	375,312
	cost penalty	cpr=0.05	605	21,303,197	439,949

Table 6: Number of broken contracts and contractors' and contractees' profits accumulated in 500 rounds in the 32+40 scenario for different commitment levels.

systems. The work described in this paper just does the first steps toward a comprehensive understanding of the limitations and benefits of such a combination. Some examples of open research issues that need to be addressed in the future are the following:

- The extension of the proposed approach toward scenarios in which both the contractees and the contractors are allowed to breach contracts.
- The extension toward parallel auctions.
- The extension toward multi-unit and combinatorial auctions.
- The extension toward learning agents (besides learning parameters like rr_{ji}, it would be possible to learn information about other agents, e.g. the costs $C[CE_j, i]$ of fellow contractors) and more adaptive protocols.
- The exploration of other bidding mechanisms and formulas than those described in 2.3.
- The use of other standard auctioning protocols such as Dutch and Vickrey auctions in combination with leveled commitment contracting, and the comparison of the different variants.

We think that the importance of automated task assignment in multiagent systems, the broad applicability range of multiagent task assignment based on auctioning and leveled commitment contracting, and the encouraging initial experimental results and key observations reported in this paper justify to explore these and related open issues. Our current work concentrates on the last mentioned item.

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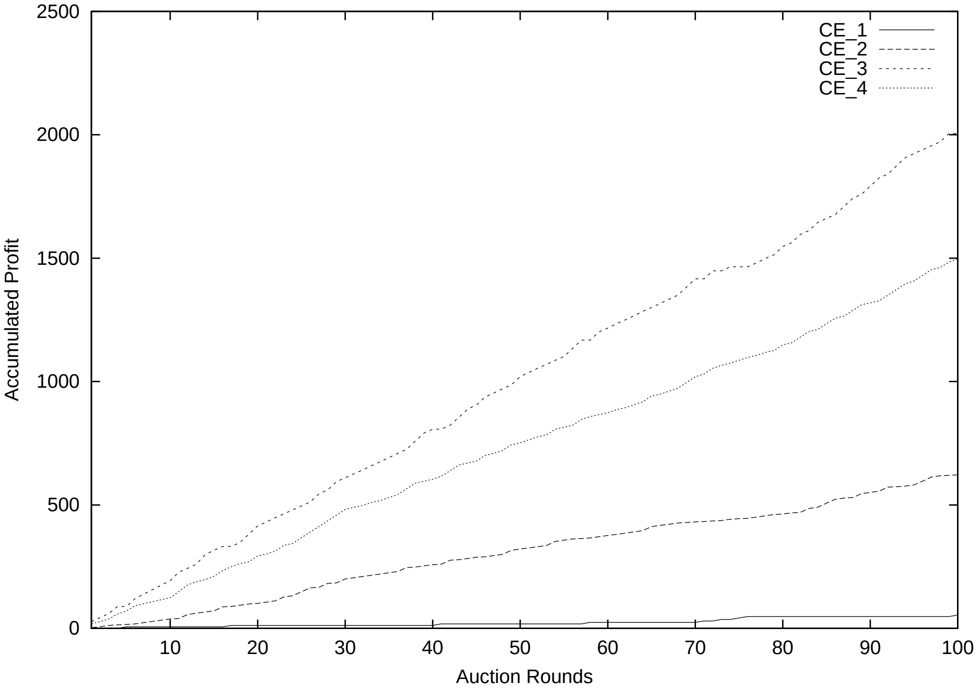

Figure 4: Accumulated profit in the 3+4 scenario with full commitment contracting.

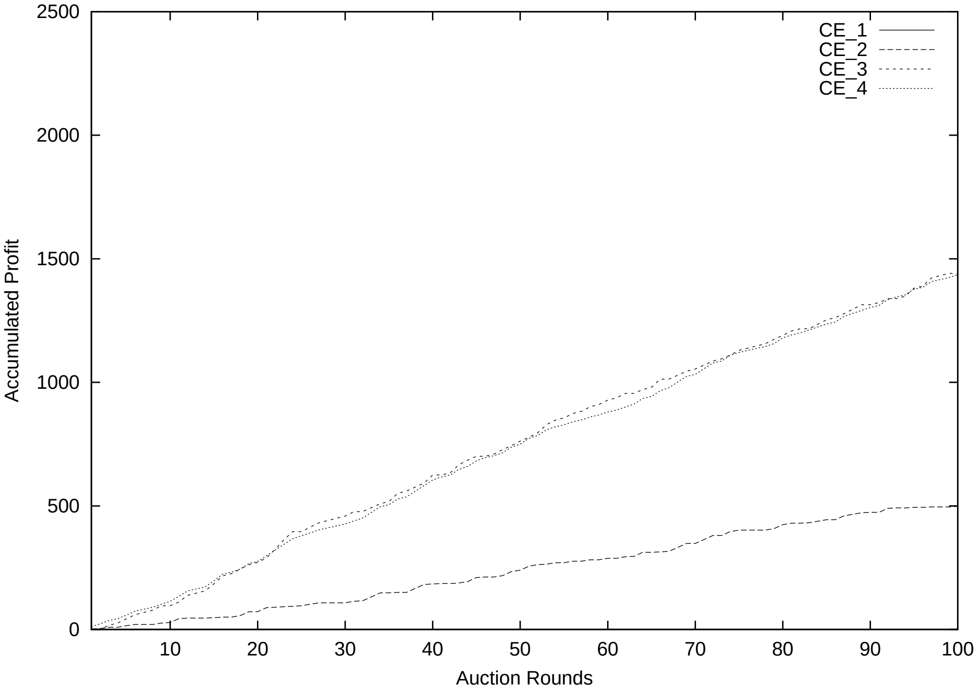

Figure 5: Accumulated profit in the 3+4 scenario with price penalty $ppr = 1.00$.

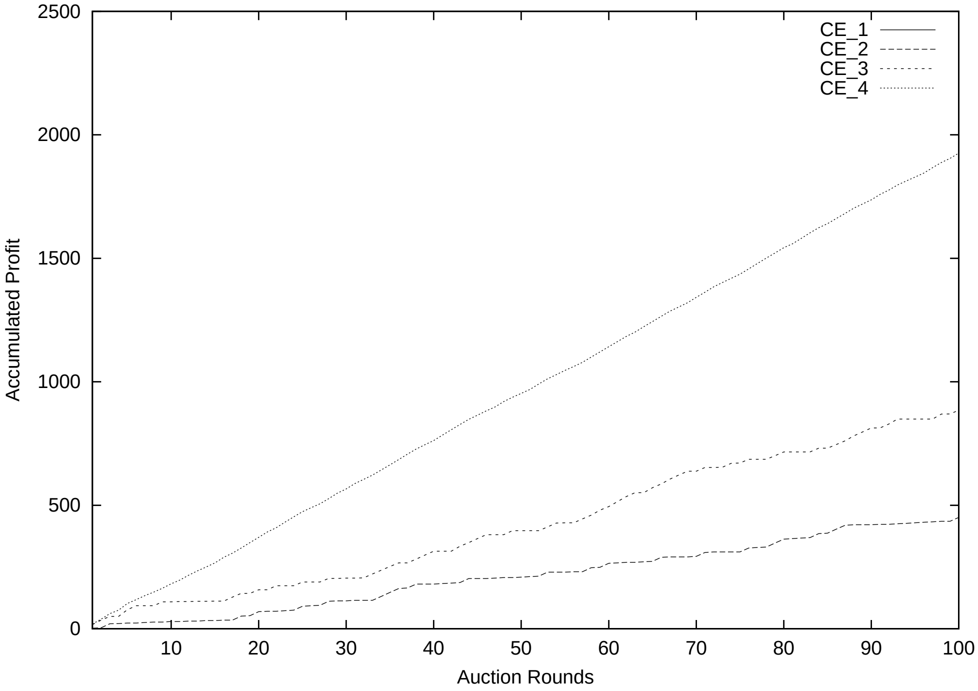

Figure 6: Accumulated profit in the 3+4 scenario with price penalty $ppr = 0.50$.

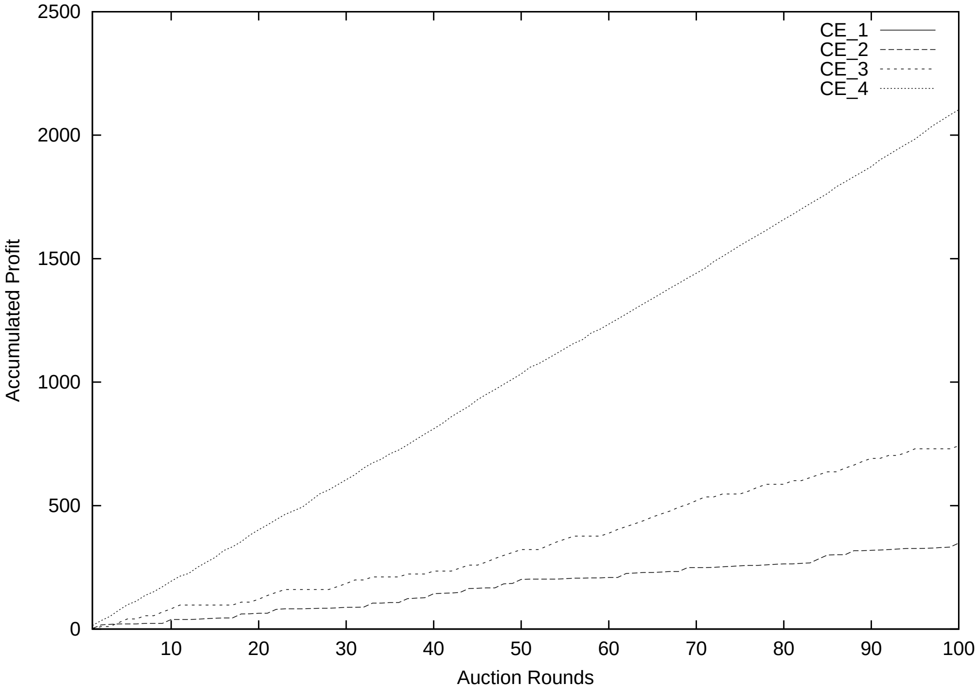

Figure 7: Accumulated profit in the 3+4 scenario with price penalty $ppr = 0.25$.

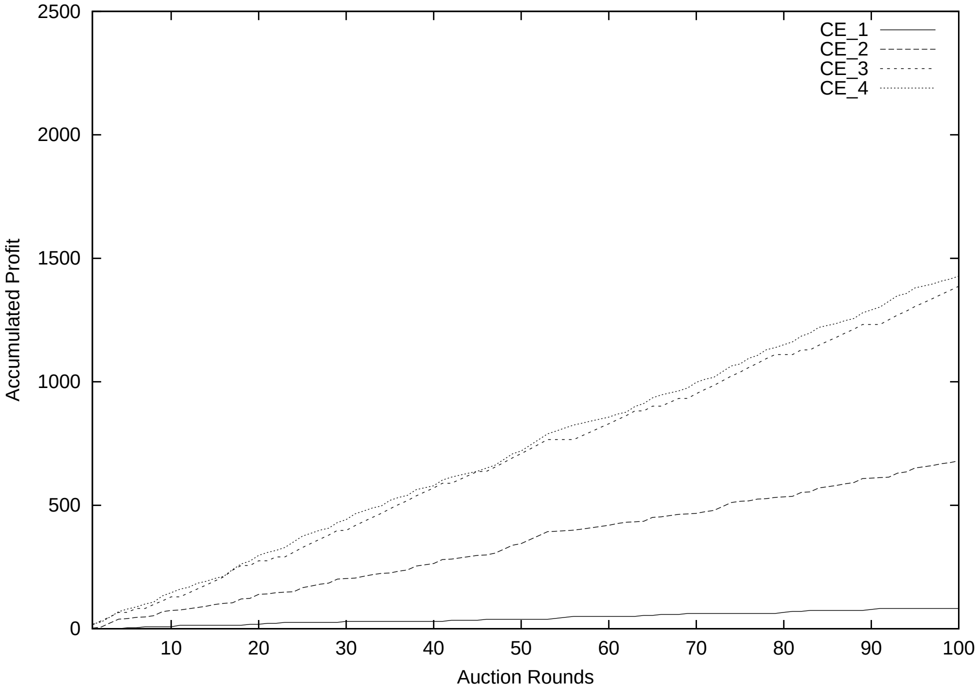

Figure 8: Accumulated profit in the 3+4 scenario with cost penalty $cpr = 0.15$.

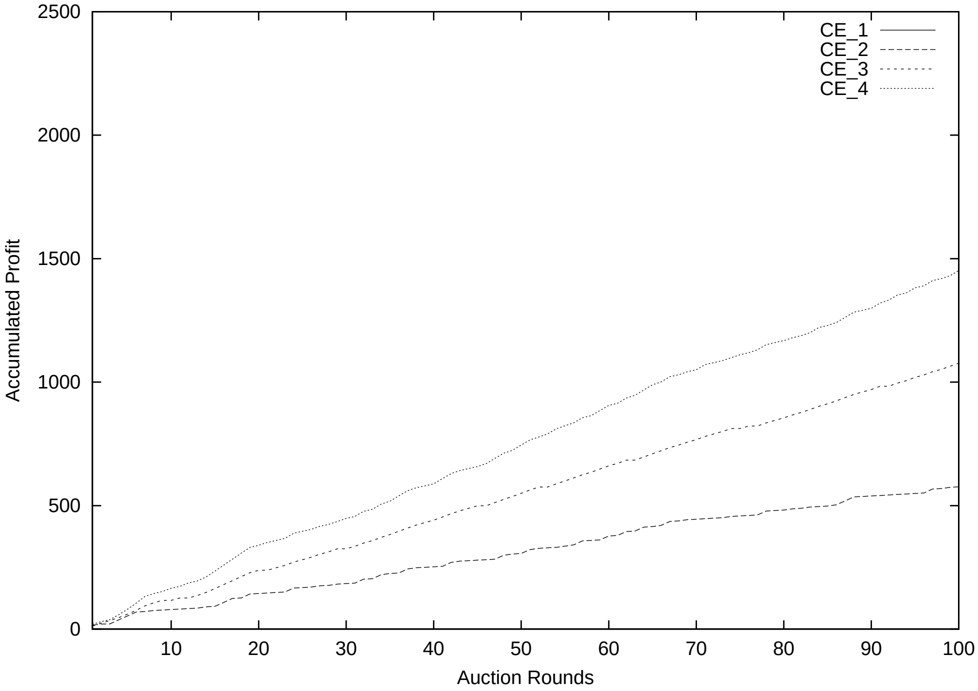

Figure 9: Accumulated profit in the 3+4 scenario with cost penalty $cpr = 0.10$.

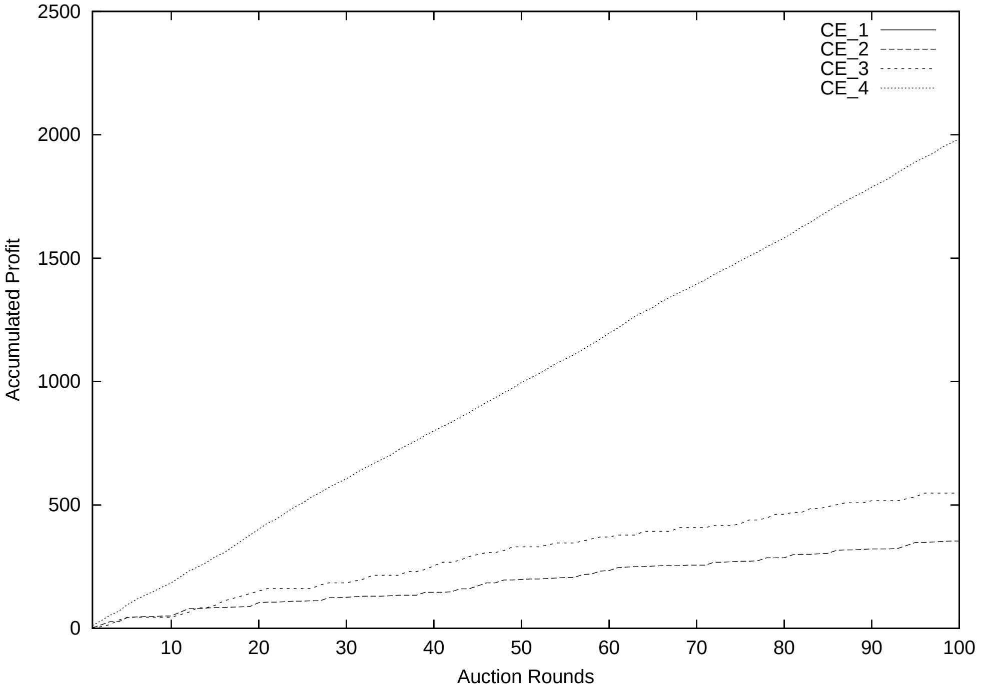

Figure 10: Accumulated profit in the 3+4 scenario with cost penalty $cpr = 0.05$.

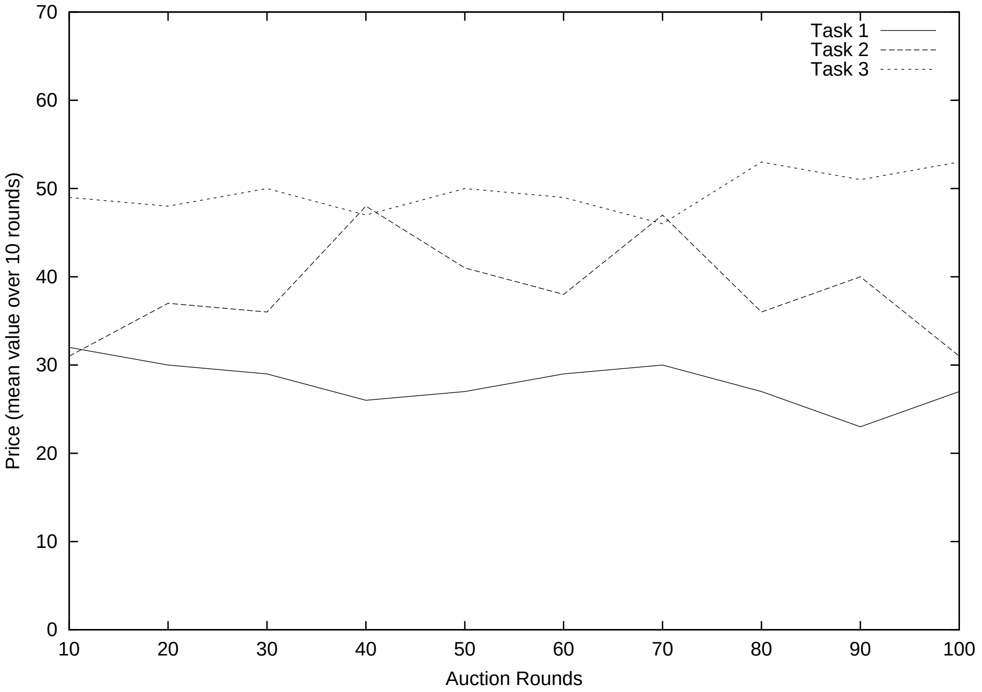

Figure 11: Price development in the 3+4 scenario with full commitment contracting.

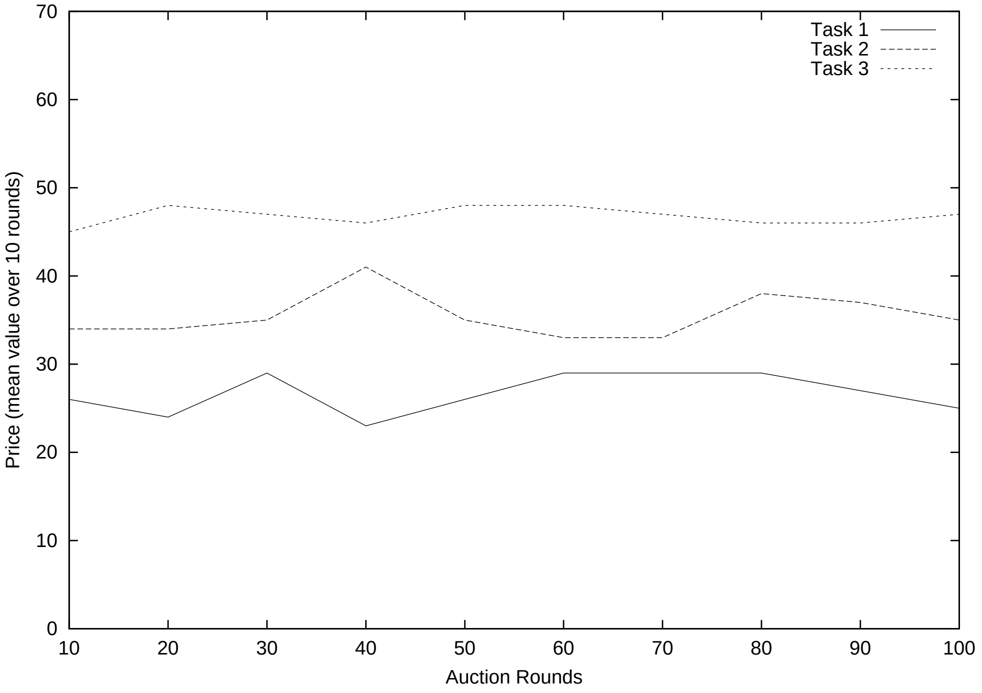

Figure 12: Price development in the 3+4 scenario with price penalty $ppr = 1.00$.

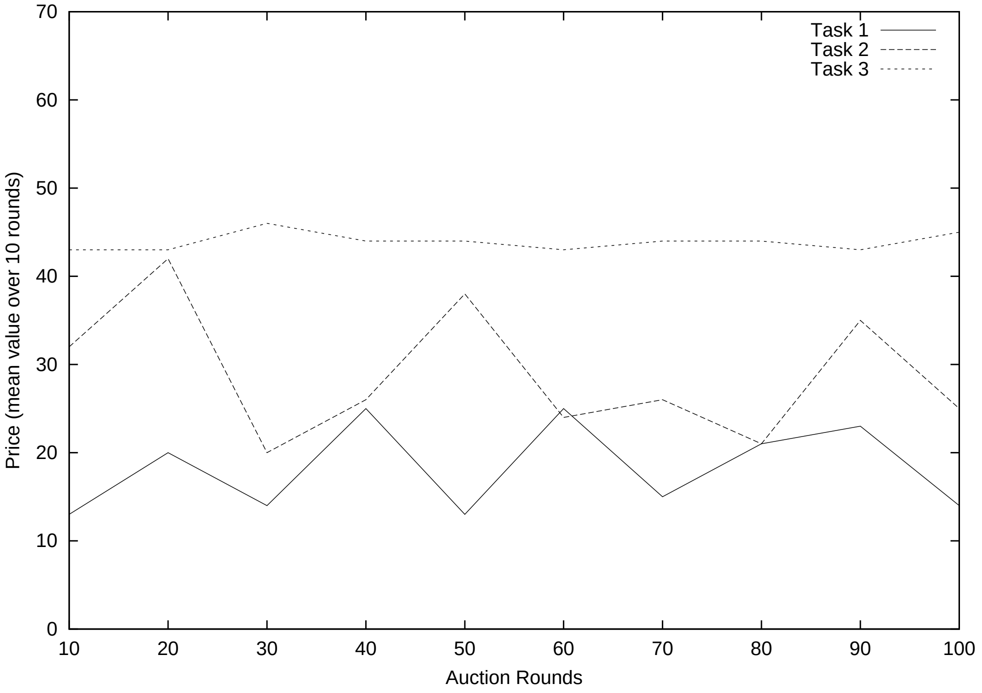

Figure 13: Price development in the 3+4 scenario with price penalty $ppr = 0.50$.

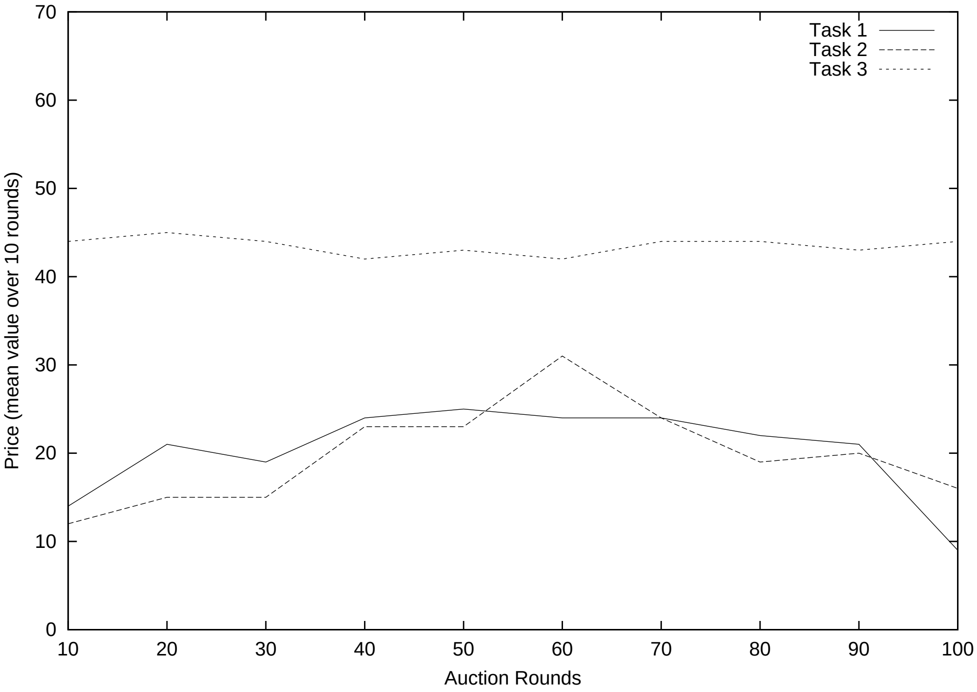

Figure 14: Price development in the 3+4 scenario with price penalty $ppr = 0.25$.

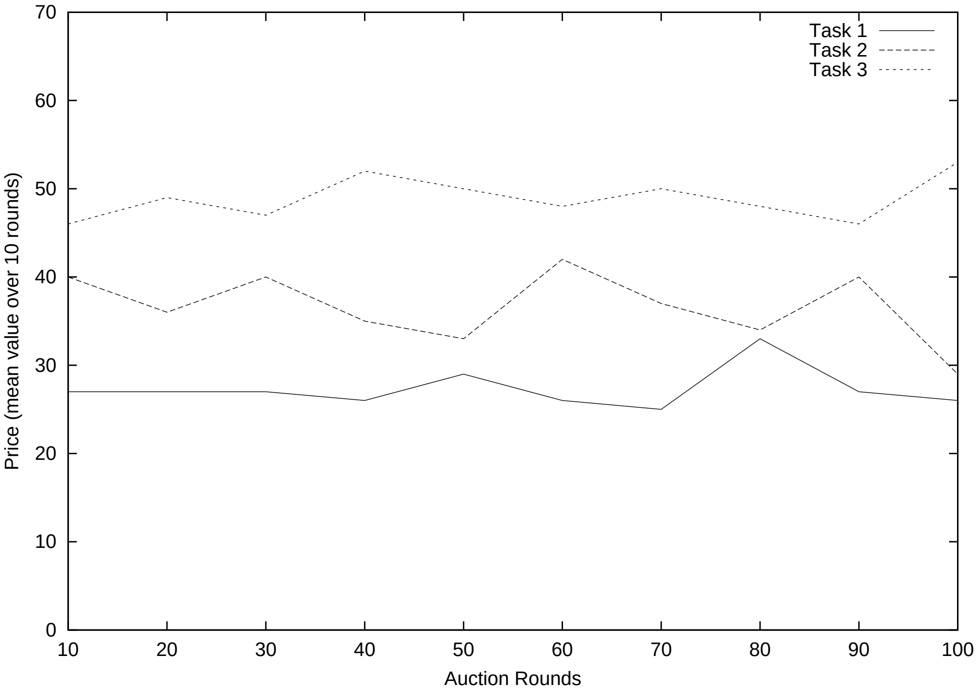

Figure 15: Price development in the 3+4 scenario with cost penalty $cpr = 0.15$.

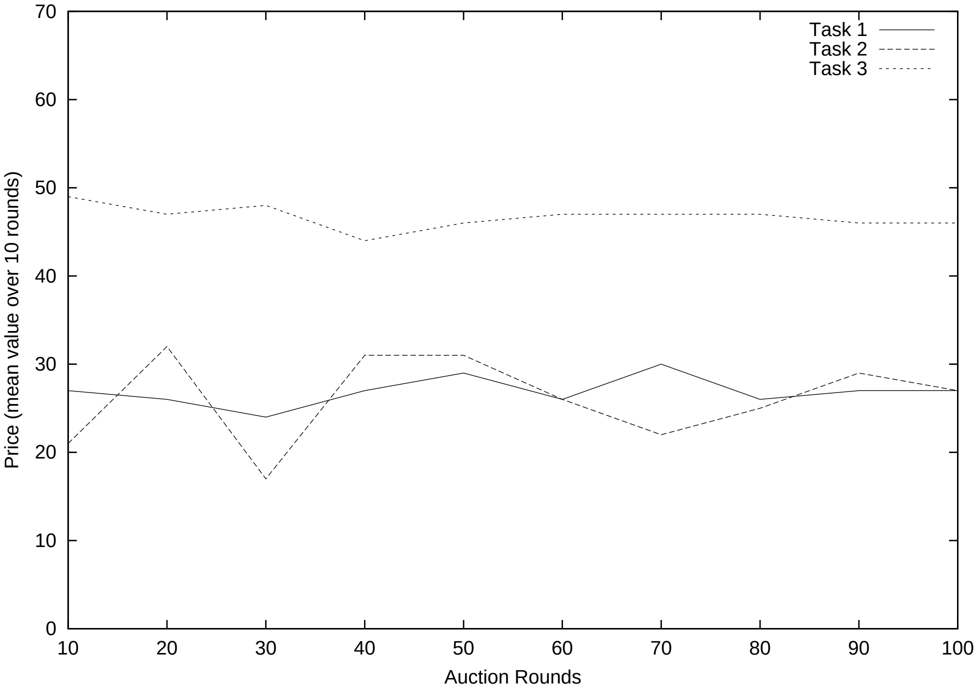

Figure 16: Price development in the 3+4 scenario with cost penalty $cpr = 0.10$.

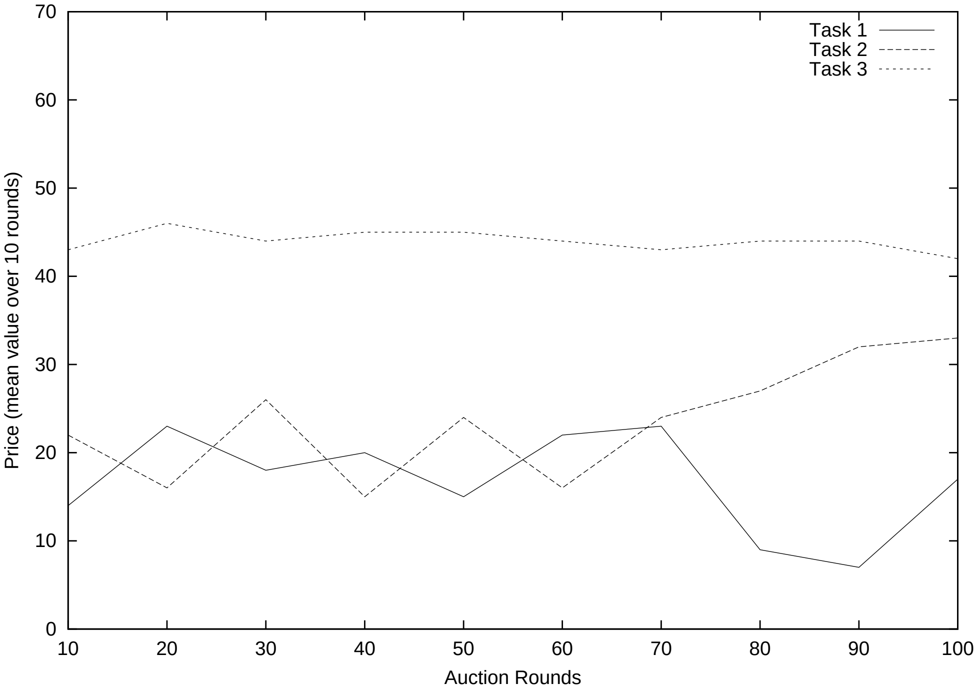

Figure 17: Price development in the 3+4 scenario with cost penalty $cpr = 0.05$.