Collaborative Giving

Effective Distribution of Individual Contributions

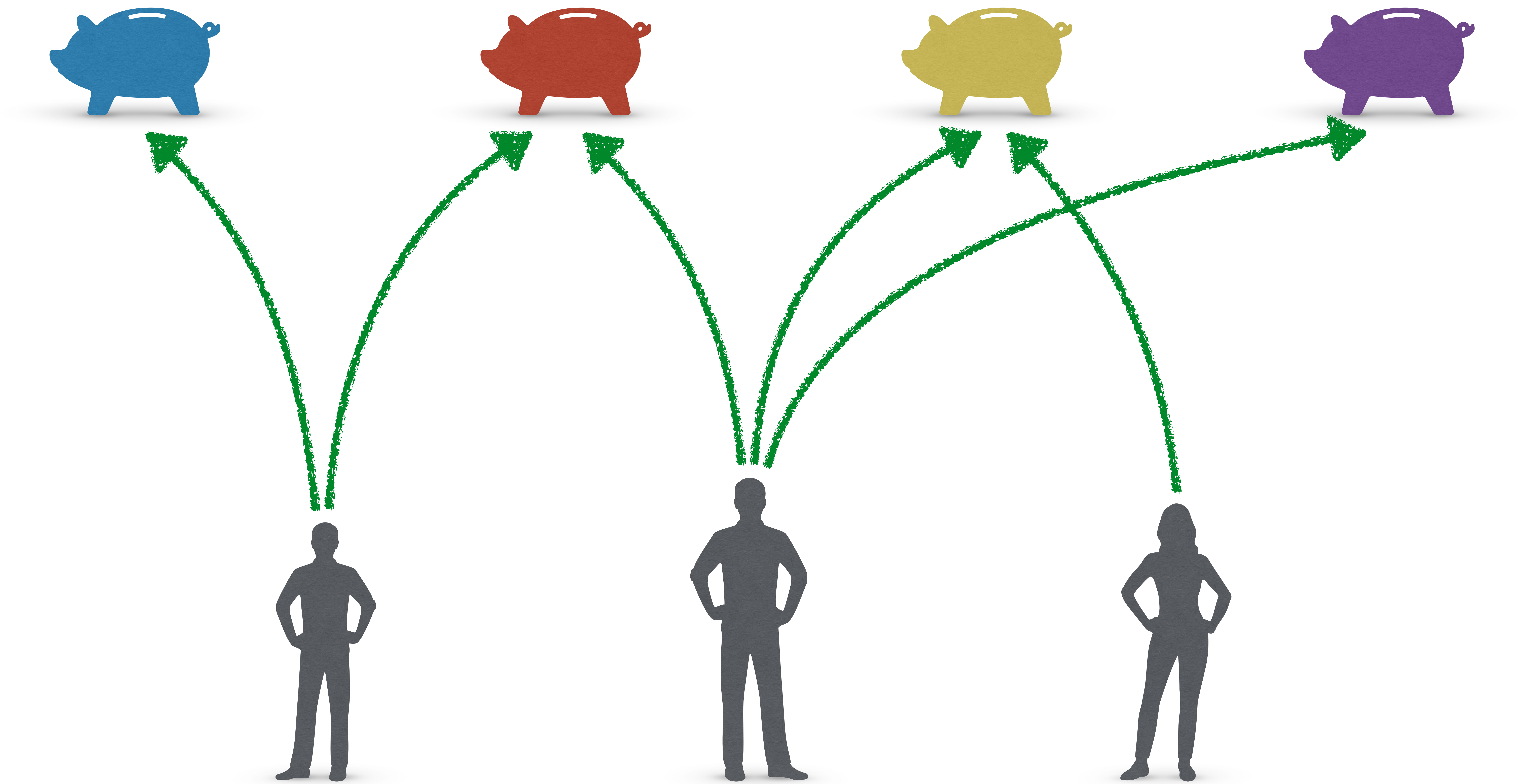
Felix Brandt

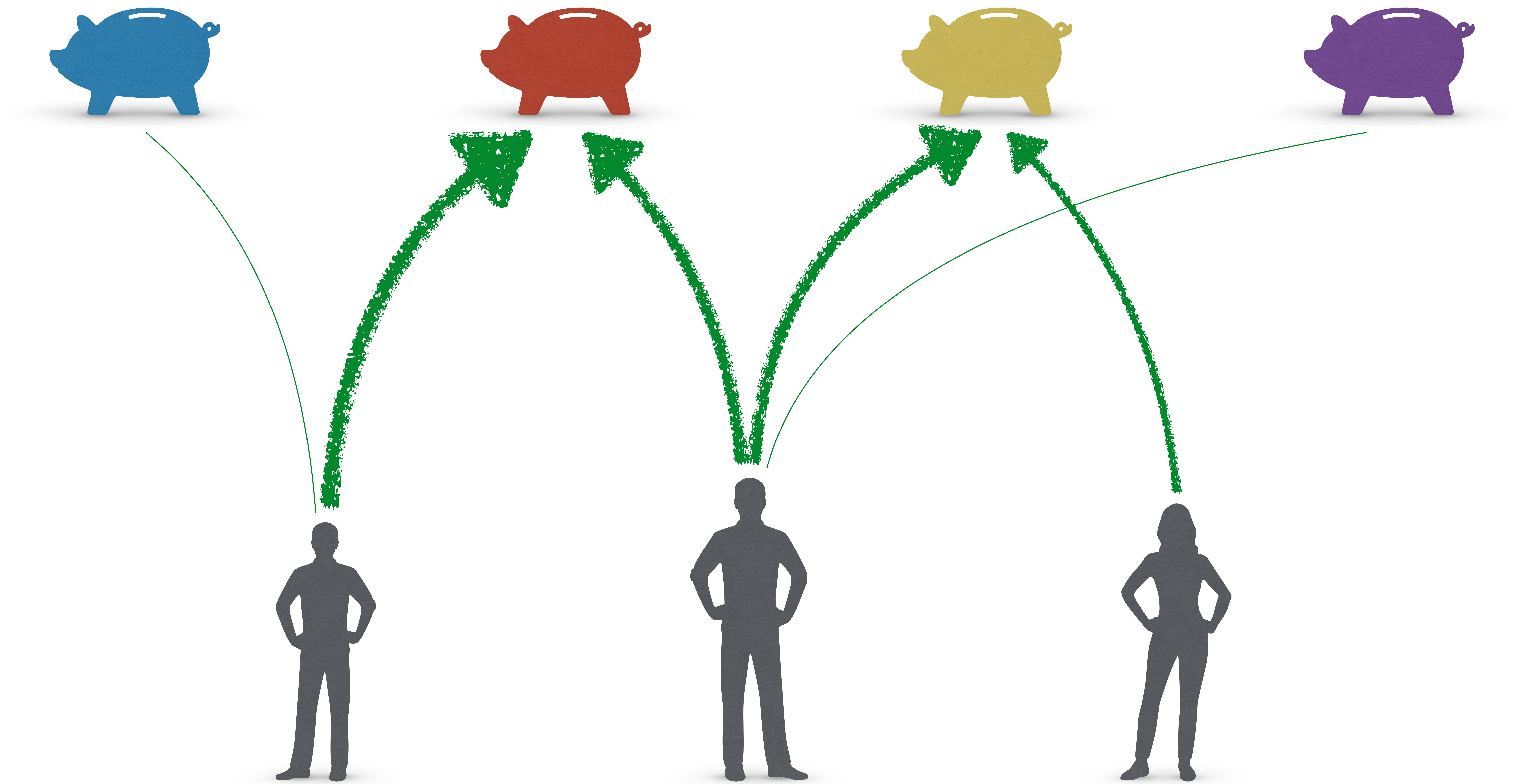
Algorithmics of Fair Division and Social Choice
NUS/IMS Singapore, December 2024

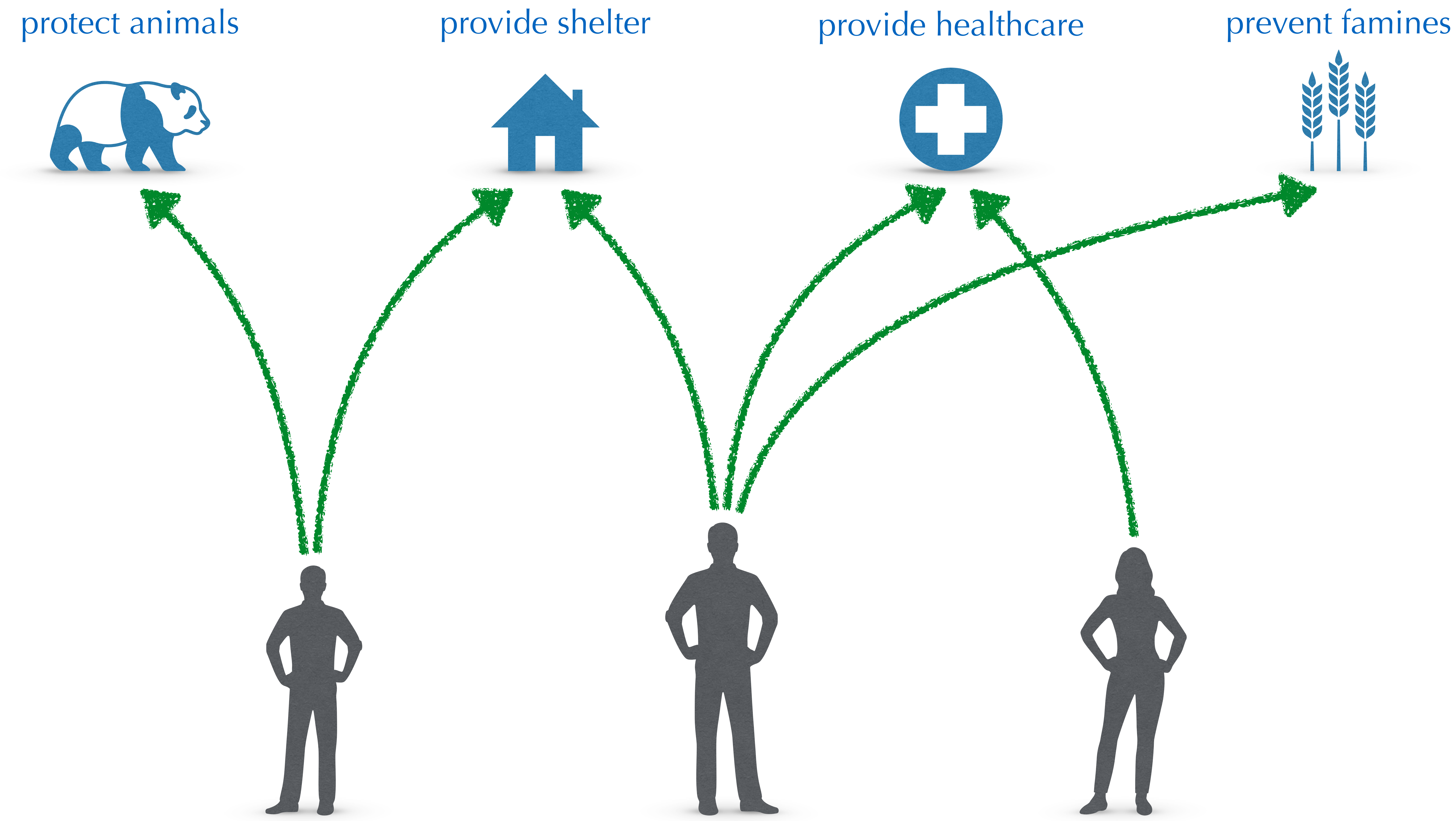
DSS
Decision Sciences & Systems

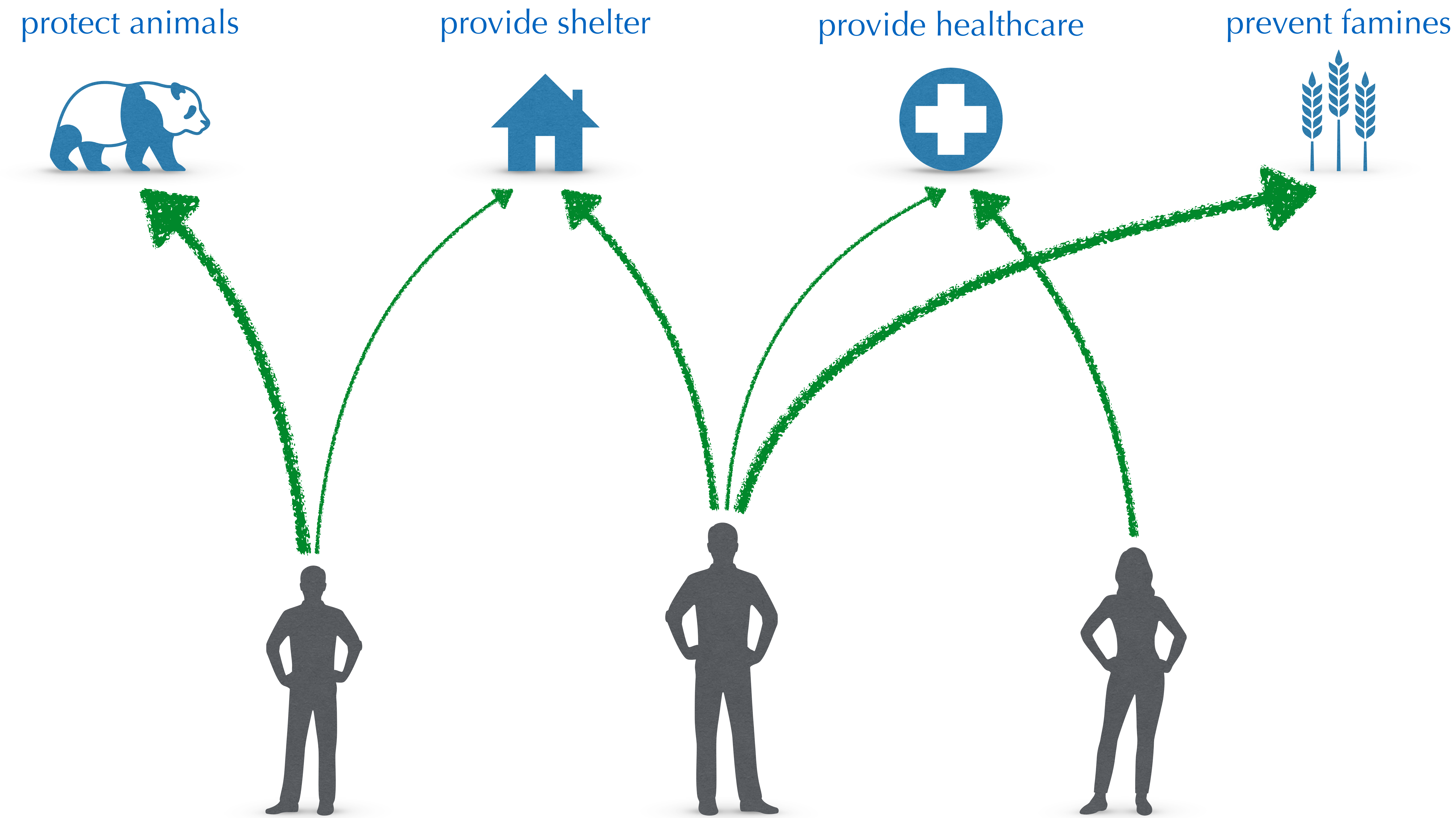
I will present results from these papers:

- Aziz, Bogomolnaia, and Moulin. *Fair mixing: the case of dichotomous preferences*. Presented at **ACM-EC 2019** (ACM Transactions on Economics and Computation, 2020).
- Aziz, Brandl, and B. *Universal Pareto dominance and welfare for plausible utility functions*. Presented at **ACM-EC 2014** (Journal of Mathematical Economics, 2015).
- Bogomolnaia, Moulin, and Stong: *Collective Choice under Dichotomous Preferences*. Journal of Economic Theory, 2005.
- Brandl, B., Greger, Peters, Stricker, and Suksompong. *Funding Public Projects—A Case for the Nash Product Rule*. Presented at **WINE 2021** (Journal of Mathematical Economics, 2022).
- Brandl, B., Peters, and Stricker. *Distribution rules under dichotomous preferences: Two out of three ain't bad*. Presented at **ACM-EC 2021**.
- B., Greger, Segal-Halevi, and Suksompong. *Optimal budget aggregation with single-peaked preferences*. Presented at **ACM-EC 2024**.
- B., Greger, Segal-Halevi, and Suksompong. *Coordinating charitable donations*. 2024. Presented at **ACM-EC 2023**.
- Duddy. *Fair sharing under dichotomous preferences*. Mathematical Social Sciences, 2015.
- Freeman, Pennock, Peters, and Vaughan. *Truthful aggregation of budget proposals*. Presented at **ACM-EC 2019** (Journal of Economic Theory, 2021).






protect animals
provide shelter
provide healthcare
prevent famines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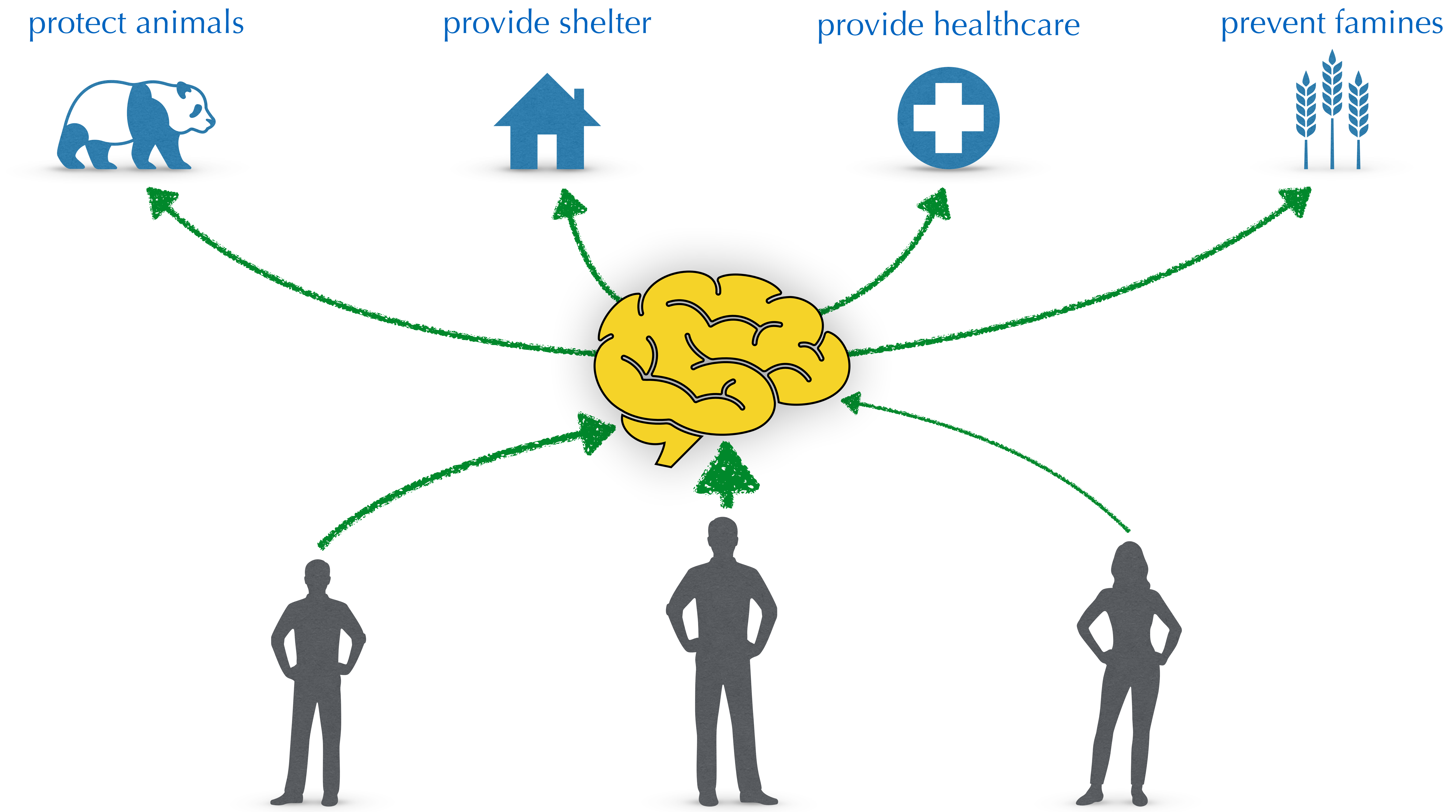
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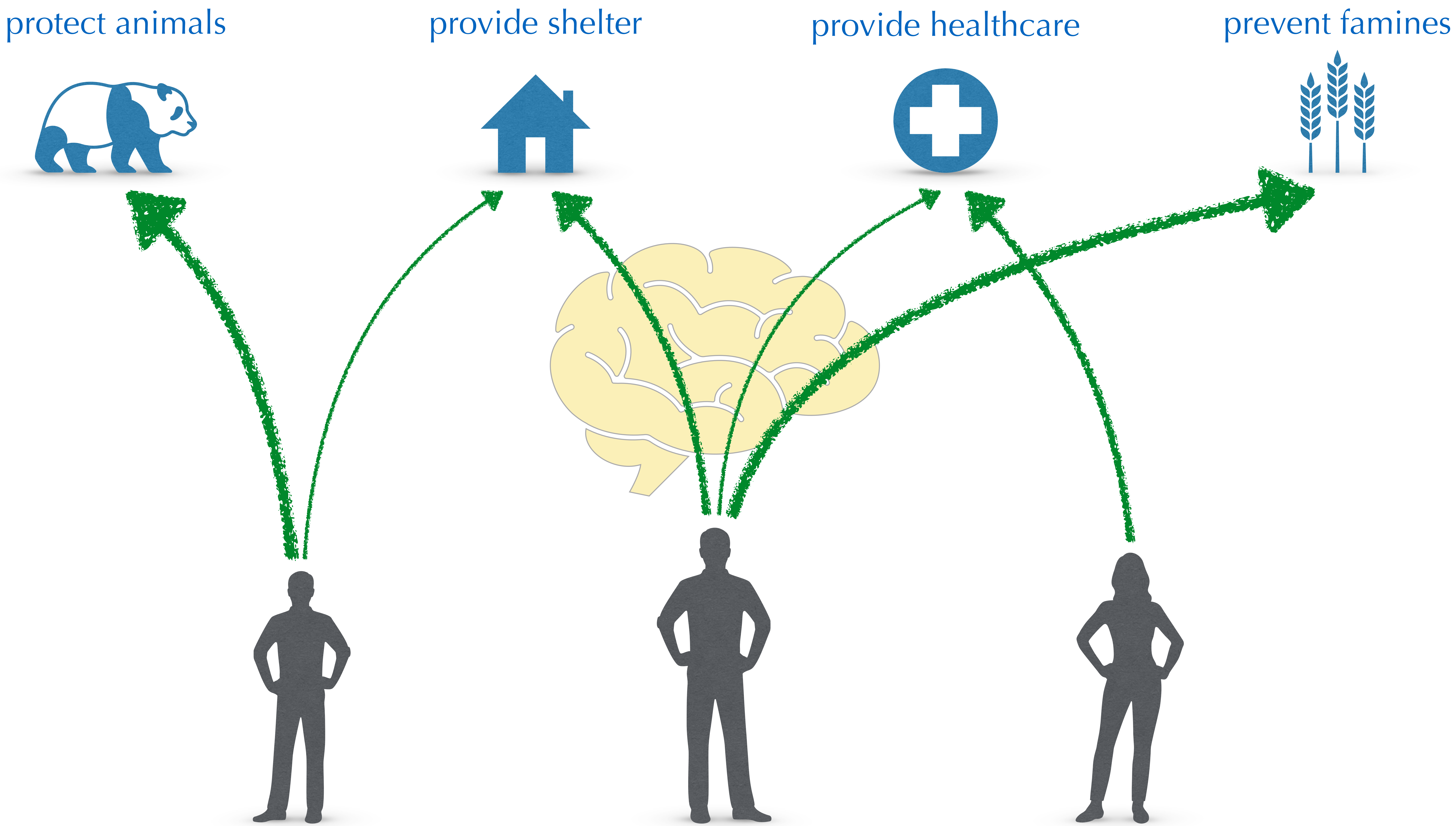
protect animals
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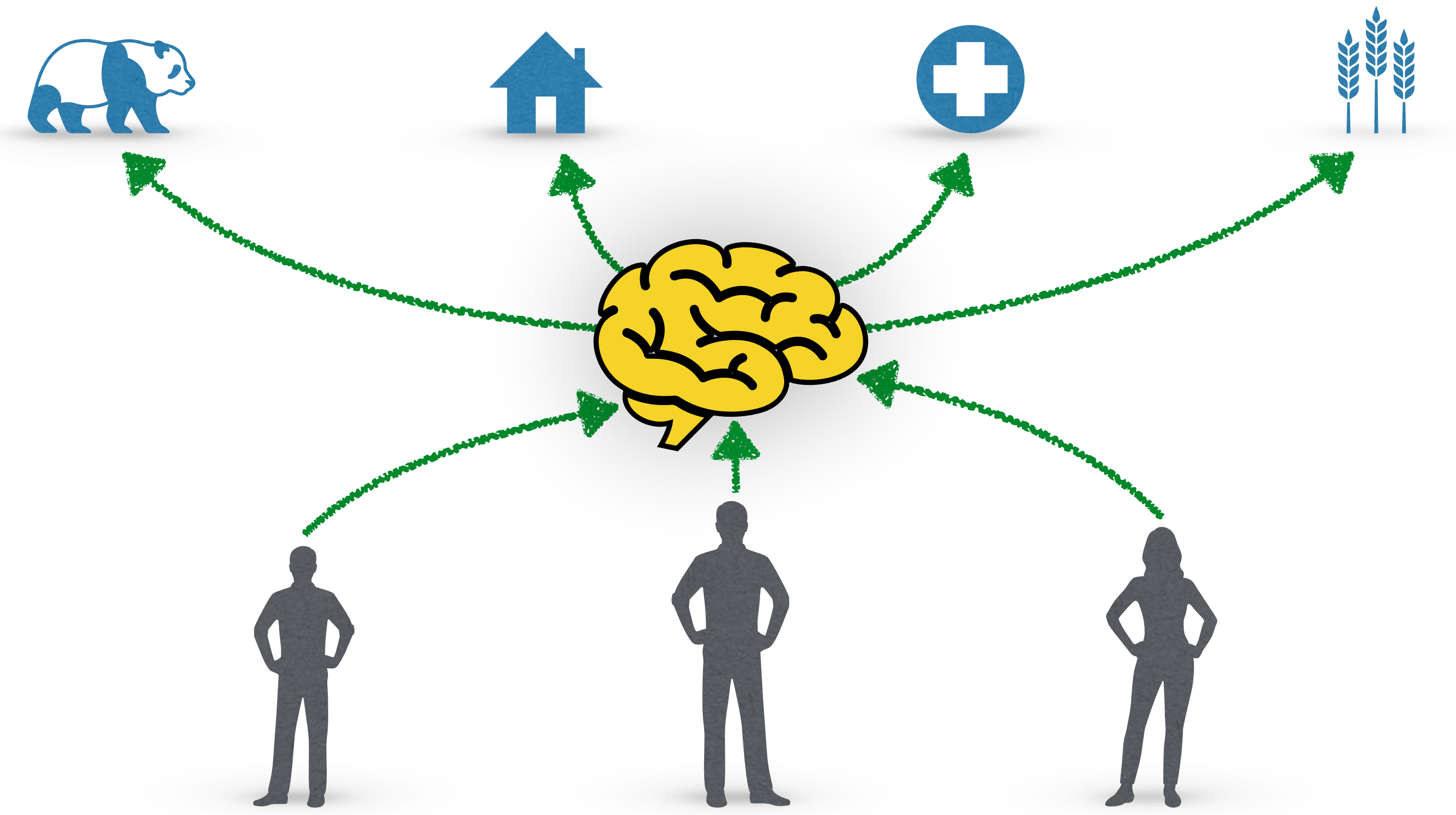
prevent famines

Potential Applications

- Cinque per mille
  - Italian citizens can select one of over 70k non-profit organizations.
  - Revenue Agency will divert 0.5% of citizen's income tax to this organization.
  - 2022: €510m
- AmazonSmile
  - Customers can select one of over 1m non-profit organizations.
  - Amazon donates 0.5% of customer's purchase price to this organization.
  - 2013–2023: $400m
- Employee charity matching programs
  - Microsoft (2022): $250m to 32k organizations
  - Apple (2011–2022): $880m to 44k organizations
- Private charity by groups of donors

Application Scenarios

- Fully centralized
  - Central authority owns individual contributions, collects preferences,
  - then distributes endowment.
- Centralized
  - Authority collects preferences and contributions,
  - then distributes endowment.
- Decentralized
  - Authority collects preferences,
  - then advises donors how to distribute their contributions.
- Fully decentralized
  - Donors independently distribute their contributions by observing previous donations.



The Model

- $N = \{1, \ldots, n\}$ is a set of agents.
- A is a set of m public goods (e.g., charities).
  - Public goods are non-excludable and non-rivalrous.
- Each agent $i \in N$ contributes amount $C_i > 0$ of a divisible and homogeneous resource (e.g., money) to a common pool.
  - $C = \sum_{i \in N} C_i$ is called the endowment.
- An individual distribution $\delta_i \in [0, C_i]^A$ is a function with $\sum_{x \in A} \delta_i(x) = C_i$.
  - The set of all distributions of C_i is denoted by $\Delta(C_i)$.
- $\delta = \sum_{i \in N} \delta_i \in \Delta(C)$ is the collective distribution of the endowment C.

The Model (ctd.)

- Agent i receives utility $u_i(\delta) \in \mathbb{R}$ from collective distribution δ.
  - $\mathscr{U} \subseteq \mathbb{R}^{\Delta(C)}$ denotes the set of admissible utility functions.
  - $u_i \in \mathscr{U}$ for all $i \in N$.
- A distribution rule f maps a utility profile $U = (u_i)_{i \in N} \in \mathscr{U}$ to $(\delta_i)_{i \in N}$.
  - We will often refer to the collective distribution δ implicitly returned by $f(U)$.

Related Models

- Private provision of public goods (e.g., Bergstrom, Blume, and Varian, 1986)
  - agents distribute their wealth between a private and a public good
  - no preferences over different public goods
- Probabilistic social choice/ fair mixing (e.g., Gibbard, 1977; Bogomolnaia et al., 2005)
  - ordinal, linear, or dichotomous preferences
  - exogenous fixed "endowment" of probability mass 1
- Participatory budgeting (e.g., Cabannes, 2004)
  - typically fixed costs for projects, which are either fully funded or not at all
  - exogenous endowment
- Budget aggregation (e.g., Freeman et al., 2021)
  - norm-based preferences (typically, ℓ_1)
  - exogenous endowment

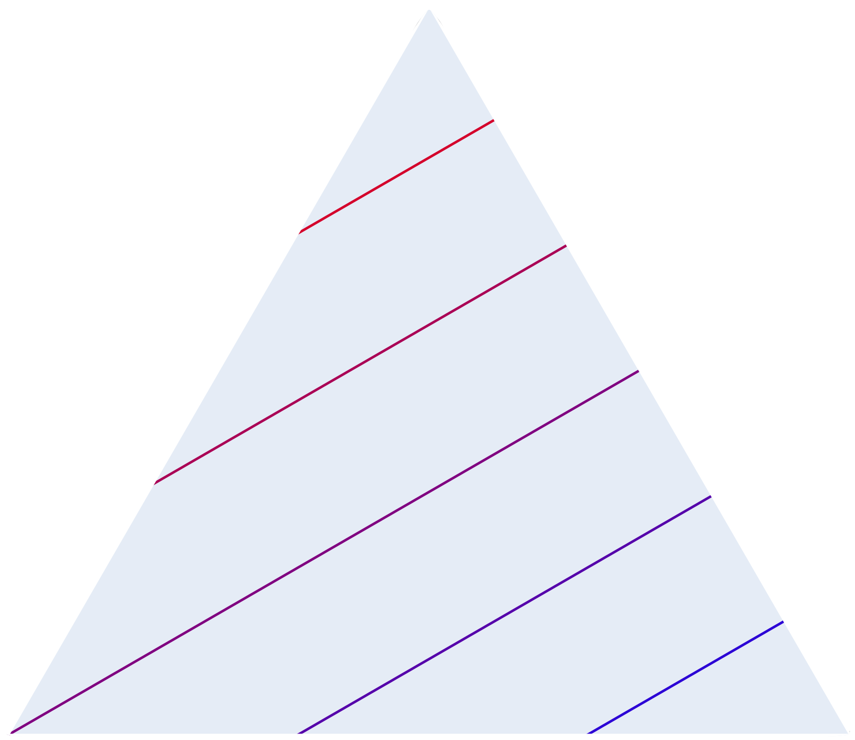
Four Desirable Properties

- The collective distribution δ returned by f is efficient.
  - There is no $\delta' \in \Delta(C)$ with $u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and $u_i(\delta') > u_i(\delta)$ for some $i \in N$.
  - Example:
    Utilitarian rule. δ maximizes $\sum_{i \in N} C_i \cdot u_i(\delta)$.
- f is strategyproof if $u_i(f(u_1, \ldots, u_n)) \geq u_i(f(u_1, \ldots, u_i', \ldots, u_n))$ for all $i \in N$ and $u_1, \ldots, u_n, u_i' \in \mathcal{U}$.
  - Examples:
    Dictatorial rules. δ maximizes $u_i(\delta)$ for some fixed $i \in N$.

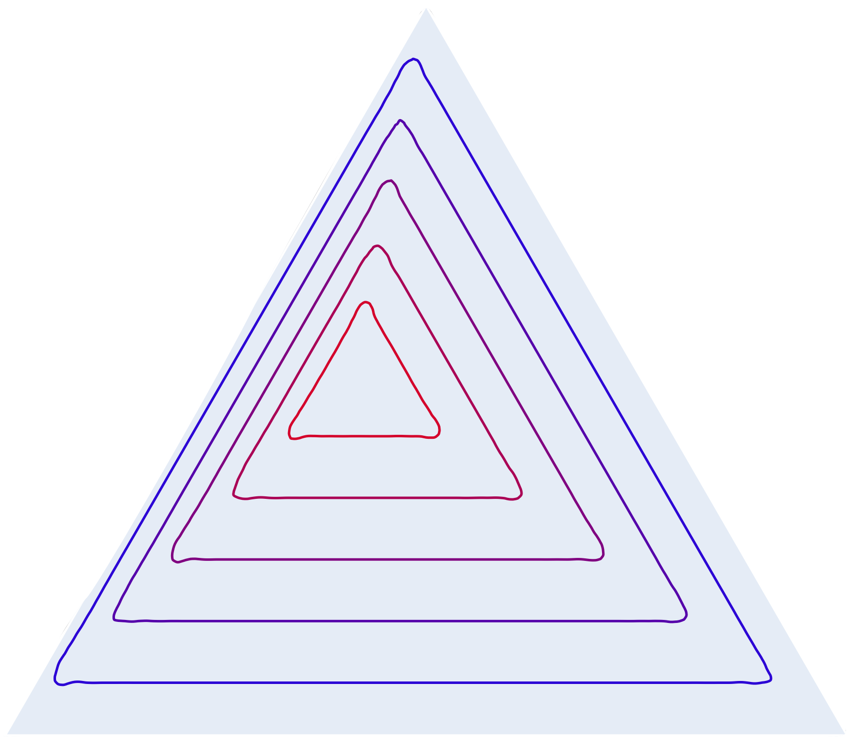
Four Desirable Properties (ctd.)

- The individual distributions $(\delta_i)_{i\in N}$ returned by f form a Nash equilibrium.
  - $u_i(\delta) = \max_{\delta_i^* \in \Delta(C_i)} u_i(\delta - \delta_i + \delta_i^*)$ for all $i \in N$ and.
  - crucial for settings where the authority merely issues recommendations
  - Equilibrium distributions exist under fairly general assumptions (Debreu, 1952).
- f is contribution incentive-compatible.
  - Agents should be better off participating than abstaining and spending their contribution optimally.
  - $u_i(f(U)) \geq \max_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ for all $U \in \mathcal{U}^N$ and $i \in N$.
    - $f_{-i}(U)$ distributes the amount $\sum_{j\in N\setminus\{i\}} C_j$ based on $(u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n)$
  - crucial for settings with voluntary participation

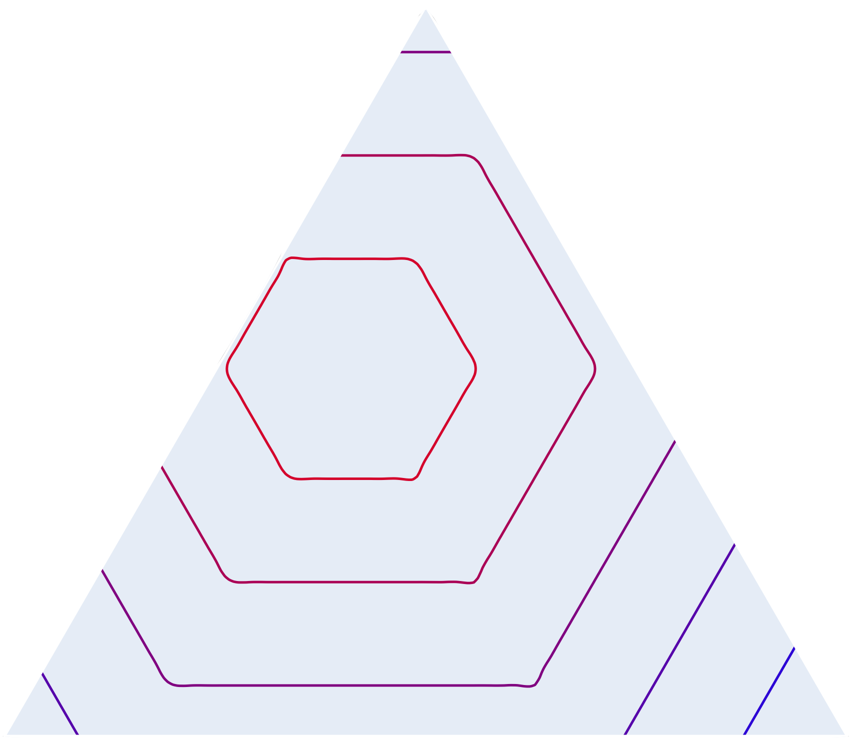
Potential Utility Functions



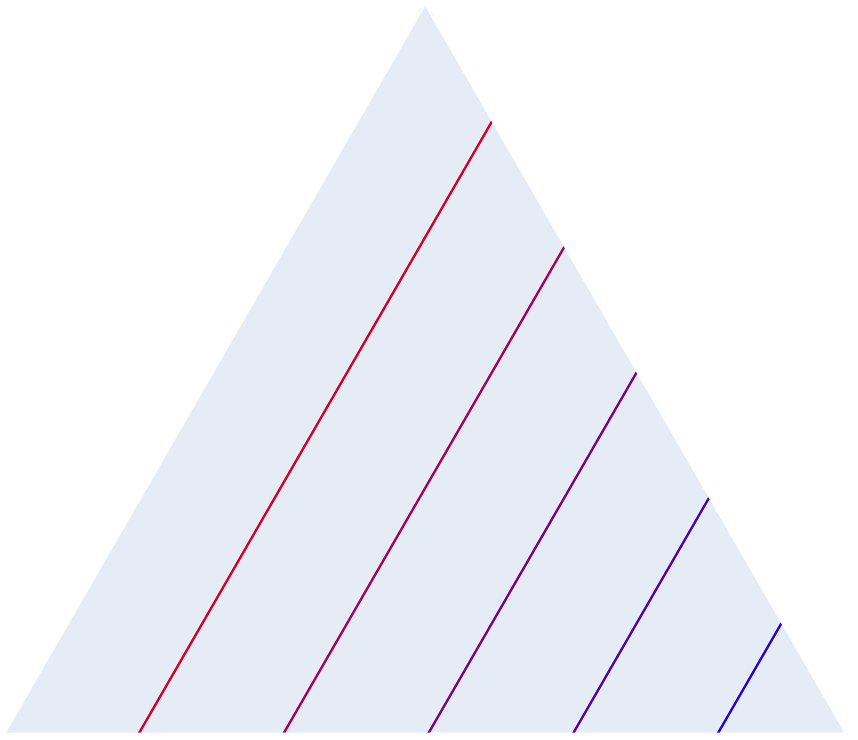
linear



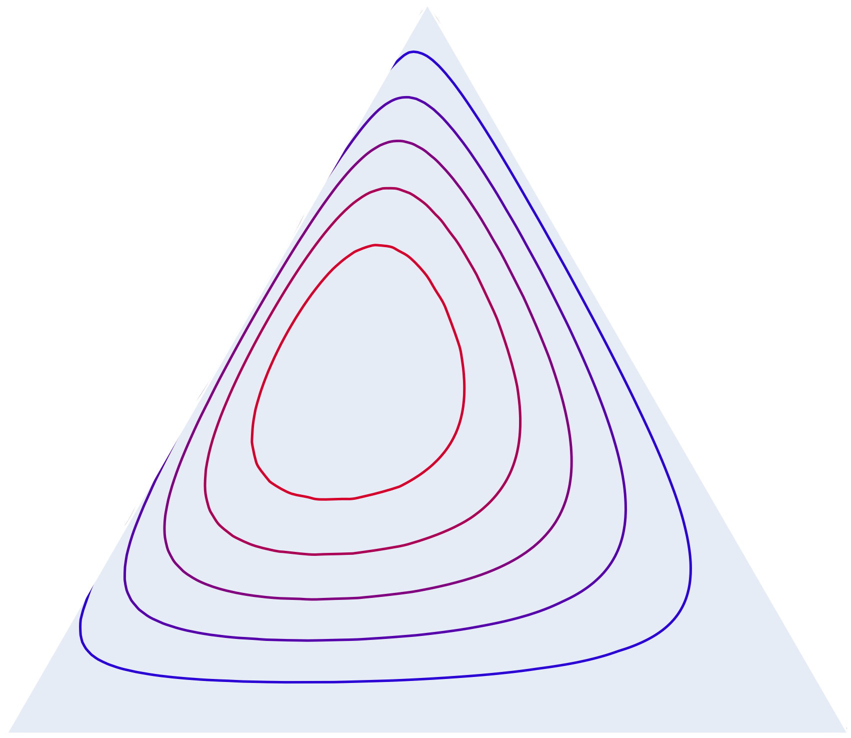
Leontief



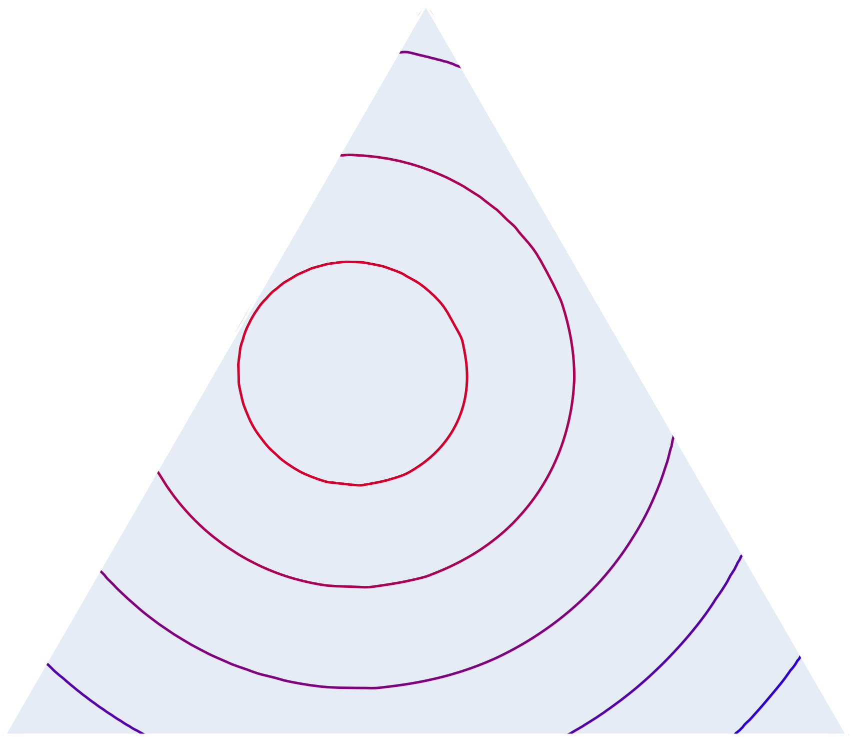
ℓ_1 disutilities



dichotomous

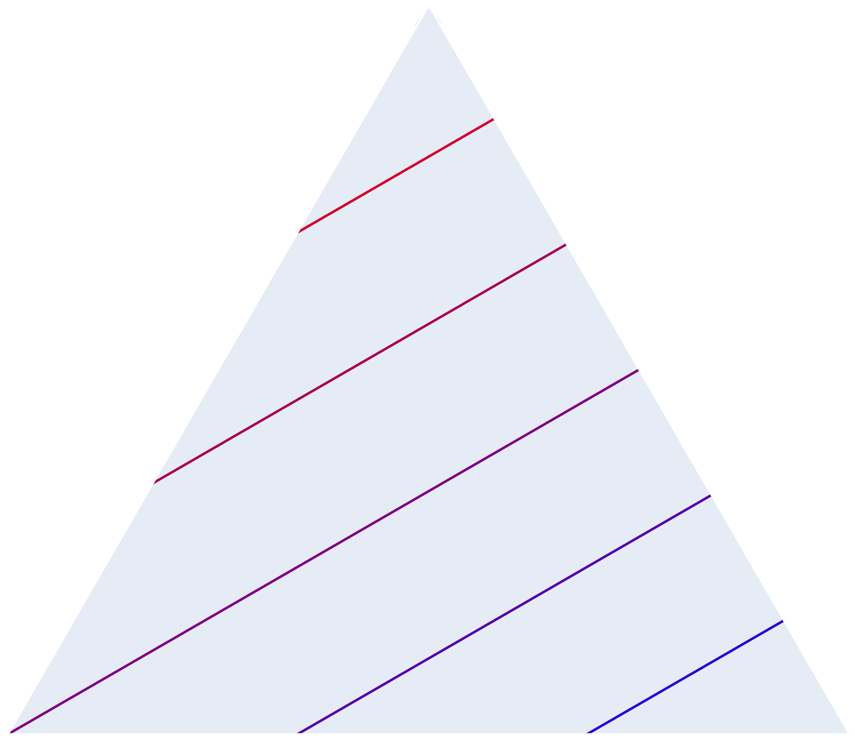


Cobb-Douglas



ℓ_2 disutilities

Linear Utilities



- $$u_i(\delta) = \sum_{x \in A} \delta(x) \cdot v_i(x)$$
  - Charities are substitutes with constant marginal rates of substitution.
- Example:

$v_i(x) = 1$ $v_i(x) = 0$

uncoordinated

	a	b	c	C_i	u_i
δ_1	0.5	0.5		1	**1.5**
δ_2		0.5	0.5	1	**1.5**
δ	0.5	1	0.5	2	

efficient

	a	b	c	C_i	u_i
δ_1		1		1	**2**
δ_2		1		1	**2**
δ		2		2	

Uncoordinated rule: Each δ_i independently maximizes $u_i((C/C_i) \cdot \delta_i)$.

Limitations under Linear Utilities

- **Theorem (Hylland, 1980)**: Only *dictatorial rules* are strategyproof and efficient.
- **Proposition**: Unique equilibrium distributions can be inefficient.

$v_i(x) = 1.5$

	a	b	c	C_i	u_i
δ_1	1			1	**1.5**
δ_2			1	1	**1.5**
δ	1		1		

	a	b	c	C_i	u_i
δ_1		1		1	**2**
δ_2		1		1	**2**
δ		2			

- **Theorem (Gibbard, 1977)**: When agents have unique top-ranked charities, then only the *uncoordinated rule* is anonymous, unanimous, and strategyproof.
- **Proposition (Brandl et al., 2022)**: No efficient rule satisfies contribution incentive-compatibility.

Proof

- **Proposition (Brandl et al., 2022)**: No efficient rule satisfies contribution incentive-compatibility when $m \geq 4$ and $n \geq 3$.
- Proof by contradiction:
  - Contribution incentive-compatibility: $u_i(\delta) \geq 1.6$ for all $i \in N$.

$v_i(x) = 1.6$

	a	b	c	d	C_i	u_i
δ_1					1	≥3.2
δ_2					1	≥3.2
δ_3					1	≥3.2
δ	>0	>0				

Assume w.l.o.g. $u_3(\delta) = \min_{i \in N} u_i(\delta)$

$\delta(a) > 0 \wedge \delta(b) > 0$. Efficiency is violated!

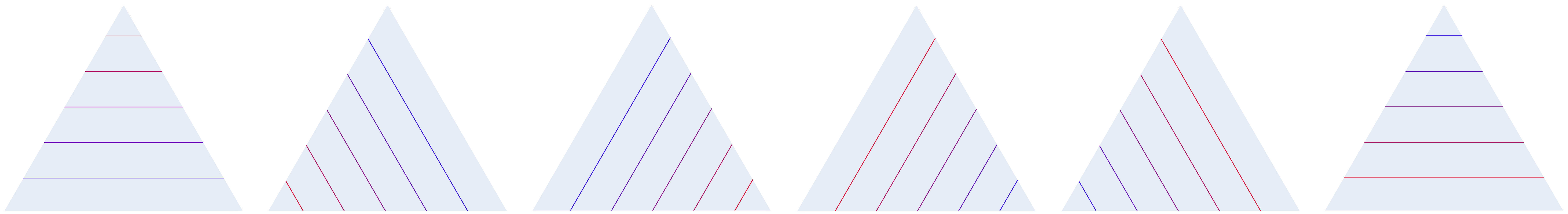
	a	b	c	d	C_i	u_i
δ'_1					1	
δ'_2					1	≥1.6
δ'		0		≥1.6		

Efficiency: $\delta(a) = 0 \vee \delta(b) = 0$

W.l.o.g. $\delta(b) = 0$

$\delta'(d) \geq 1.6$

Dichotomous Utilities



- A linear utility function u_i is dichotomous if $v_i(x) \in \{0,1\}$ for all $x \in A$.
  - Each agent i approves a non-empty set of projects $A_i \subseteq A$ and $u_i(\delta) = \sum_{x \in A_i} \delta(x)$.
- Positive share: $u_i(f(U)) > 0$ for all $i \in N$ and $U \in \mathcal{U}^N$.
  - much weaker than both contribution incentive-compatibility and being in equilibrium
  - minimal requirement to incentivize customers to participate in Amazon Smile: Amazon should donate money to at least one approved charity of each customer.

UTIL

	a	b	c	d	C_i	u_i
δ_1	1				1	5
δ_2	1				1	5
δ_3	1				1	0
δ_4	1				1	0
δ_5	1				1	5
δ	5					

CUT

	a	b	c	d	C_i	u_i
δ_1	1				1	3.5
δ_2	1				1	3.5
δ_3		0.5	0.5		1	1.5
δ_4		0.5		0.5	1	1.5
δ_5	1				1	3
δ	3	1	0.5	0.5		

NASH

	a	b	c	d	C_i	u_i
δ_1	1				1	3
δ_2	1				1	3
δ_3		1			1	2
δ_4		1			1	2
δ_5	1				1	3
δ	3	2				

- Utilitarian rule (*UTIL*) **violates positive share!**
  - δ maximizes $\sum_{i\in N} C_i \cdot u_i(\delta)$
- Conditional utilitarian rule (*CUT*) **violates efficiency!**
  - among all δ_i that maximize $u_i((C/C_i)\cdot\delta_i)$, pick those that maximize $\sum_{j\in N\setminus\{i\}} C_j \cdot u_j((C/C_i)\cdot\delta_i)$
- Nash product rule (*NASH*) **violates strategyproofness!**
  - δ maximizes $\prod_{i\in N} u_i(\delta)^{C_i}$ (or, equivalently, $\sum_{i\in N} C_i \log u_i(\delta)$)

	a	b	c	d	C_i	u_i
δ_1	1				1	4.4
δ_2	1				1	3.8
δ_3		0.4	0.6		1	1.2
δ_4	0.8	0.2			1	4.4
δ_5	1				1	3.8
δ	3.8	0.6	0.6			

Two Out of Three Ain't Bad

	UTIL	*CUT*	*NASH*
efficient	✓	-	✓
strategyproof	✓	✓	-
positive share	-	✓	✓

- **Theorem (Brandl et al., 2021)**: No distribution rule satisfies efficiency, strategyproofness, and positive share when $m \geq 4$, $n \geq 6$.
  - confirms a conjecture by Bogomolnaia, Moulin, and Stong (2005)
  - significantly weaker notion of strategyproofness suffices: manipulation only "counts" if $u_i(\delta') = C$.

A FULL PROOF OF THEOREM

A.1 Assuming $f(\mathcal{A}_1)$ has support bc or abc leads to contradiction.

	A_1	A_2	A_3	A_4	A_5	A_6	possible supports	dominated supports
Profile 1	b	c	ab	ac	bd	cd	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrowtail bc$
Profile 2	b	c	abc	ac	bd	cd	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 3	b	c	bc	ac	bd	cd	$\underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 4	bc	c	bc	ac	bd	cd	$cd, \underline{bc}, bcd$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail bc$
Profile 5	bc	c	bc	ac	bd	acd	$cd, \underline{bc}, \underline{bcd}$	$a \leftarrowtail c,\ ab \leftarrowtail bc,\ ad \leftarrowtail cd$
Profile 6	bc	c	bc	ac	bd	ad	$cd, acd, \underline{bcd}$	$ab \leftarrowtail cd$
Profile 7	bc	c	bc	ac	bcd	ad	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail cd$
Profile 8	bc	c	bc	ac	cd	ad	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$
Profile 9	bc	c	cd	ac	cd	ad	$ac, \underline{cd}, acd$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$
Profile 10	bc	c	cd	abc	cd	ad	$ac, \underline{cd}, \underline{acd}$	$b \leftarrowtail c,\ ab \leftarrowtail ac,\ bd \leftarrowtail ac$
Profile 11	bc	c	cd	ab	cd	ad	$ac, \underline{acd}, abc$	$bd \leftarrowtail ac$
Profile 12	bc	c	cd	ab	cd	acd	ac, bc, abc	$d \leftarrowtail c,\ ad \leftarrowtail ac,\ bd \leftarrowtail ac$

A Weaker Impossibility

- **Theorem (Brandl et al., 2021):** No anonymous and neutral distribution rule satisfies efficiency, strategyproofness, and positive share when $m \geq 4$, $n \geq 5$.
  - Proof:

	a	b	c	d	C_i	u_i
δ_1					1	
δ_2					1	
δ_3					1	>0
δ_4					1	<C
δ_5					1	
δ		>0	>0			

	a	b	c	d	C_i	u_i
δ'_1					1	
δ'_2					1	
δ'_3					1	
δ'_4					1	C
δ'_5					1	
δ'			0	0		

Anonymity and neutrality: $\delta(b) = \delta(c)$.
Positive share: $\delta(b), \delta(c) > 0$. Hence, $u_4(\delta) < C$

Anonymity and neutrality: $\delta'(c) = \delta'(d)$.
Efficiency: $\delta'(c), \delta'(d) = 0$. Hence, $u_4(\delta') = C > u_4(\delta)$.

Further Results

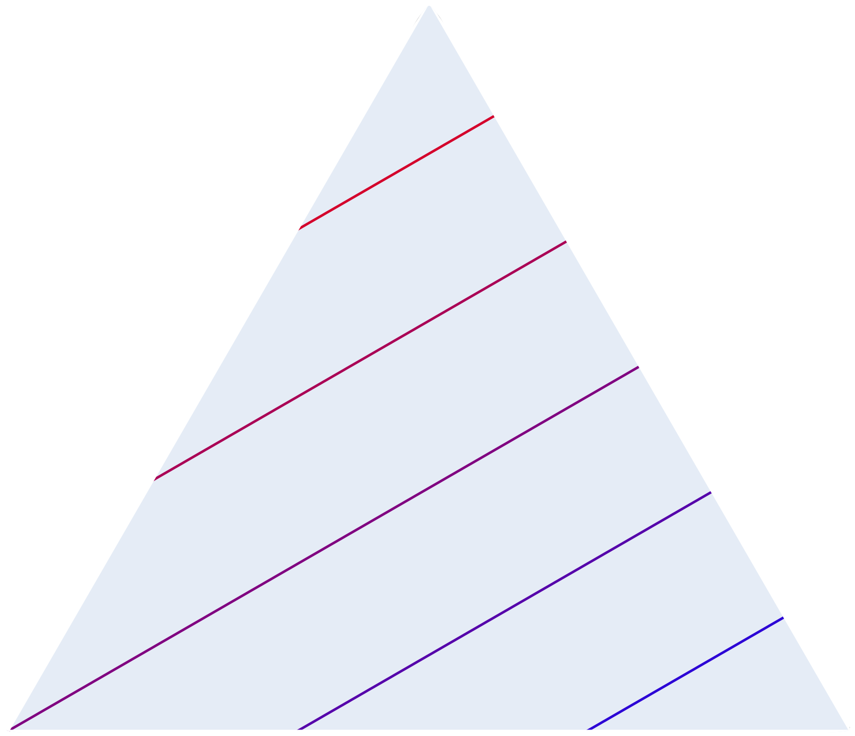
- Core fair share
  - $\forall S \subseteq N \; \nexists \delta' \in \Delta\left(\sum_{j\in S} C_j\right) \; \forall \delta'' \in \Delta\left(\sum_{j\in N\setminus S} C_j\right)$ s.t. $\forall i \in N\colon u_i(\delta' + \delta'') \geq u_i(\delta)$ and $\exists i \in N\colon u_i(\delta' + \delta'') > u_i(\delta)$.
- **Theorem (Aziz et al., 2020)**: *NASH* satisfies core fair share, *CUT* does not.
  - **Theorem (Duddy, 2015)**: *CUT* satisfies group fair share, a weakening of core fair share.
  - **Theorem (Brandl et al., 2022)**: A distribution rule satisfies group fair share iff it returns equilibrium distributions.
  - Positive share is weaker than group fair share.
- **Theorem (Brandl et al., 2022)**: *NASH* and *CUT* always return equilibrium distributions and satisfy contribution incentive-compatibility.

Summary and Open Problems

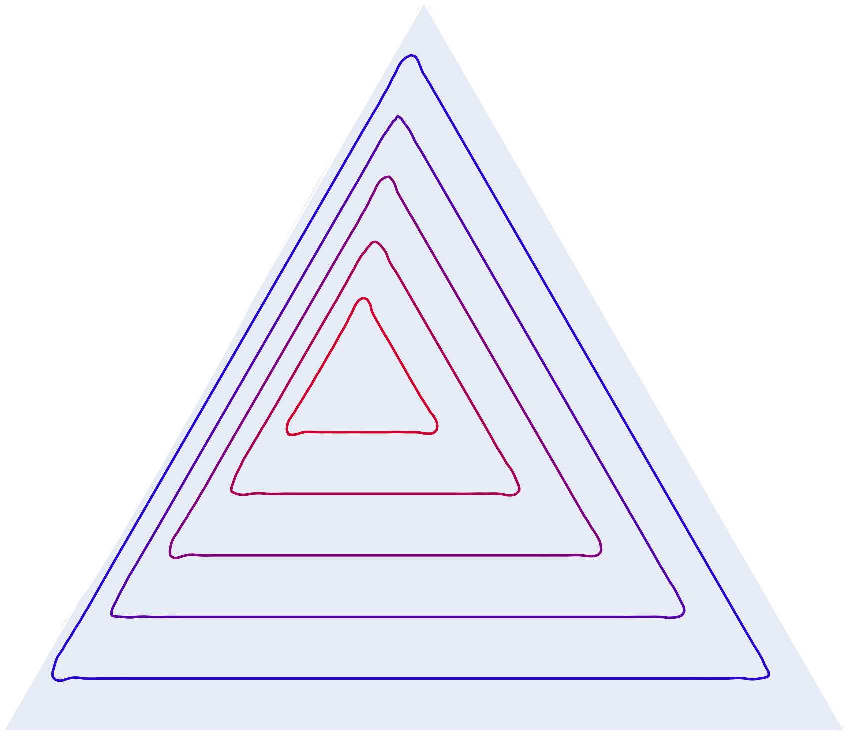
	UTIL	*CUT*	*NASH*	No rule!
efficiency	✓	-	✓	↯
strategyproofness	✓	✓	-	↯
contribution incentive-compatibility	-	✓	✓	
core fair share	-	-	✓	
group fair share (=in equilibrium)	-	✓	✓	
positive share	-	✓	✓	↯

- Quantify efficiency failures of *CUT* and strategyproofness failures of *NASH*
- Pending axiomatic characterizations:
  - *CUT* using contribution incentive-compatibility and strategyproofness
  - *NASH* using contribution incentive-compatibility and efficiency
  - *NASH* using core fair share

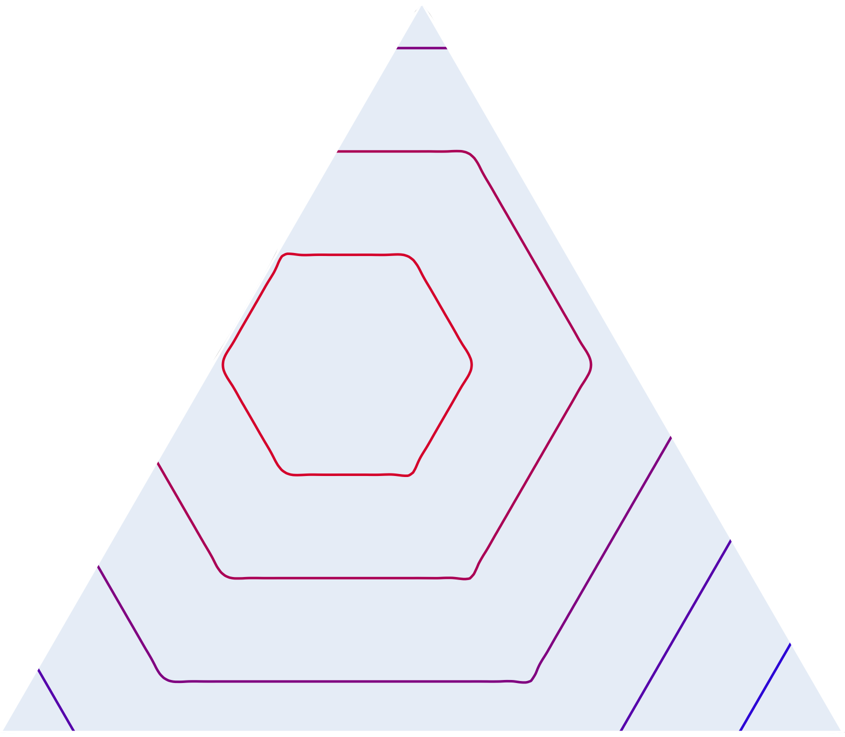
Potential Utility Functions



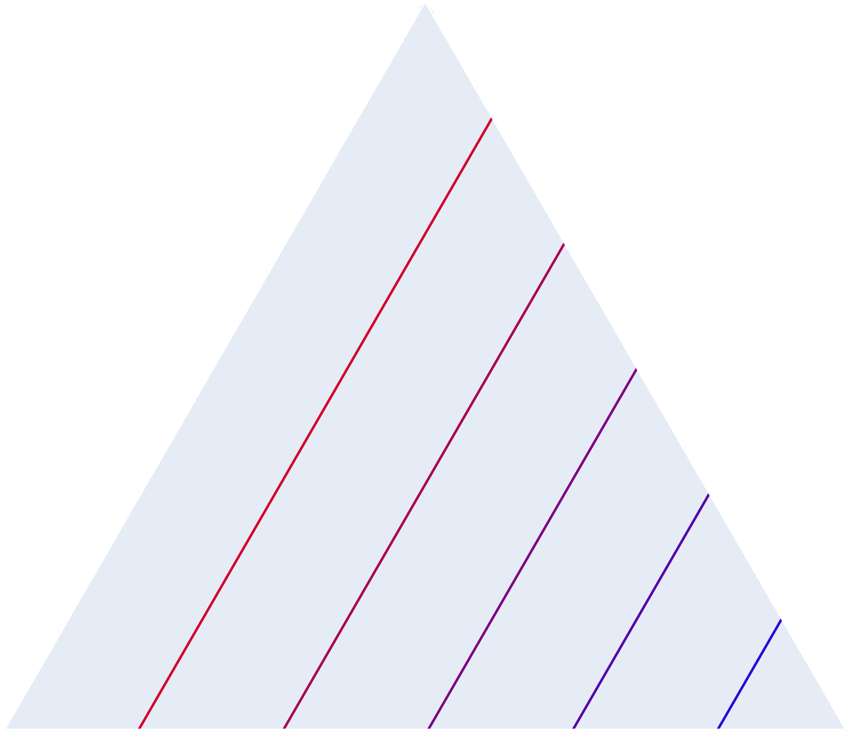
linear



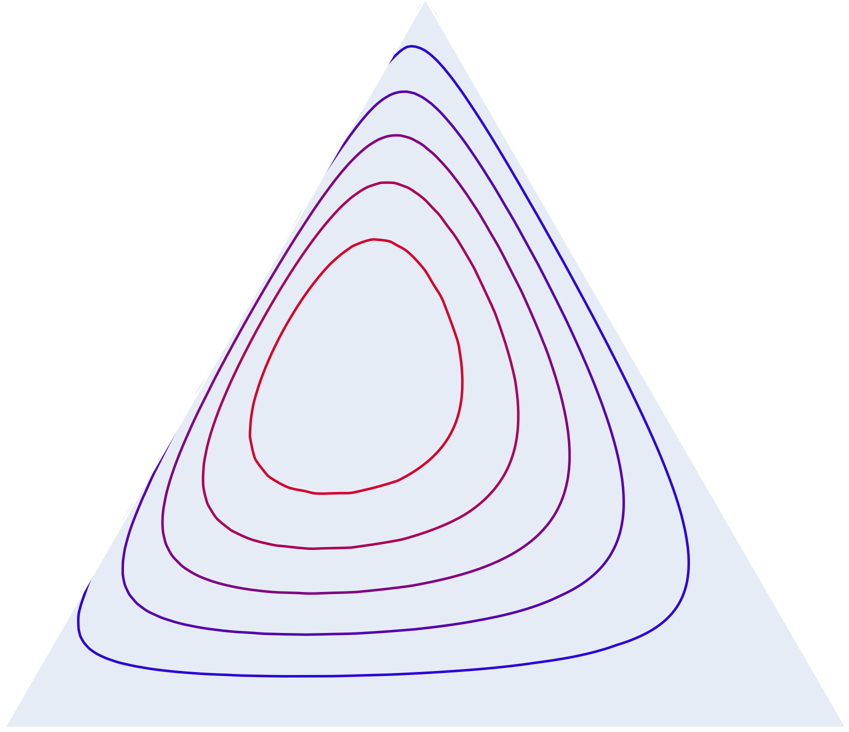
Leontief



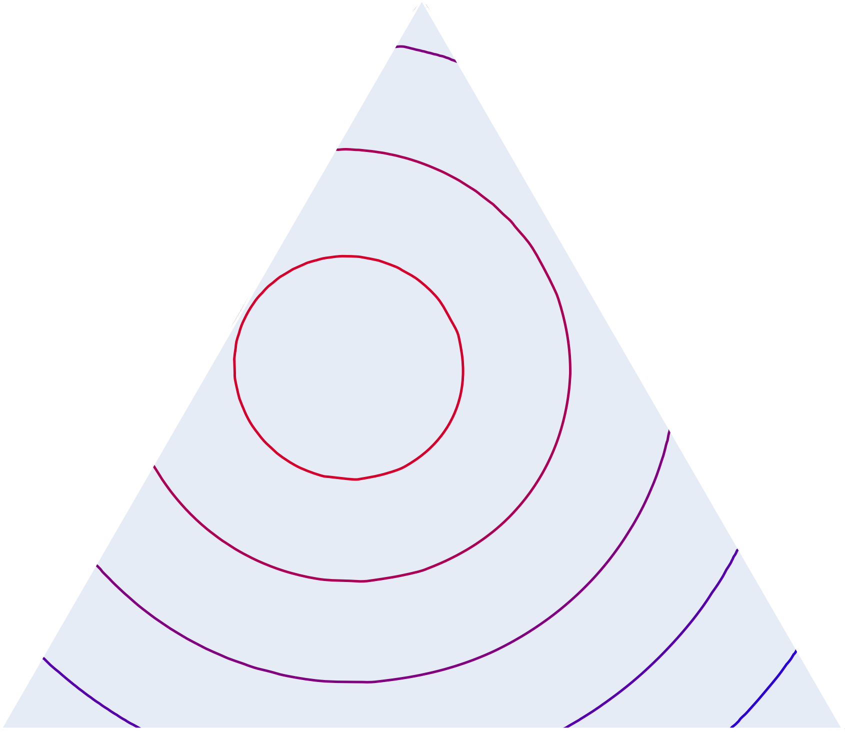
ℓ_1 disutilities



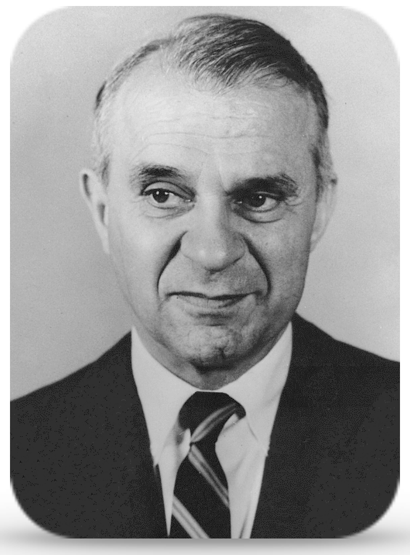
dichotomous



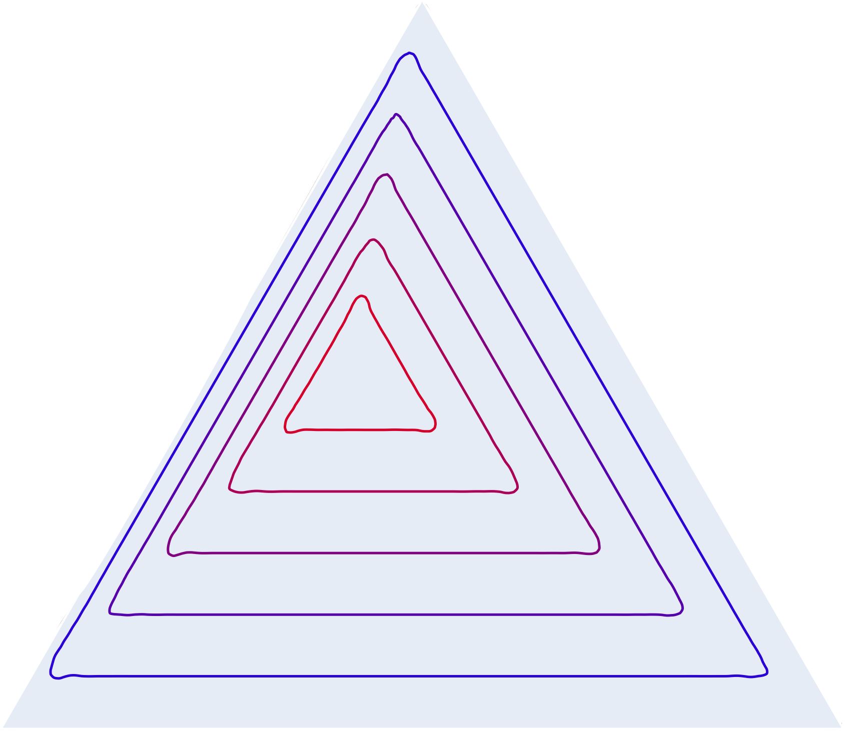
Cobb-Douglas



ℓ_2 disutilities


Wassily Leontief

Leontief Utilities



- $u_i(\delta) = \min_{x \in A:\, v_i(x) > 0} \frac{\delta(x)}{v_i(x)}$
  - $v_i(x) \geq 0$ for all $x \in A$ and $v_i(x) > 0$ for some $x \in A$.
- Charities are complements rather than substitutes.
- **Theorem (B. et al., 2023)**: Each utility profile admits a unique equilibrium. This distribution maximizes Nash welfare and thus is efficient.
- *EDR* (equilibrium distribution rule) returns the equilibrium distribution.
- Example:

	a	b	c	C_i	u_i
δ_1	1	0.5		1.5	**1**
δ_2		0.5	1	1.5	**1**
δ	1	1	1	3	

- Healthcare example
  - Charity a supports patients with common disease
  - Each charity b_i supports patients with some rare disease.
- Unique equilibrium for linear utilities.

$v_i(b_i)=1$ $v_i(a)=2$

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	30											30
δ_2	30											30
δ_3	30											30
δ_4	30											30
δ_5	30											30
δ_6	30											30
δ_7	30											30
δ_8	30											30
δ_9	30											30
δ_{10}	30											30
δ	**300**											

- Healthcare example
  - Charity a supports patients with common disease
  - Each charity b_i supports patients with some rare disease.
- Uncoordinated rule for Leontief utilities

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	20	10										30
δ_2	20		10									30
δ_3	20			10								30
δ_4	20				10							30
δ_5	20					10						30
δ_6	20						10					30
δ_7	20							10				30
δ_8	20								10			30
δ_9	20									10		30
δ_{10}	20										10	30
δ	**200**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	

- Healthcare example
  - Charity a supports patients with common disease
  - Each charity b_i supports patients with some rare disease.
- **EDR** (unique equilibrium for Leontief utilities)

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	5	25										30
δ_2	5		25									30
δ_3	5			25								30
δ_4	5				25							30
δ_5	5					25						30
δ_6	5						25					30
δ_7	5							25				30
δ_8	5								25			30
δ_9	5									25		30
δ_{10}	5										25	30
δ	**50**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	

Equilibrium Distribution Rule

- The tradeoffs present in the case of linear and dichotomous utilities vanish!
- **Theorem (B. et al., 2023)**: *EDR* is (group-)strategyproof.
- **Theorem (·)**: The equilibrium is rational-valued and can be computed in polynomial time via convex programming and a separation oracle.
- **Theorem (·)**: EDR has nice monotonicity properties:
  - Agent increases contribution $\Rightarrow$ funding of no charity decreases
  - Agent increases weight for charity $\Rightarrow$ funding of charity does not decrease
  - For dichotomous utilities, both properties are violated by *NASH*.

Further Results

- **Theorem (B. et al., 2024)**: *EDR* is the only rule that satisfies group-strategyproofness, core fair share, and continuity.
- For all $U \in \mathcal{U}^N$ and $i \in N$:
  - $u_i(f(U)) \geq \max_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ (Contribution incentive-compatibility)
  - $u_i(f(U)) > \min_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ (Strict participation)
  - $f_{-i}(U)$ distributes the amount $\sum_{j \in N \setminus \{i\}} C_j$ based on $(u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n)$.
- Strict participation is weaker than contribution incentive-compatibility.
- **Theorem (B. et al., 2023)**: *EDR* satisfies strict participation.
- **Proposition**: No distribution rule satisfies contribution incentive-compatibility for Leontief utilities.

Proof

- **Proposition**: No distribution rule satisfies contribution incentive-compatibility.
- Proof by contradiction:

	a	b	c	d	C_i	u_i
δ_1					6	≥ 4
δ_2					6	≥ 3
δ	≥ 4	≥ 4	≥ 3	≥ 3		

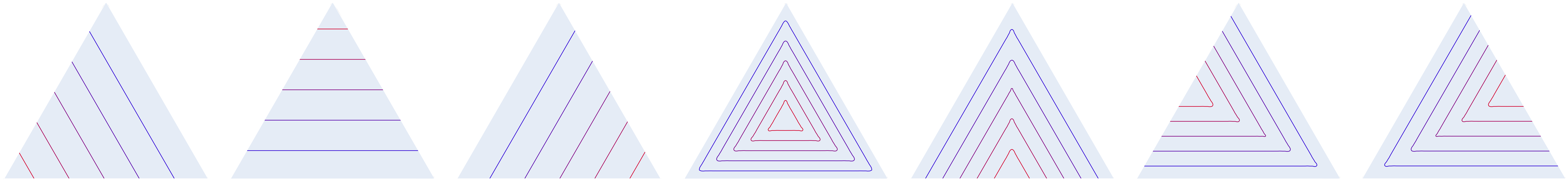
	a	b	c	d	C_1	u_1
δ_1	3	3			6	3

	a	b	c	d	C_2	u_2
δ_2		2	2	2	6	2

$u_1(\delta) \geq 4$ and $u_2(\delta) \geq 3$

$4 + 4 + 3 + 3 = 14 \geq C = 6 + 6 = 12$ ↯

Binary Weights



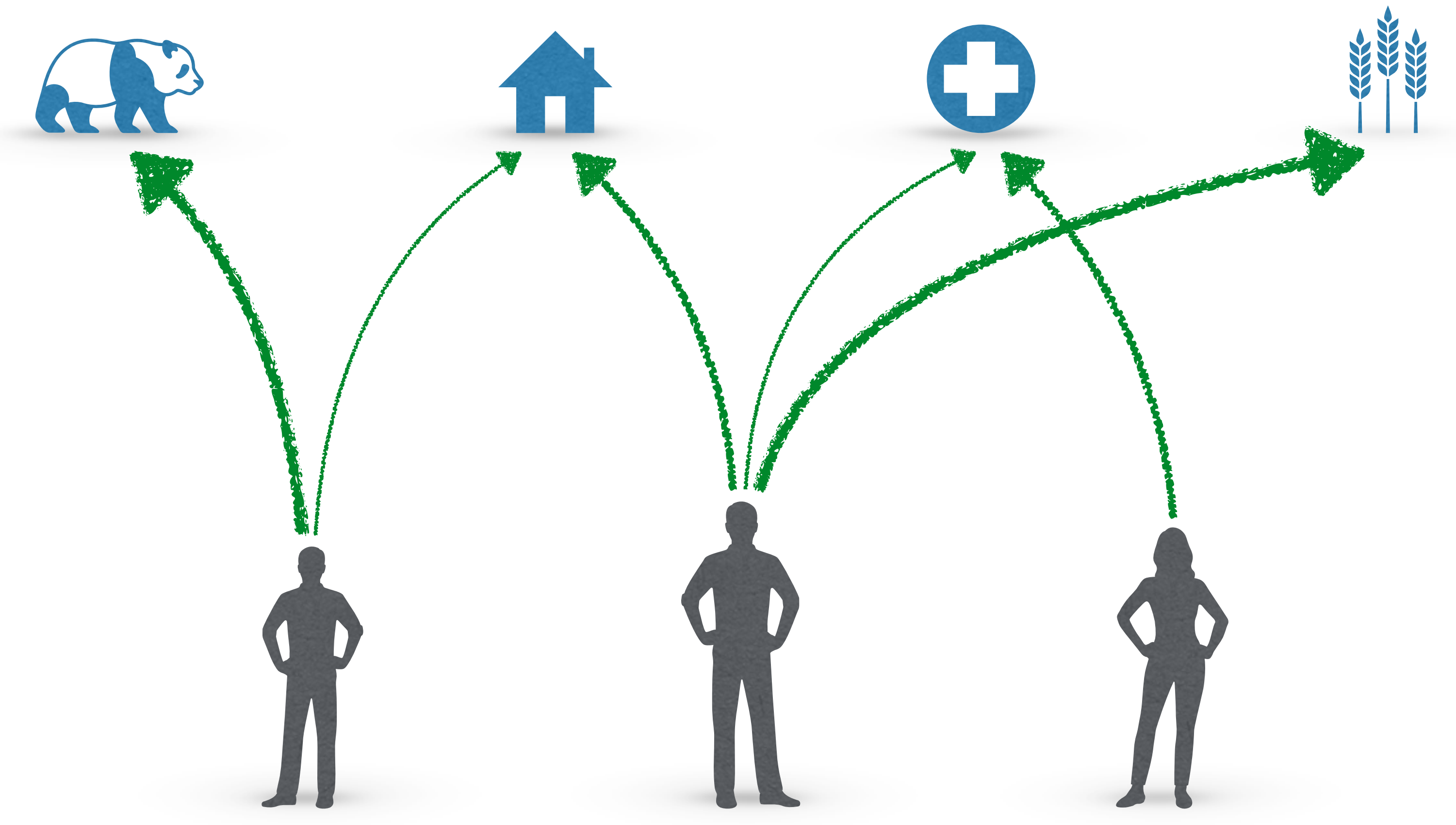
- A Leontief utility function u_i has binary weights if $v_i(x) \in \{0,1\}$ for all $x \in A$.
- **Theorem (B. et al., 2023)**: For binary weights, *EDR* coincides with egalitarian rules:

  Among all $\delta \in \Delta(C)$ with $\sum_{x \in A:\, v_i(x)>0} \delta_i(x) = C_i$, *EDR* lexicographically maximizes both

  - $\min_{x \in A} \delta(x)$ (the minimal contribution to a charity), and
  - $\min_{i \in N} u_i(\delta)$ (the minimal utility of an agent).
- For binary weights, *EDR* can thus be computed via linear programming.

Spending Dynamics

- Each agent has set aside a, say, *monthly* budget for charitable activities.
- Agents become active in round-robin order.
- Each agent observes the accumulated distribution of the last $n-1$ rounds and then distributes her own contribution myopically optimal.
- **Theorem (B. et al., 2023)**: The collective distribution of the last n rounds converges to *EDR*.
- Even with occasional changes to preferences and contributions, the relative overall distribution keeps converging towards the equilibrium distribution.



	a	b	c	d	C_i	u_i
δ_1	18	18			36	**18**
δ_2		6	24	24	54	**24**
δ_3			18		18	**42**
δ_1	21	15			36	**21**
δ_2		14	11	29	54	**29**
δ_3			18		18	**29**
δ_1	25	11			36	**25**
	⋮	⋮	⋮	⋮		
δ	27	27	27	27	108	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Thank you
for your contribution.
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Tops Tops Tops Tops
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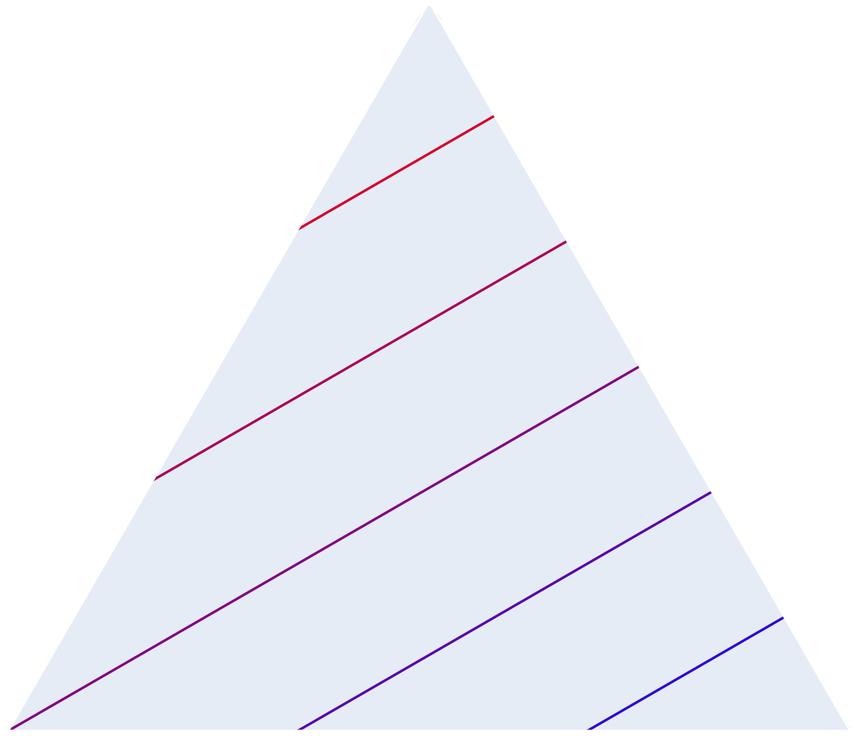
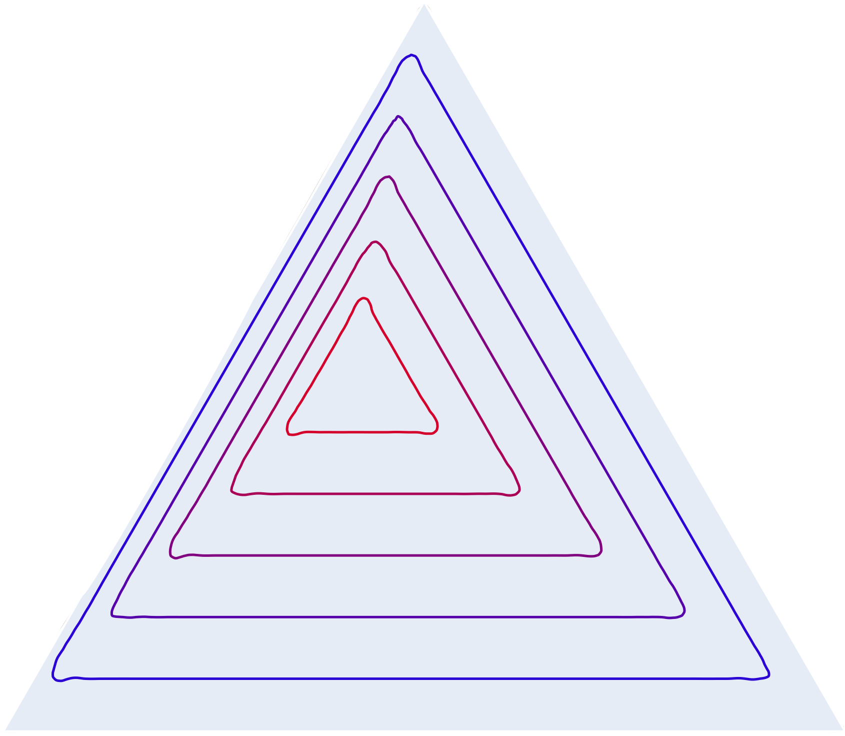
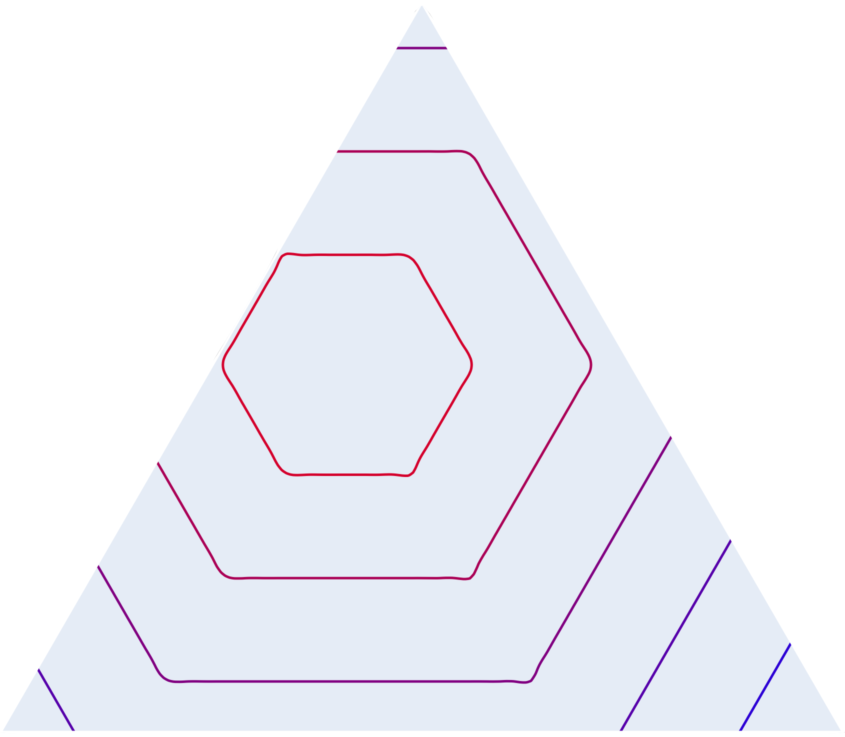
Potential Utility Functions



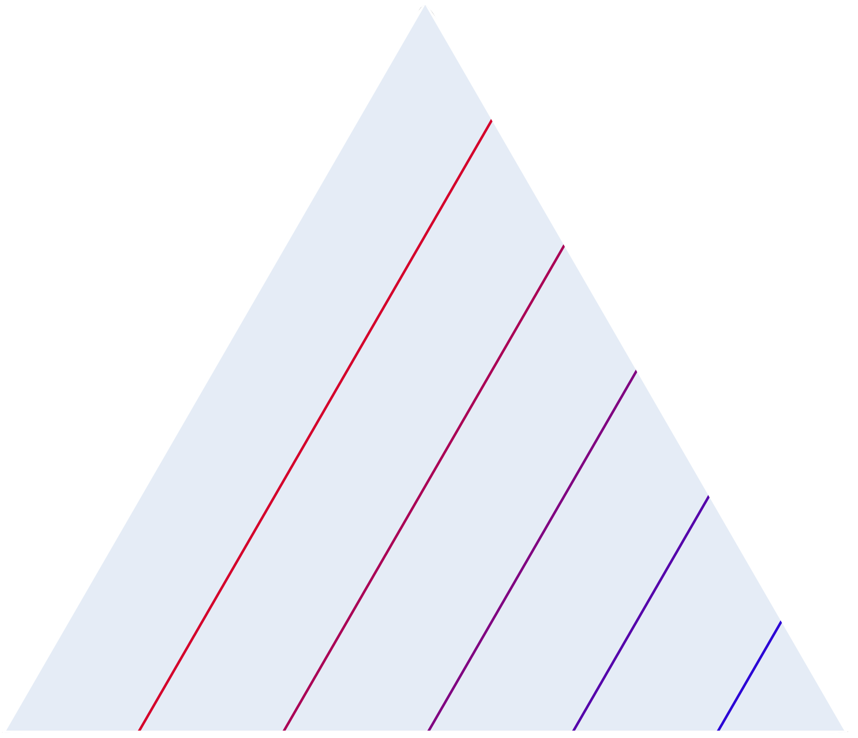
linear



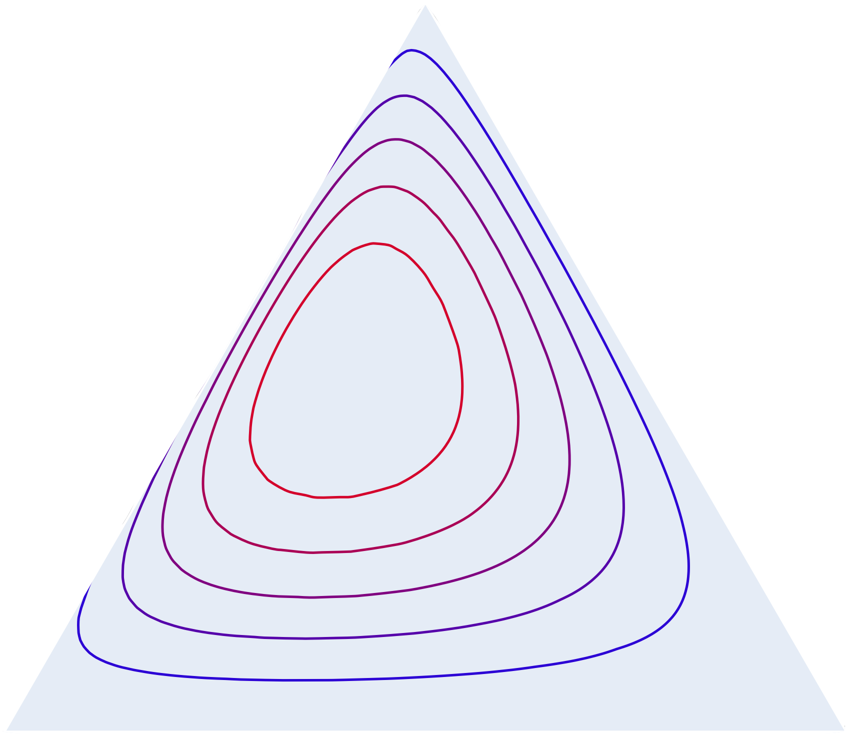
Leontief



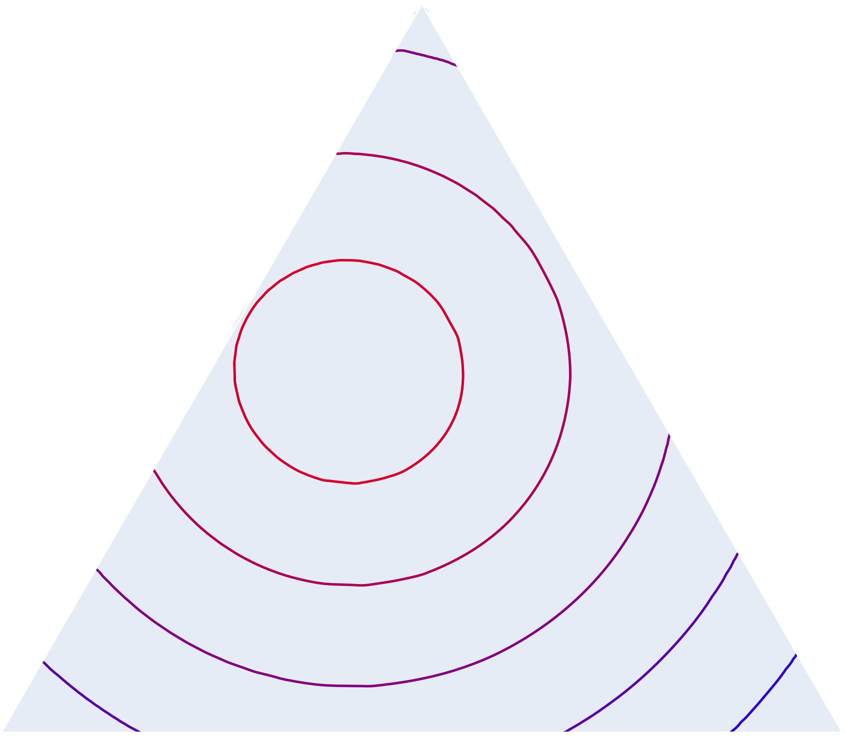
ℓ_1 disutilities



dichotomous



Cobb-Douglas

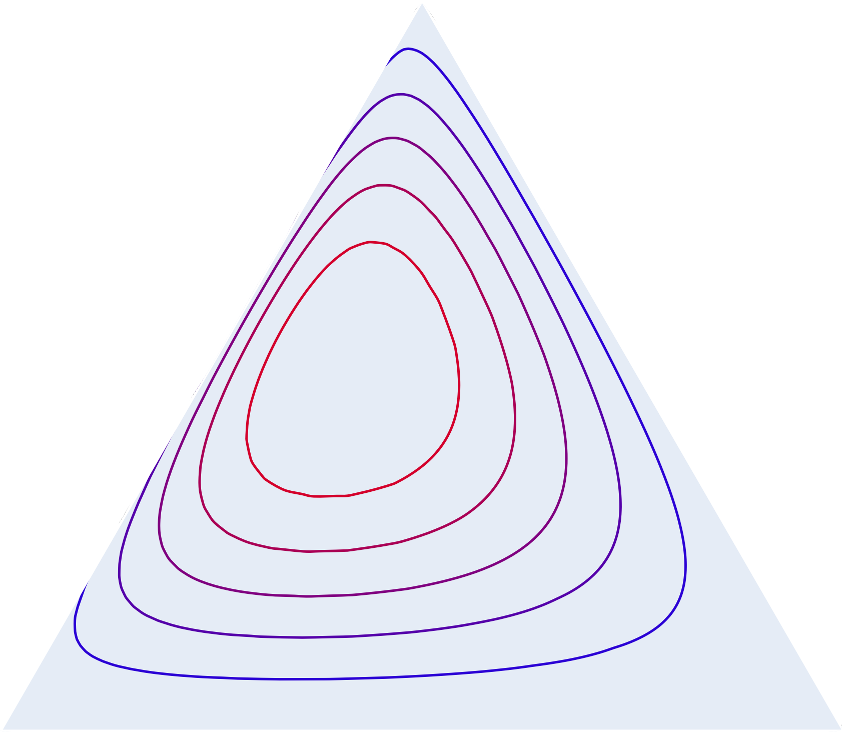


ℓ_2 disutilities

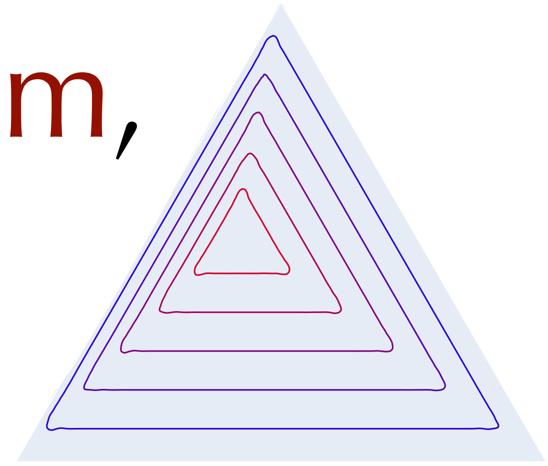

Charles Cobb


Paul Douglas

Cobb-Douglas Utilities



- $u_i(\delta) = \prod_{x \in A} \delta(x)^{v_i(x)}$ (Equivalently, $u_i(\delta) = \sum_{x \in A} v_i(x) \cdot \log \delta(x)$)
  - $v_i(x) \geq 0$ for all $x \in A$ and $v_i(x) > 0$ for some $x \in A$.
- **Theorem (B. et al., 2023)**: Each utility profile admits a unique equilibrium, which coincides with the equilibrium for Leontief utility functions using the same weights.



- Convergence of the spending dynamics also holds for Cobb-Douglas.
- However, efficiency and strategyproofness break down.

A Prisoners' Dilemma

- The equilibrium distribution can be inefficient for Cobb-Douglas utilities.
  - There is $\delta' \in \Delta(C)$ with $u_i(\delta') > u_i(\delta)$ for all $i \in N$.

equilibrium

	a	b	c	C_i	u_i
δ_1	4	2		6	**16**
δ_2		2	4	6	**16**
δ	4	4	4	12	

uncoordinated rule

	a	b	c	C_i	u_i
δ_1'	3	3		6	**18**
δ_2'		3	3	6	**18**
δ'	3	6	3	12	

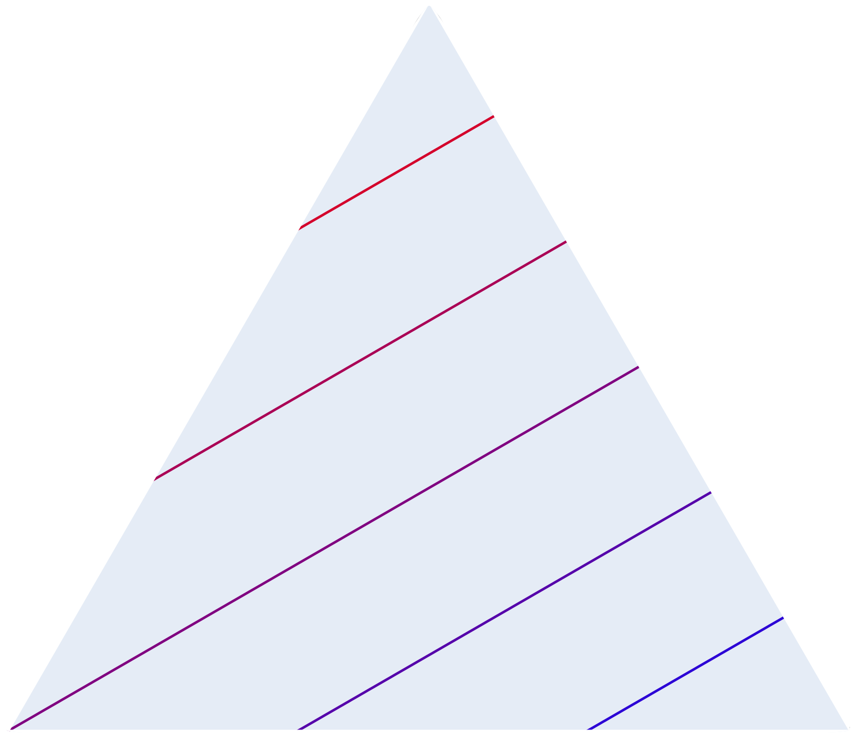
- For Leontief utility functions, equilibrium distributions are always efficient!

	(0,3,3)	(0,2,4)
(3,3,0)	(3,6,3)	(3,5,4)
(4,2,0)	(4,5,3)	(4,4,4)

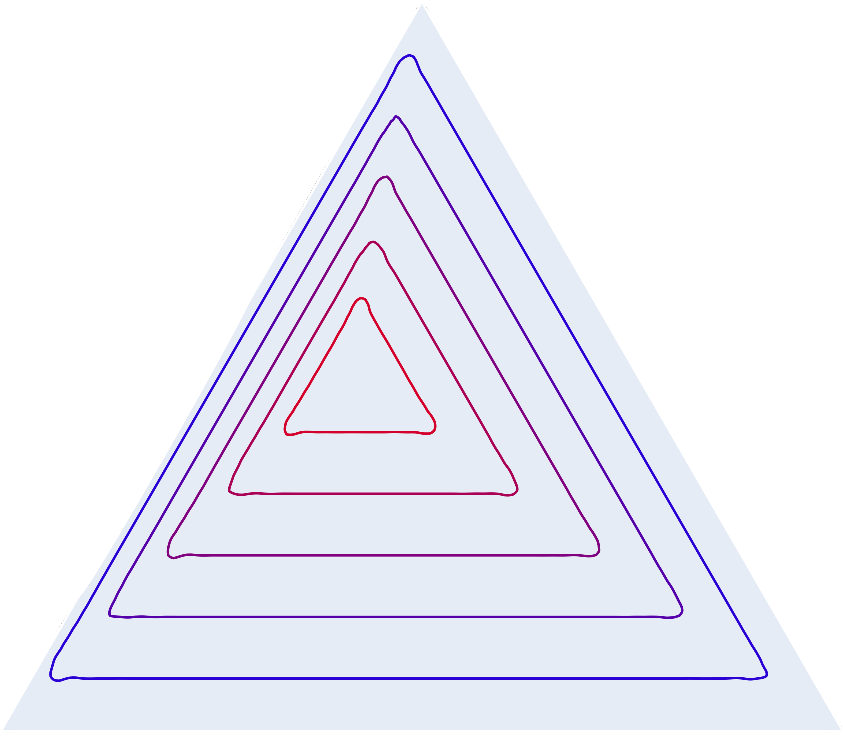
Cobb-Douglas	(0,3,3)	(0,2,4)
(3,3,0)	**18,18**	15,20
(4,2,0)	20,15	**16,16**

Leontief	(0,3,3)	(0,2,4)
(3,3,0)	**3,3**	3,4
(4,2,0)	4,3	**4,4**

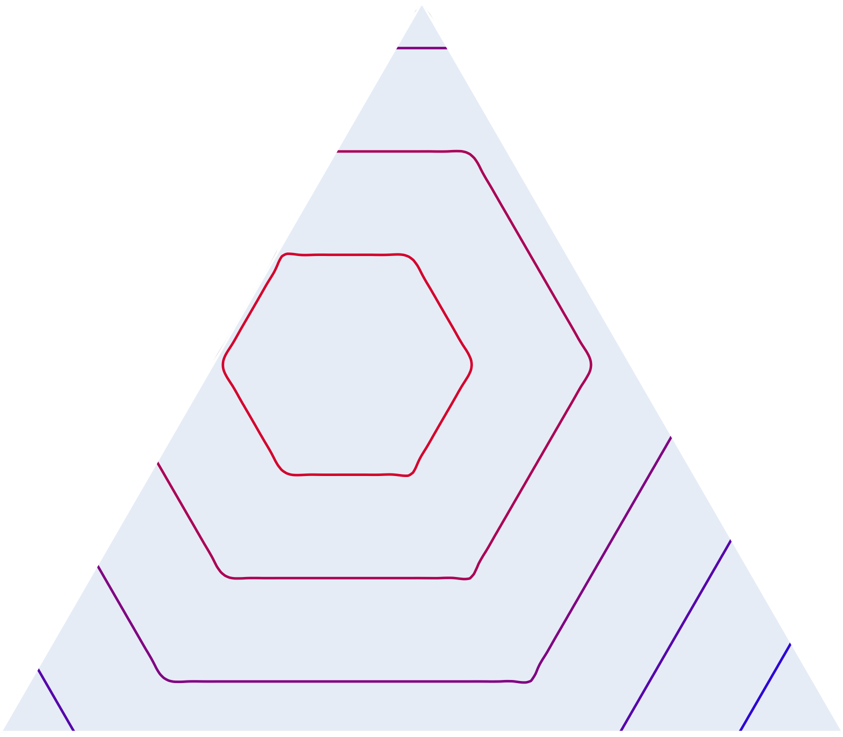
Potential Utility Functions



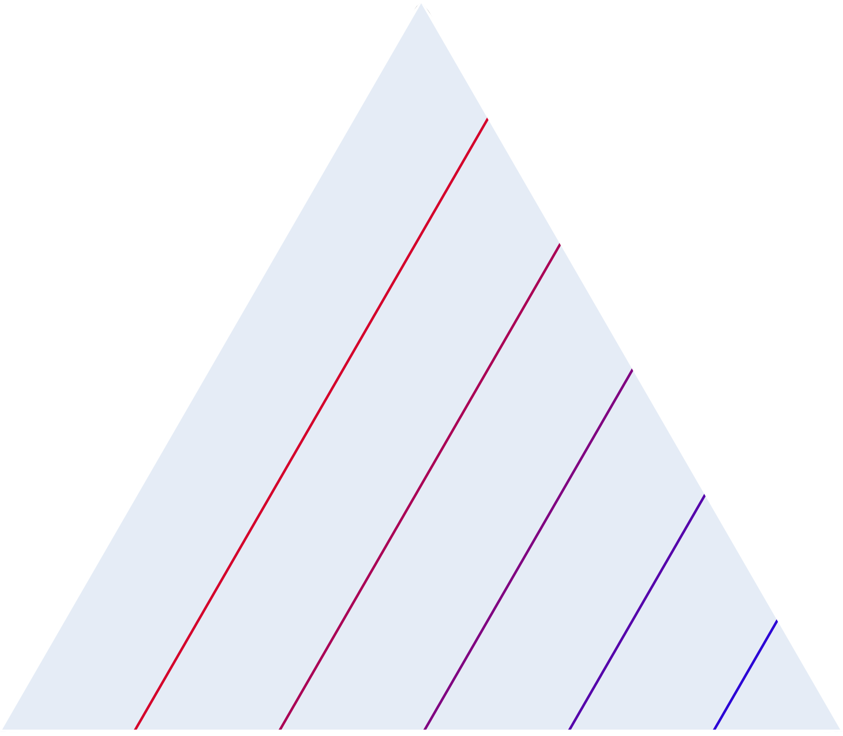
linear



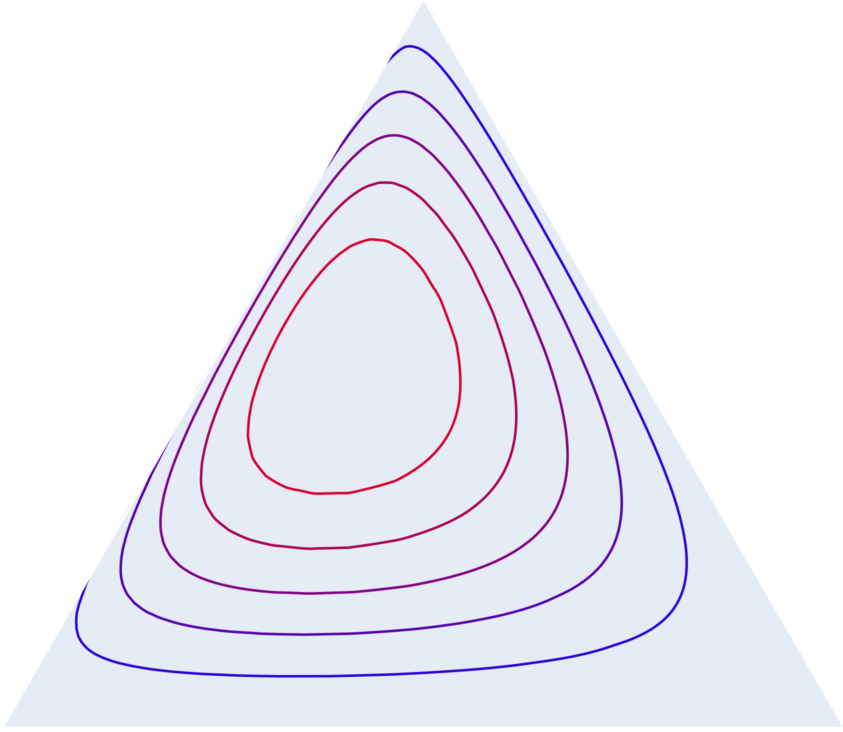
Leontief



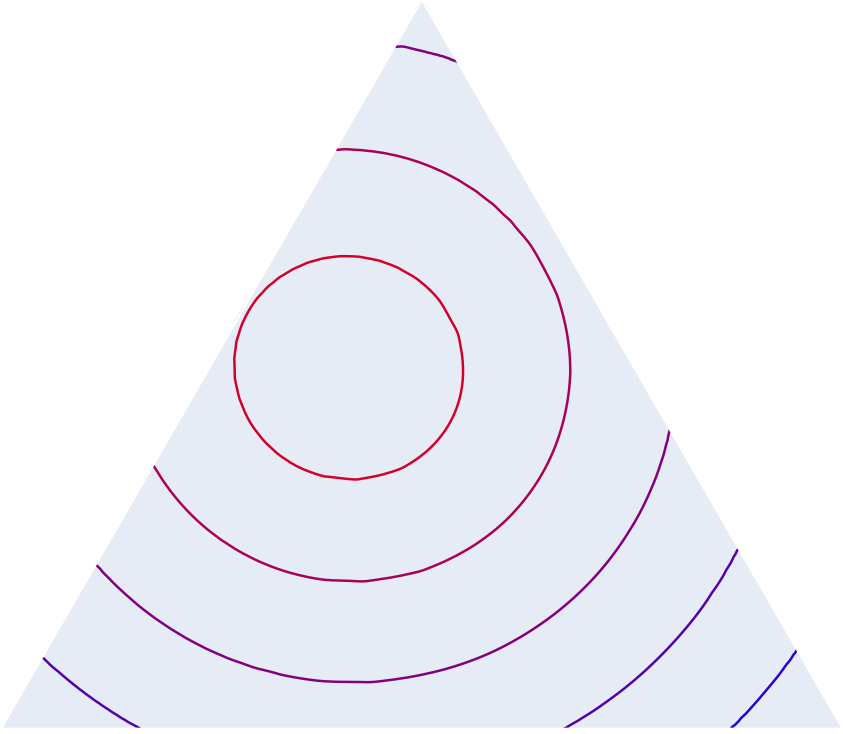
ℓ_1 disutilities



dichotomous

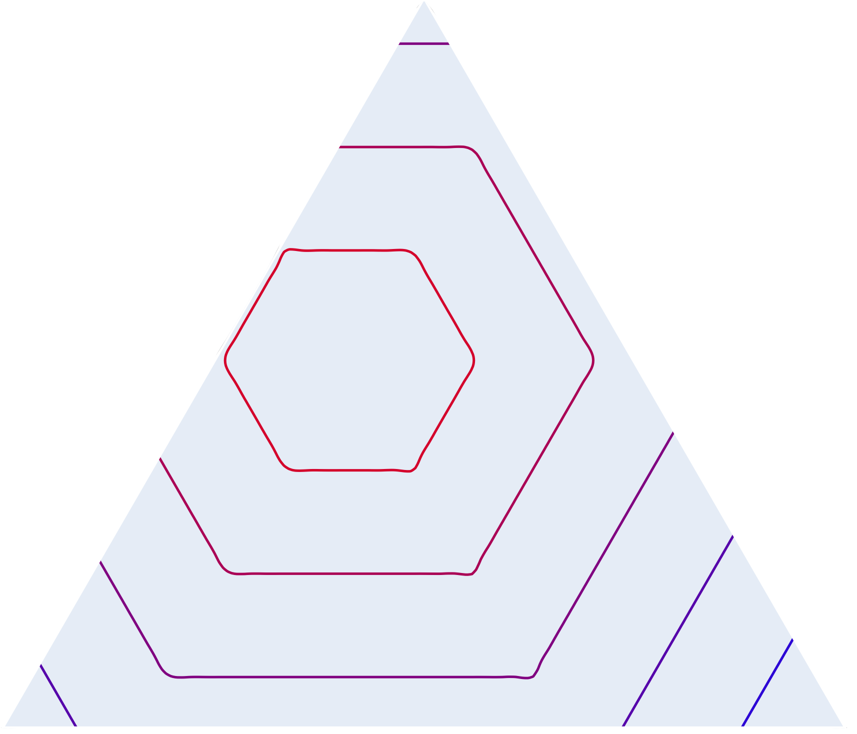


Cobb-Douglas



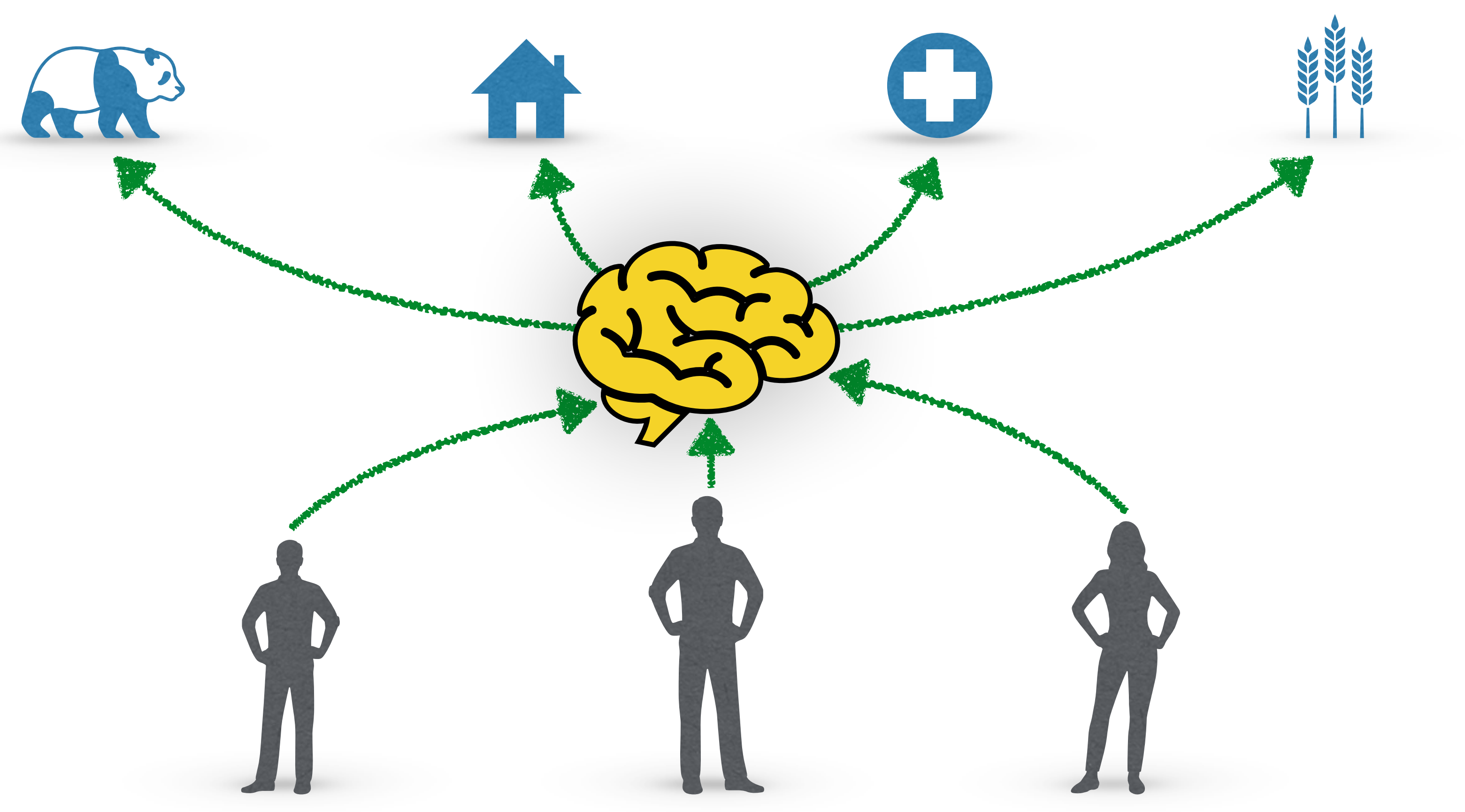
ℓ_2 disutilities

$\ell 1$ Disutilities



- $u_i(\delta) = -\sum_{x \in A} |v_i(x) - \delta(x)|$
  - $v_i(x) \geq 0$ for all $x \in A$ and $\sum_{x \in A} v_i(x) = C$.
- **Theorem (Linder et al., 2008)**: The utilitarian rule (with careful tie-breaking) satisfies efficiency and strategyproofness.
- **Theorem (Freeman et al., 2021)**: The "independent markets" rule satisfies strategyproofness and proportionality (a weakening of core fair share) but fails efficiency.
- **Theorem (B. et al., 2024)**: No distribution rule satisfies efficiency, strategyproofness, and proportionality.

Summary



- Donor coordination can increase the efficiency of charitable giving.
- For linear utilities, results are mostly negative.
- For dichotomous utilities, desirable properties need to be traded off and *CUT* and *NASH* are attractive rules.
- For Leontief utilities, *EDR* satisfies virtually all desirable properties.
- Equilibrium existence, uniqueness, and convergence extend to Cobb-Douglas utility functions.
  - Efficiency and strategyproofness break down.
- Early mixed results for ℓ_1 disutilities.
  - Equilibrium distributions and spending dynamics for ℓ_1 are largely unexplored.

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