Efficiently Computing Nash Equilibria in Budget-Aggregation Games

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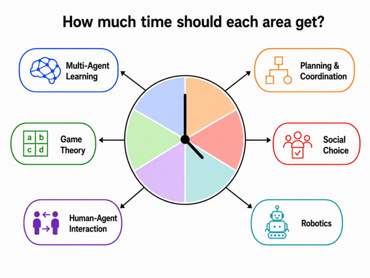
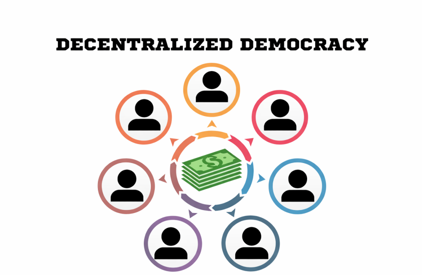
What is Budget Aggregation?

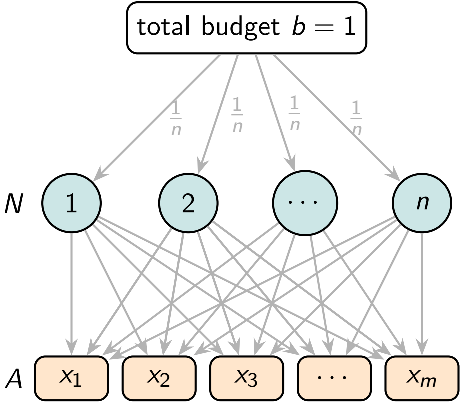
Allocating a **divisible resource** amomg **public projects**, respecting citizens' preferences.





Citizens express their preferences by distributing their **individual shares** among projects.

Budget-Aggregation Games



Agent i's strategy space:
$S_i = \{\delta_i \in [0, \frac{1}{n}]^m : \sum_{x \in A} \delta_{i,x} = \frac{1}{n}\}$

Outcome space:
$\Delta(A) = \{\delta \in [0, 1]^m : \sum_{x \in A} \delta_x = 1\}$

- Agent i's utility is written as $u_i : \Delta(A) \to \mathbb{R}$.
- **Remark:** the division of the budget can be hypothetical.

Nash Equilibrium

A strategy profile $(\delta_i^*)_{i \in N}$ is a *Nash equilibrium* if for all $i \in N$,

$$u_i(\delta^*) = \max_{\delta_i \in S_i} u_i \left(\sum_{j \in N \setminus i} \delta_j^* + \delta_i \right).$$

Properties:

- **Individual optimality**: in equilibrium, no agent would like to distribute their share differently.
- **Fairness**: any distribution in equilibrium satisfies *individual fair share*.

In a Nash equilibrium, an agent obtains at least as much utility as she could guarantee for herself by allocating her budget independently of the others.

Computing a Nash Equilibrium

We survey existing results on computing Nash equilibria:

- In general bi-matrix games, it is PPAD-hard to compute an ϵ-approximate NE, even for two players.[1]
- Our game belongs to the class of *aggregative games*: NE not efficiently computable in general.[2]

Our result:

➡ For budget-aggregation games, NE can be computed efficiently for common utility models.

[1] X. Chen and X. Deng. Settling the complexity of two-player Nash equilibrium. In Proceedings of the 47th Annual IEEE Symposium on Foundations of Computer Science (FOCS), pages 261–272, 2006

[2] Yakov Babichenko. 2018. Fast convergence of best-reply dynamics in aggregative games. Mathematics of Operations Research 43, 1 (2018), 333–346

Our Results

For **four common utility models** (linear, Leontief, ℓ_1, and binary symmetric separable), we provide:

- Characterizations of equilibrium strategies
- Efficient algorithms for computing NE
- Efficient algorithms to verify whether a distribution δ is decomposable into individual NE distributions $(\delta_i)_{i \in N}$
- Generalization to agents with different weights

Linear utilities

$u_i(\delta) = \sum_{x \in A} v_{i,x} \delta(x)$

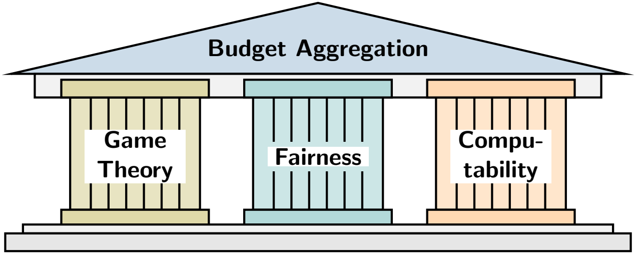
valuations $v_{i,x}$

	a	b	c
$i = 1$	3	1	0
$i = 2$	0	2	4

$A_i = \arg\max_{x \in A} v_{i,x}$

$(\delta_i)_{i \in N}$ in equilibrium iff $\mathrm{supp}(\delta_i) \subseteq A_i$ for all $i \in N$

Conclusion



Future Work:

- Does there exist a superclass of utility models that gives an efficient computation of NEs?
- Can we impose other axioms to select specific equilibrium distributions?