
Collaborative Giving

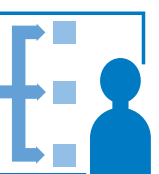
Effective Distribution of Individual Contributions

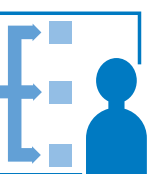
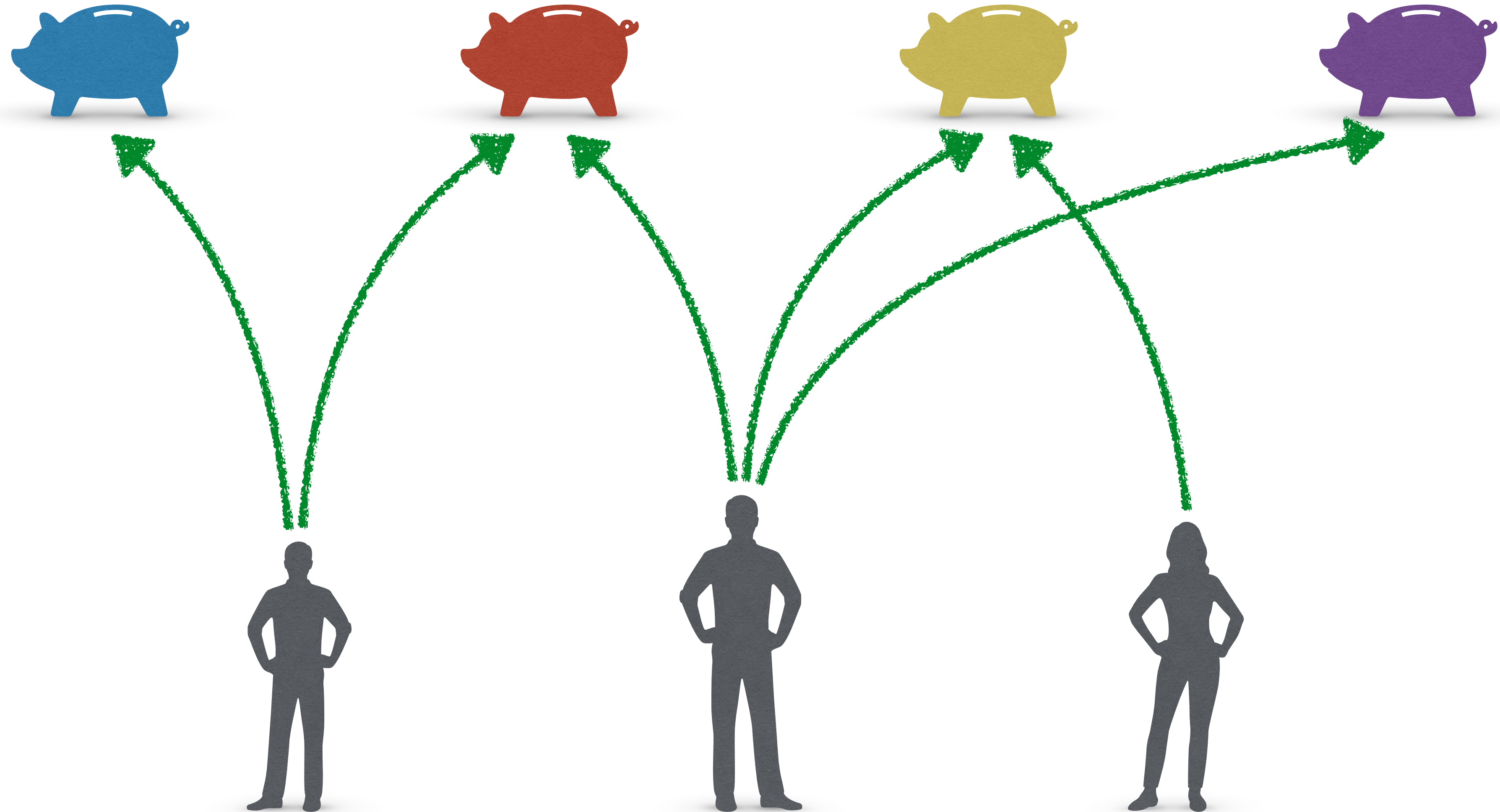
Felix Brandt

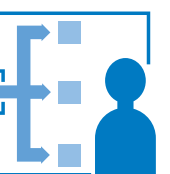
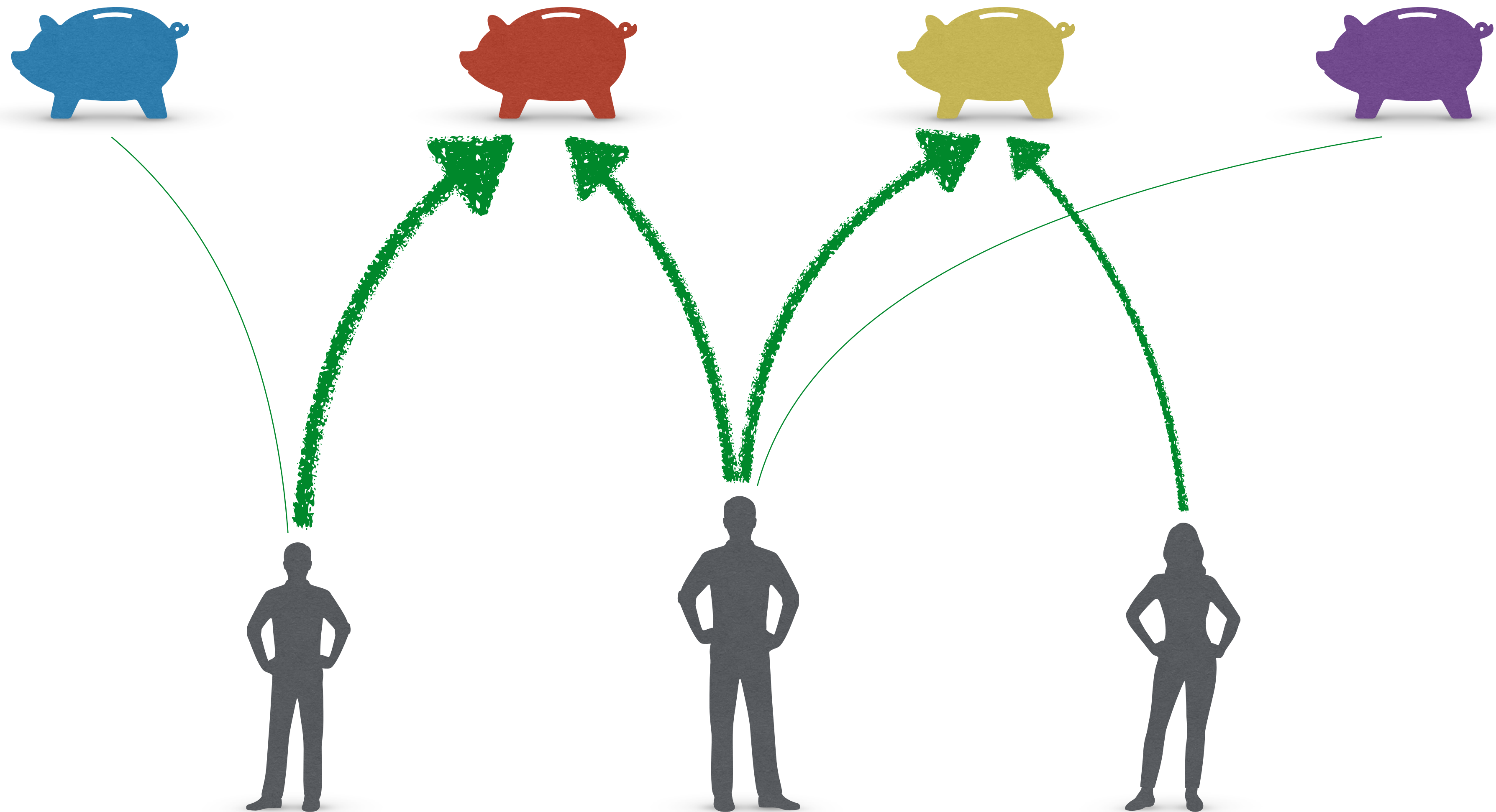
Algorithmics of Fair Division and Social Choice
NUS/IMS Singapore, December 2024

I will present results from these papers:

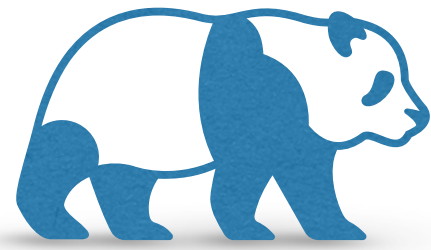
- Aziz, Bogomolnaia, and Moulin. *Fair mixing: the case of dichotomous preferences*. Presented at **ACM-EC 2019** (ACM Transactions on Economics and Computation, 2020).
- Aziz, Brandl, and B. *Universal Pareto dominance and welfare for plausible utility functions*. Presented at **ACM-EC 2014** (Journal of Mathematical Economics, 2015).
- Bogomolnaia, Moulin, and Stong: *Collective Choice under Dichotomous Preferences*. Journal of Economic Theory, 2005.
- Brandl, B., Greger, Peters, Stricker, and Suksompong. *Funding Public Projects—A Case for the Nash Product Rule*. Presented at **WINE 2021** (Journal of Mathematical Economics, 2022).
- Brandl, B., Peters, and Stricker. *Distribution rules under dichotomous preferences: Two out of three ain't bad*. Presented at **ACM-EC 2021**.
- B., Greger, Segal-Halevi, and Suksompong. *Optimal budget aggregation with single-peaked preferences*. Presented at **ACM-EC 2024**.
- B., Greger, Segal-Halevi, and Suksompong. *Coordinating charitable donations*. 2024. Presented at **ACM-EC 2023**.
- Duddy. *Fair sharing under dichotomous preferences*. Mathematical Social Sciences, 2015.
- Freeman, Pennock, Peters, and Vaughan. *Truthful aggregation of budget proposals*. Presented at **ACM-EC 2019** (Journal of Economic Theory, 2021).







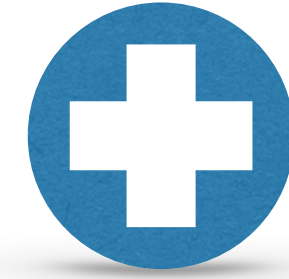
protect animals



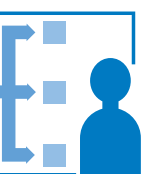
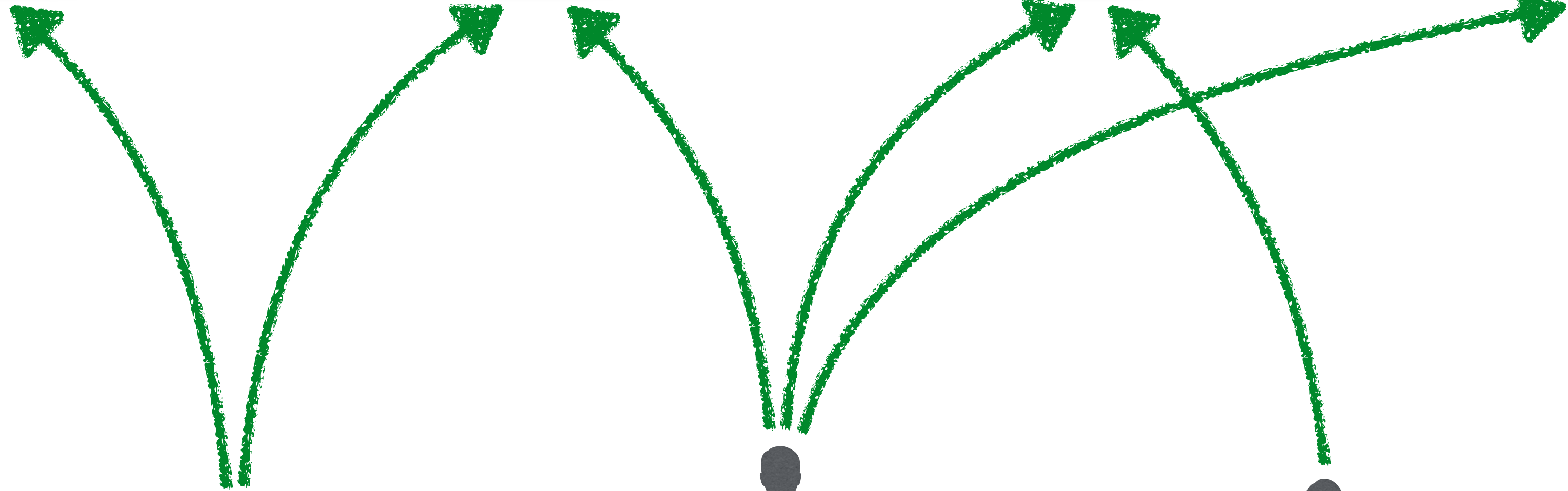
provide shelter



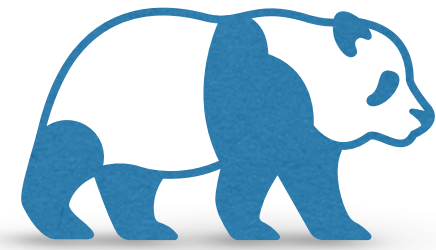
provide healthcare



prevent famines



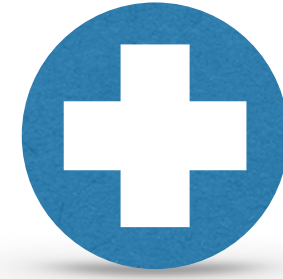
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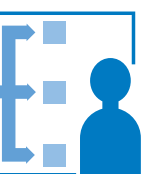
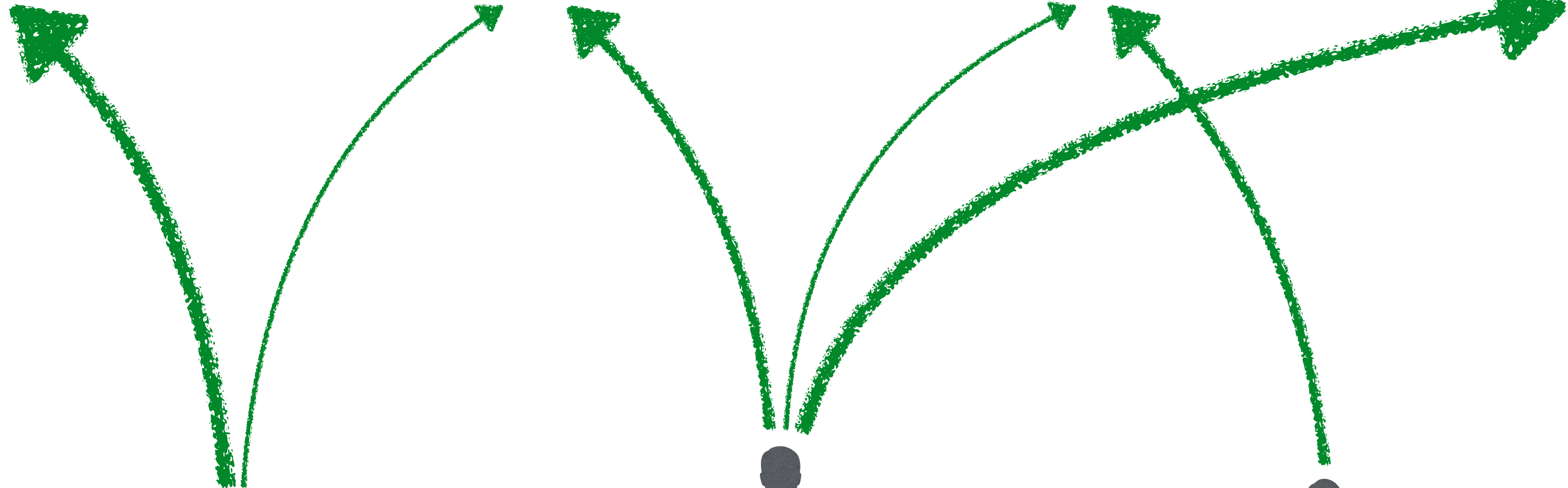
provide shelter



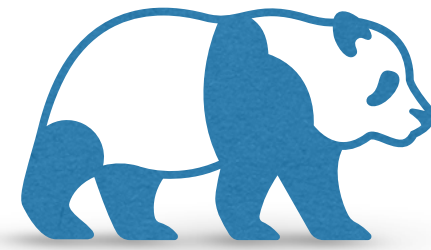
provide healthcare



prevent famines



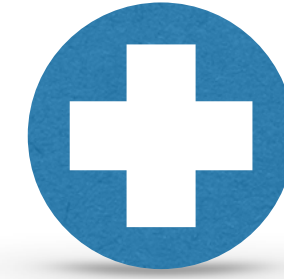
protect animals



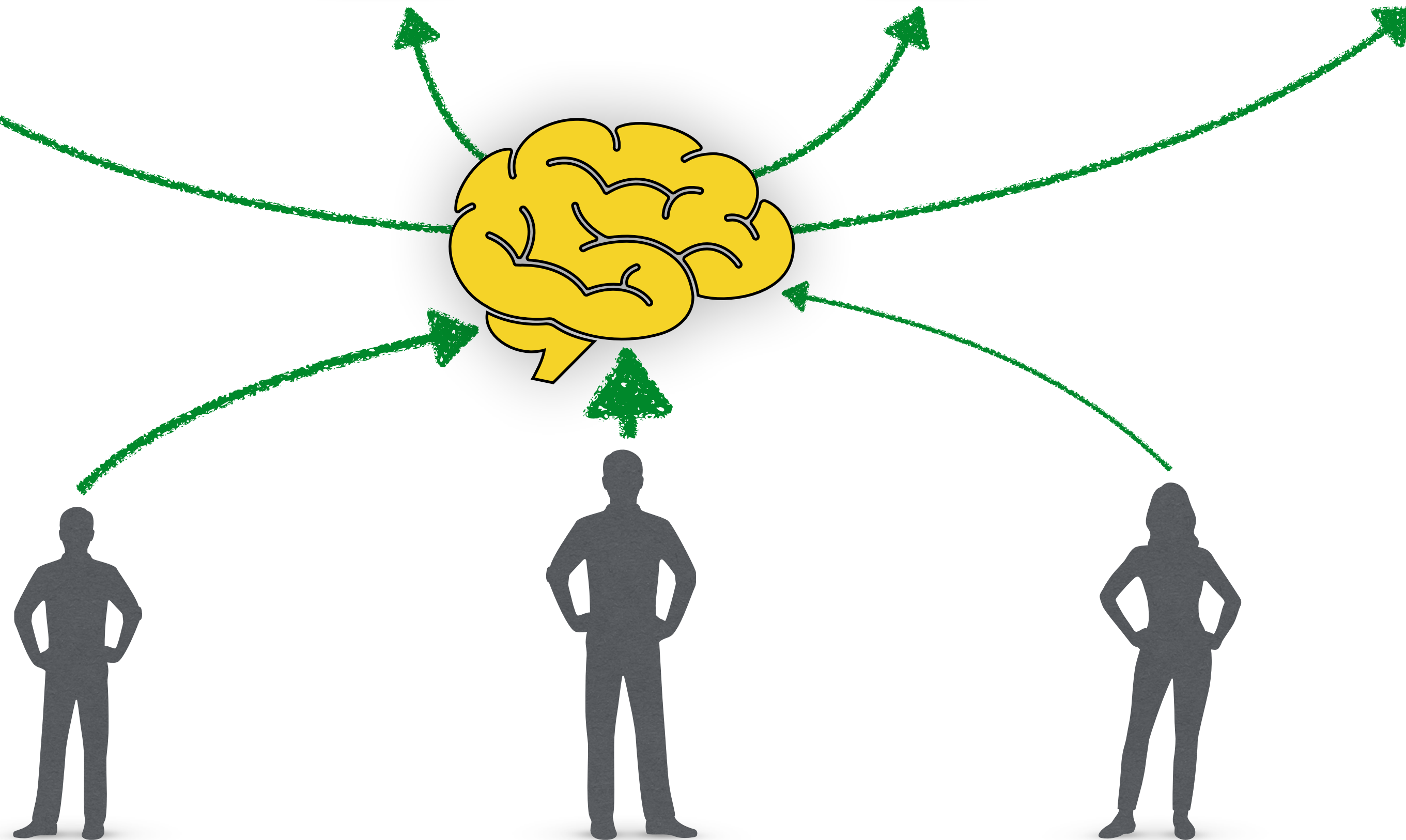
provide shelter



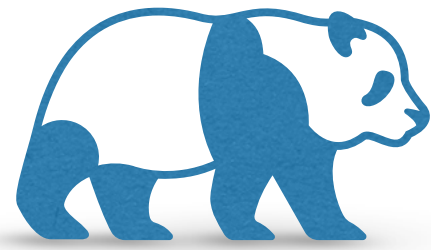
provide healthcare



prevent famines



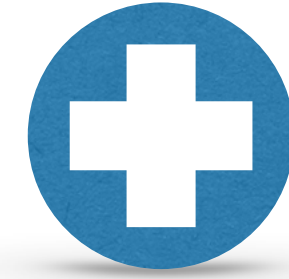
protect animals



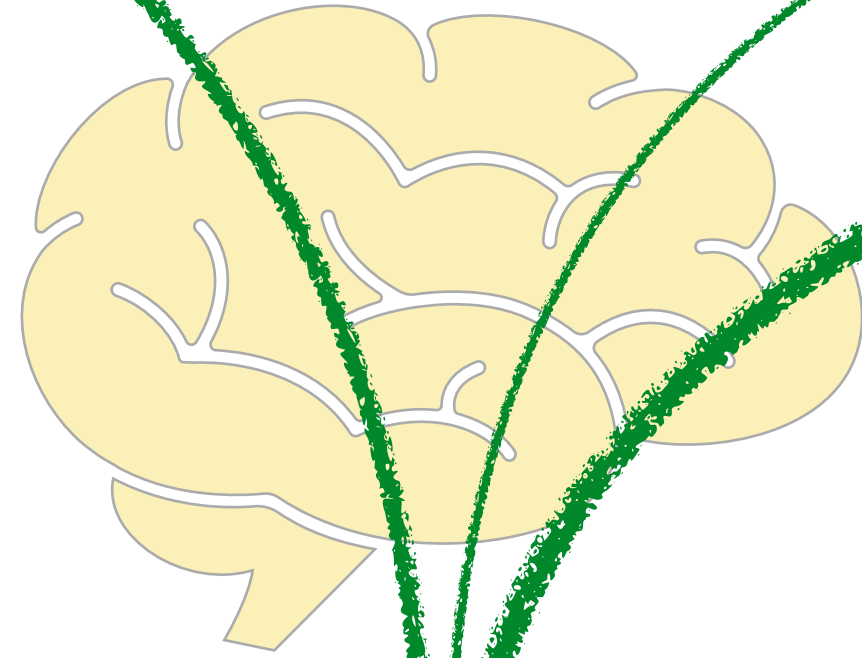
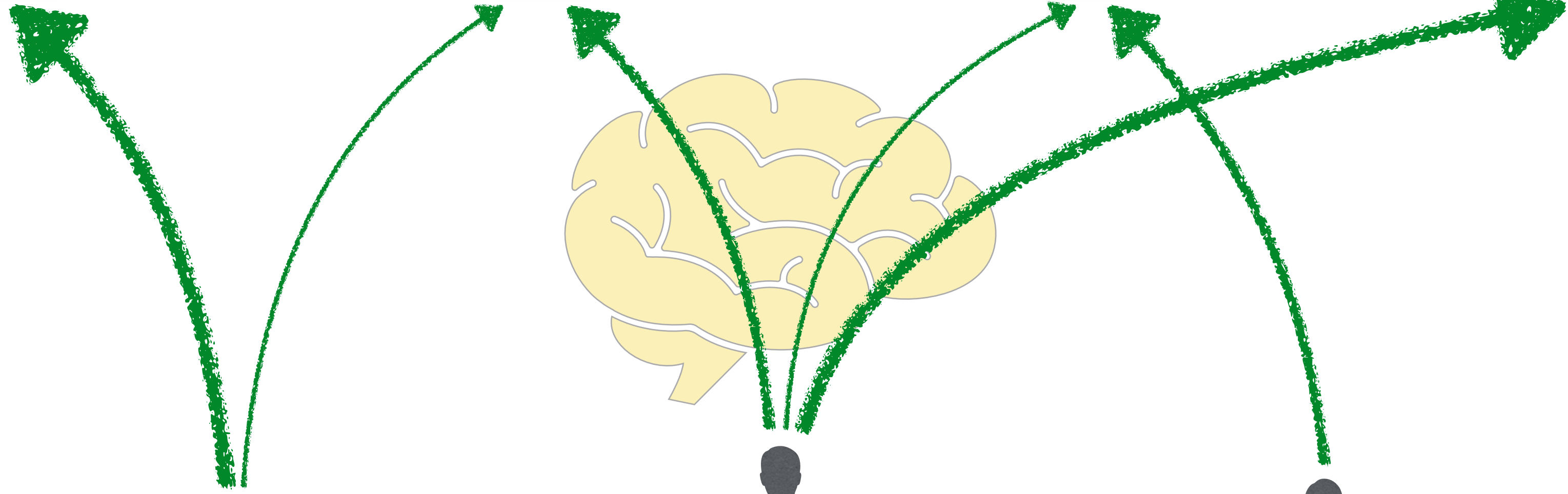
provide shelter



provide healthcare



prevent famines



Potential Applications

- ▶ **Cinque per mille**

- ▶ Italian citizens can select one of over 70k non-profit organizations.
- ▶ Revenue Agency will divert 0.5% of citizen's income tax to this organization.
- ▶ 2022: €510m



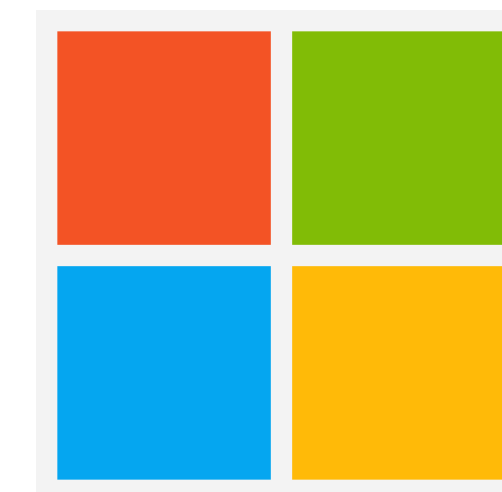
- ▶ **AmazonSmile**

- ▶ Customers can select one of over 1m non-profit organizations.
- ▶ Amazon donates 0.5% of customer's purchase price to this organization.
- ▶ 2013–2023: \$400m

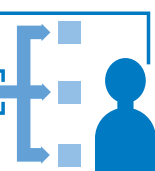


- ▶ **Employee charity matching programs**

- ▶ Microsoft (2022): \$250m to 32k organizations
- ▶ Apple (2011–2022): \$880m to 44k organizations



- ▶ **Private charity** by groups of donors



Application Scenarios

- ▶ **Fully centralized**

- ▶ Central authority owns individual contributions, collects preferences,
- ▶ then distributes endowment.

- ▶ **Centralized**

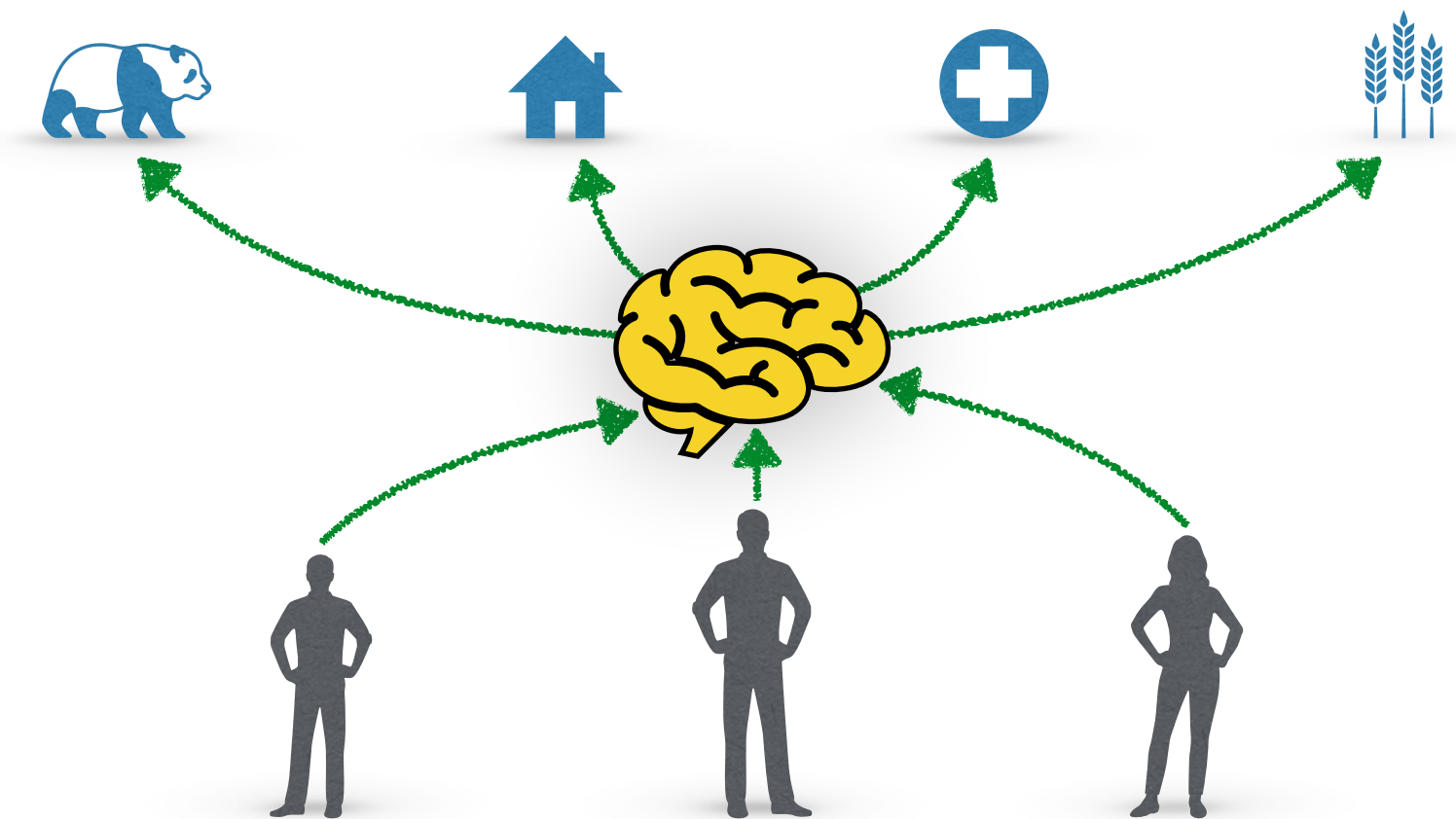
- ▶ Authority collects preferences and contributions,
- ▶ then distributes endowment.

- ▶ **Decentralized**

- ▶ Authority collects preferences,
- ▶ then advises donors how to distribute their contributions.

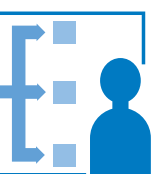
- ▶ **Fully decentralized**

- ▶ Donors independently distribute their contributions by observing previous donations.



The Model

- ▶ $N = \{1, \dots, n\}$ is a set of **agents**.
- ▶ A is a set of m **public goods** (e.g., charities).
 - ▶ Public goods are non-excludable and non-rivalrous.
- ▶ Each agent $i \in N$ contributes amount $C_i > 0$ of a **divisible and homogeneous resource** (e.g., money) to a common pool.
 - ▶ $C = \sum_{i \in N} C_i$ is called the **endowment**.
- ▶ An **individual distribution** $\delta_i \in [0, C_i]^A$ is a function with $\sum_{x \in A} \delta_i(x) = C_i$.
 - ▶ The set of all distributions of C_i is denoted by $\Delta(C_i)$.
- ▶ $\delta = \sum_{i \in N} \delta_i \in \Delta(C)$ is the **collective distribution** of the endowment C .



The Model (ctd.)

- ▶ Agent i receives **utility** $u_i(\delta) \in \mathbb{R}$ from collective distribution δ .
 - ▶ $\mathcal{U} \subseteq \mathbb{R}^{\Delta(C)}$ denotes the set of admissible utility functions.
 - ▶ $u_i \in \mathcal{U}$ for all $i \in N$.
- ▶ A **distribution rule** f maps a utility profile $U = (u_i)_{i \in N} \in \mathcal{U}$ to $(\delta_i)_{i \in N}$.
 - ▶ We will often refer to the collective distribution δ implicitly returned by $f(U)$.



Related Models

- ▶ **Private provision of public goods** (e.g., Bergstrom, Blume, and Varian, 1986)
 - ▶ agents distribute their wealth between a private and a public good
 - ▶ no preferences over different public goods
- ▶ **Probabilistic social choice/ fair mixing** (e.g., Gibbard, 1977; Bogomolnaia et al., 2005)
 - ▶ ordinal, linear, or dichotomous preferences
 - ▶ exogenous fixed “endowment” of probability mass 1
- ▶ **Participatory budgeting** (e.g., Cabannes, 2004)
 - ▶ typically fixed costs for projects, which are either fully funded or not at all
 - ▶ exogenous endowment
- ▶ **Budget aggregation** (e.g., Freeman et al., 2021)
 - ▶ norm-based preferences (typically, ℓ_1)
 - ▶ exogenous endowment



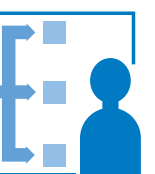
Four Desirable Properties

- ▶ The collective distribution δ returned by f is **efficient**.
 - ▶ There is no $\delta' \in \Delta(C)$ with $u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and $u_i(\delta') > u_i(\delta)$ for some $i \in N$.
 - ▶ Example:
Utilitarian rule. δ maximizes $\sum_{i \in N} C_i \cdot u_i(\delta)$.
- ▶ f is **strategyproof** if $u_i(f(u_1, \dots, u_n)) \geq u_i(f(u_1, \dots, u'_i, \dots, u_n))$ for all $i \in N$ and $u_1, \dots, u_n, u'_i \in \mathcal{U}$.
 - ▶ Examples:
Dictatorial rules. δ maximizes $u_i(\delta)$ for some fixed $i \in N$.

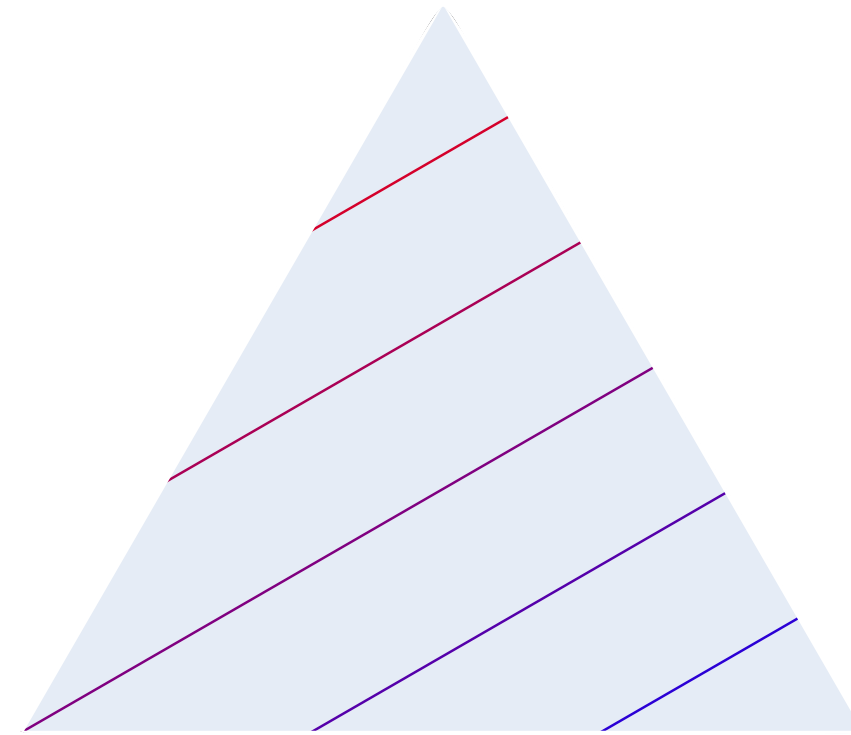


Four Desirable Properties (ctd.)

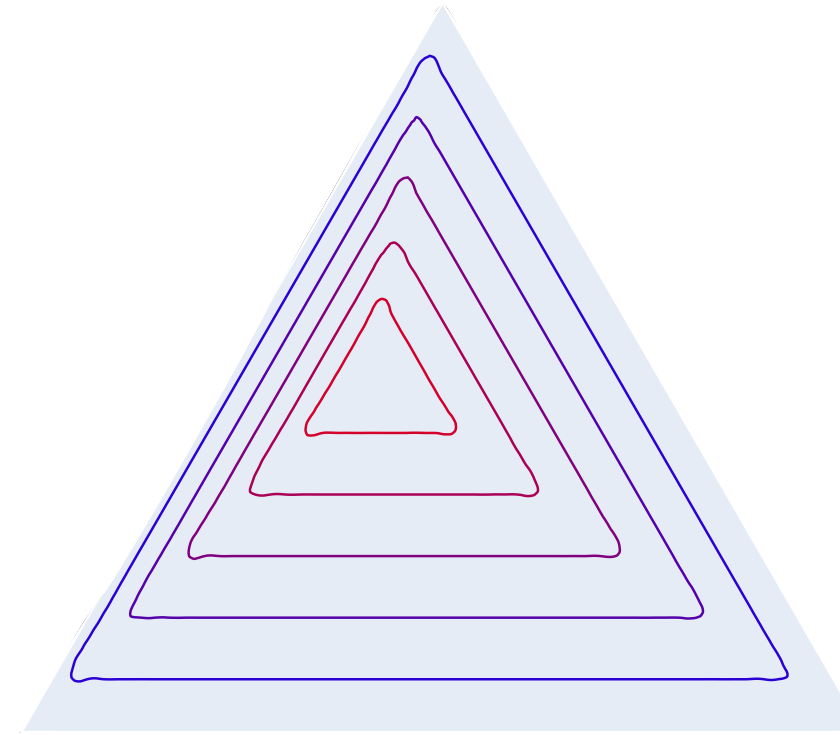
- ▶ The individual distributions $(\delta_i)_{i \in N}$ returned by f form a Nash **equilibrium**.
 - ▶ $u_i(\delta) = \max_{\delta_i^* \in \Delta(C_i)} u_i(\delta - \delta_i + \delta_i^*)$ for all $i \in N$ and.
 - ▶ crucial for settings where the authority merely issues recommendations
 - ▶ Equilibrium distributions exist under fairly general assumptions (Debreu, 1952).
- ▶ f is **contribution incentive-compatible**.
 - ▶ Agents should be better off participating than abstaining and spending their contribution optimally.
 - ▶ $u_i(f(U)) \geq \max_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ for all $U \in \mathcal{U}^N$ and $i \in N$.
 - $f_{-i}(U)$ distributes the amount $\sum_{j \in N \setminus \{i\}} C_j$ based on $(u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n)$
 - ▶ crucial for settings with voluntary participation



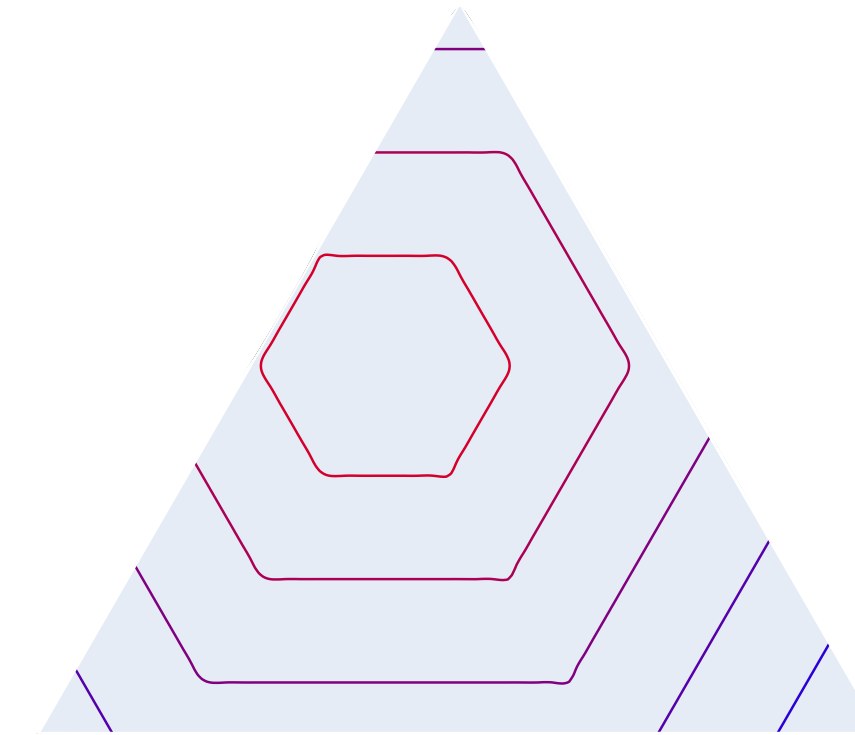
Potential Utility Functions



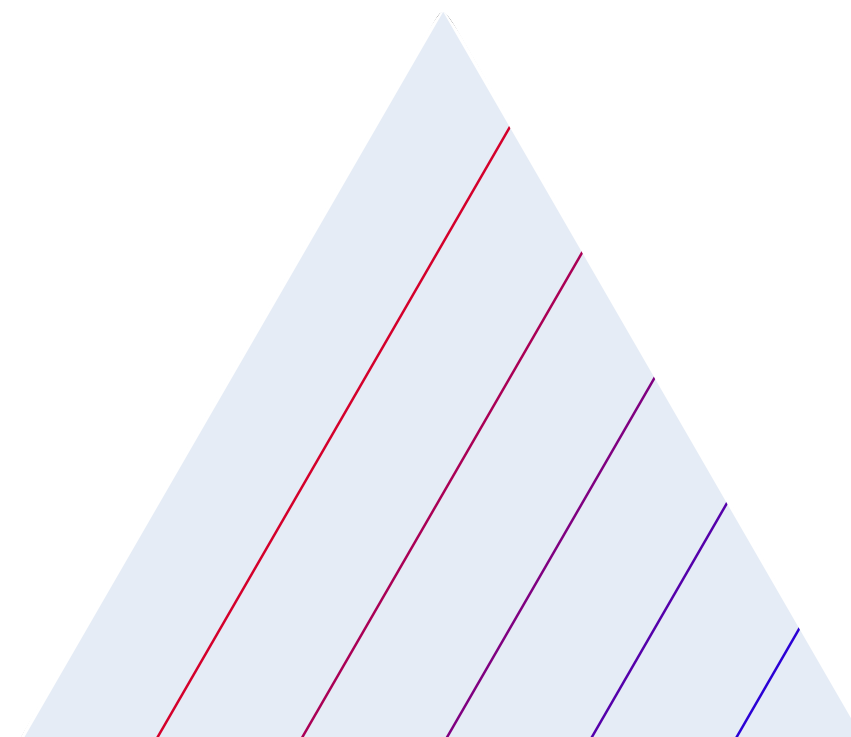
linear



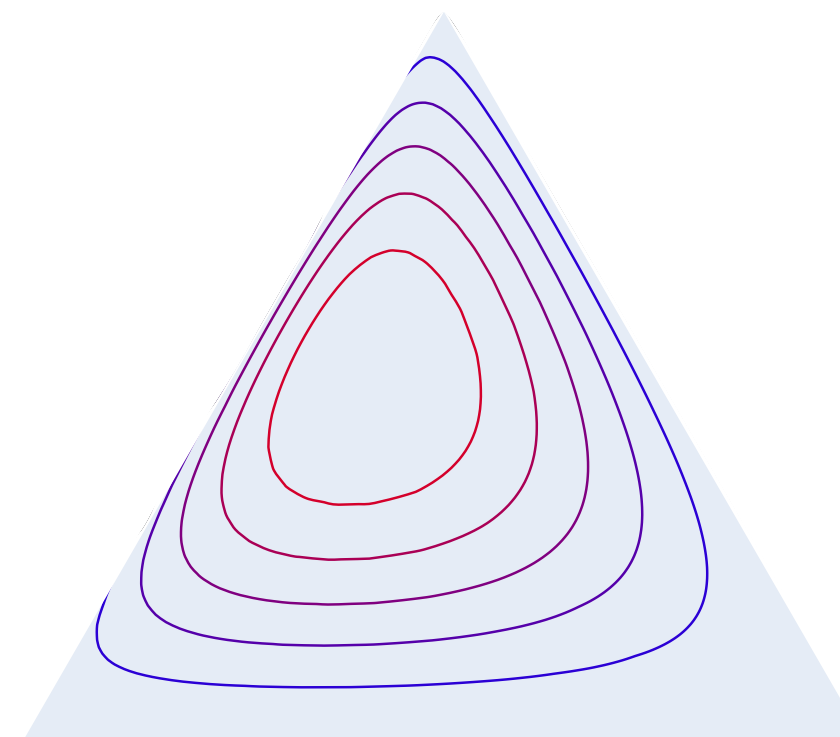
Leontief



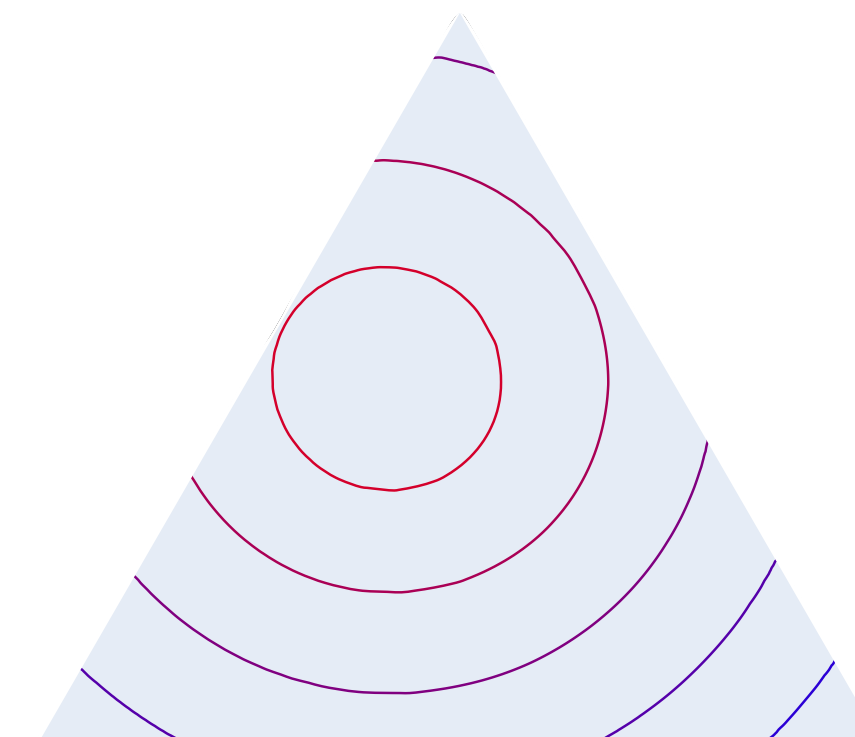
ℓ_1 disutilities



dichotomous



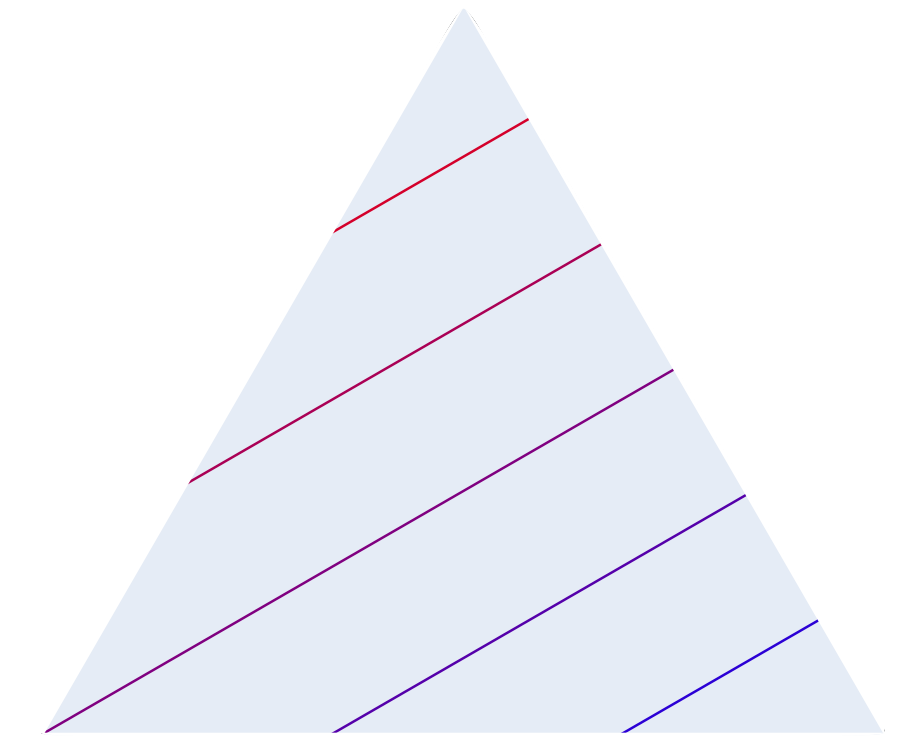
Cobb-Douglas



ℓ_2 disutilities



Linear Utilities



$$u_i(\delta) = \sum_{x \in A} \delta(x) \cdot v_i(x)$$

- Charities are substitutes with constant marginal rates of substitution.
- Example:

uncoordinated

$v_i(x) = 1$ (pointing to column 'a')

$v_i(x) = 0$ (pointing to column 'c')

	a	b	c	C_i	u_i
δ_1	0.5	0.5		1	1.5
δ_2		0.5	0.5	1	1.5
δ	0.5	1	0.5	2	

efficient

	a	b	c	C_i	u_i
δ_1		1		1	2
δ_2		1		1	2
δ		2		2	

Uncoordinated rule: Each δ_i independently maximizes $u_i((C/C_i) \cdot \delta_i)$.



Limitations under Linear Utilities

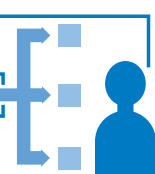
- ▶ **Theorem (Hylland, 1980):** Only *dictatorial rules* are **strategyproof** and **efficient**.
- ▶ **Proposition:** Unique **equilibrium** distributions can be **inefficient**.

$V_i(x) = 1.5$

	a	b	c	C_i	u_i
δ_1	1			1	1.5
δ_2			1	1	1.5
δ	1		1		

	a	b	c	C_i	u_i
δ_1		1		1	2
δ_2		1		1	2
δ		2			

- ▶ **Theorem (Gibbard, 1977):** When agents have unique top-ranked charities, then only the *uncoordinated rule* is anonymous, unanimous, and **strategyproof**.
- ▶ **Proposition (Brandl et al., 2022):** No efficient rule satisfies contribution incentive-compatibility.



Proof

- ▶ **Proposition (Brandl et al., 2022):** No efficient rule satisfies contribution incentive-compatibility when $m \geq 4$ and $n \geq 3$.
- ▶ Proof by contradiction:
 - ▶ Contribution incentive-compatibility: $u_i(\delta) \geq 1.6$ for all $i \in N$.

$v_i(x) = 1.6$ (with arrow pointing to cell (δ_1, a))

	a	b	c	d	C_i	u_i
δ_1					1	≥ 3.2
δ_2					1	≥ 3.2
δ_3					1	≥ 3.2
δ	>0	>0				

Assume w.l.o.g. $u_3(\delta) = \min_{i \in N} u_i(\delta)$

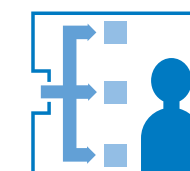
$\delta(a) > 0 \wedge \delta(b) > 0$. Efficiency is violated!

	a	b	c	d	C_i	u_i
δ'_1					1	
δ'_2					1	≥ 1.6
δ'		0		≥ 1.6		

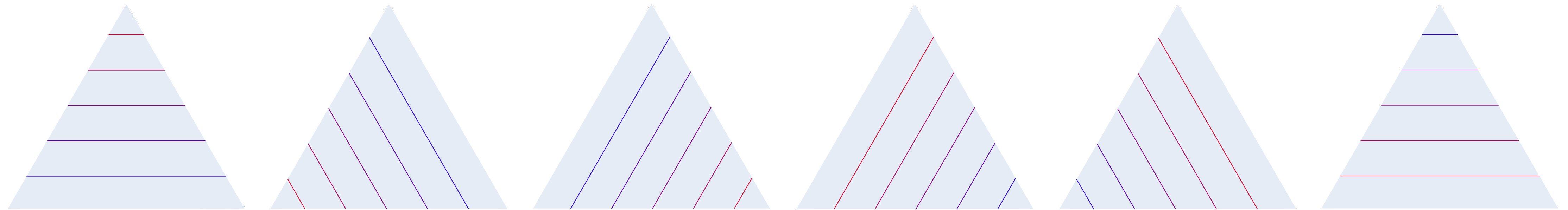
Efficiency: $\delta(a) = 0 \vee \delta(b) = 0$

W.l.o.g. $\delta(b) = 0$

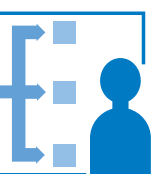
$\delta'(d) \geq 1.6$



Dichotomous Utilities



- ▶ A linear utility function u_i is **dichotomous** if $v_i(x) \in \{0,1\}$ for all $x \in A$.
 - ▶ Each agent i **approves** a non-empty set of projects $A_i \subseteq A$ and $u_i(\delta) = \sum_{x \in A_i} \delta(x)$.
- ▶ **Positive share:** $u_i(f(U)) > 0$ for all $i \in N$ and $U \in \mathcal{U}^N$.
 - ▶ much weaker than both contribution incentive-compatibility and being in equilibrium
 - ▶ minimal requirement to incentivize customers to participate in Amazon Smile:
Amazon should donate money to at least one approved charity of each customer.



UTIL

	a	b	c	d	C_i	u_i
δ_1	1				1	5
δ_2	1				1	5
δ_3	1				1	0
δ_4	1				1	0
δ_5	1				1	5
δ	5					

CUT

	a	b	c	d	C_i	u_i
δ_1	1				1	3.5
δ_2	1				1	3.5
δ_3		0.5	0.5		1	1.5
δ_4		0.5		0.5	1	1.5
δ_5	1				1	3
δ	3	1	0.5	0.5		

NASH

	a	b	c	d	C_i	u_i
δ_1	1				1	3
δ_2	1				1	3
δ_3		1			1	2
δ_4		1			1	2
δ_5	1				1	3
δ	3	2				

▸ Utilitarian rule (*UTIL*)

violates positive share!

▸ δ maximizes $\sum_{i \in N} C_i \cdot u_i(\delta)$

▸ Conditional utilitarian rule (*CUT*)

violates efficiency!

▸ among all δ_i that maximize $u_i((C/C_i) \cdot \delta_i)$,
pick those that maximize $\sum_{j \in N \setminus \{i\}} C_j \cdot u_j((C/C_i) \cdot \delta_i)$

▸ Nash product rule (*NASH*)

violates strategyproofness!

▸ δ maximizes $\prod_{i \in N} u_i(\delta)^{C_i}$ (or, equivalently, $\sum_{i \in N} C_i \log u_i(\delta)$)

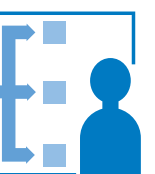
	a	b	c	d	C_i	u_i
δ_1	1				1	4.4
δ_2	1				1	3.8
δ_3		0.4	0.6		1	1.2
δ_4	0.8	0.2			1	4.4
δ_5	1				1	3.8
δ	3.8	0.6	0.6			



Two Out of Three Ain't Bad

	<i>UTIL</i>	<i>CUT</i>	<i>NASH</i>
efficient	✓	-	✓
strategyproof	✓	✓	-
positive share	-	✓	✓

- ▶ **Theorem (Brandl et al., 2021):** No distribution rule satisfies efficiency, strategyproofness, and positive share when $m \geq 4, n \geq 6$.
 - ▶ confirms a conjecture by Bogomolnaia, Moulin, and Stong (2005)
 - ▶ significantly weaker notion of strategyproofness suffices: manipulation only “counts” if $u_i(\delta') = C$.



A FULL PROOF OF THEOREM

A.1 Assuming $f(\mathcal{A}_1)$ has support bc or abc leads to contradiction.

	A_1	A_2	A_3	A_4	A_5	A_6	possible supports	dominated supports
Profile 1	b	c	ab	ac	bd	cd	$\underline{bc}, \underline{abc}, bcd$	$ad \leftarrow bc$
Profile 2	b	c	abc	ac	bd	cd	\underline{bc}, bcd	$a \leftarrow c, ab \leftarrow bc, ad \leftarrow bc$

A Weaker Impossibility

- ▶ **Theorem (Brandl et al., 2021):** No **anonymous** and **neutral** distribution rule satisfies efficiency, strategyproofness, and positive share when $m \geq 4, n \geq 5$.
- ▶ Proof:

	a	b	c	d	C_i	u_i
δ_1					1	
δ_2					1	
δ_3					1	>0
δ_4					1	$<C$
δ_5					1	
δ		>0	>0			

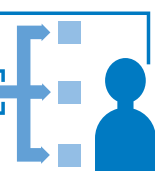
Anonymity and neutrality: $\delta(b) = \delta(c)$.

Positive share: $\delta(b), \delta(c) > 0$. Hence, $u_4(\delta) < C$

	a	b	c	d	C_i	u_i
δ'_1					1	
δ'_2					1	
δ'_3					1	
δ'_4					1	C
δ'_5					1	
δ'			0	0		

Anonymity and neutrality: $\delta'(c) = \delta'(d)$.

Efficiency: $\delta'(c), \delta'(d) = 0$. Hence, $u_4(\delta') = C > u_4(\delta)$.



Further Results

- ▶ **Core fair share**

- ▶ $\forall S \subseteq N \nexists \delta' \in \Delta(\sum_{j \in S} C_j) \forall \delta'' \in \Delta(\sum_{j \in N \setminus S} C_j)$ s.t. $\forall i \in N: u_i(\delta' + \delta'') \geq u_i(\delta)$ and $\exists i \in N: u_i(\delta' + \delta'') > u_i(\delta)$.

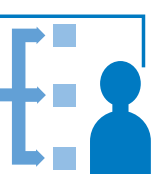
- ▶ **Theorem (Aziz et al., 2020):** *NASH* satisfies core fair share, *CUT* does not.

- ▶ **Theorem (Duddy, 2015):** *CUT* satisfies **group fair share**, a weakening of core fair share.

- ▶ **Theorem (Brandl et al., 2022):** A distribution rule satisfies group fair share iff it returns equilibrium distributions.

- ▶ **Positive share** is weaker than group fair share.

- ▶ **Theorem (Brandl et al., 2022):** *NASH* and *CUT* always return **equilibrium** distributions and satisfy **contribution incentive-compatibility**.



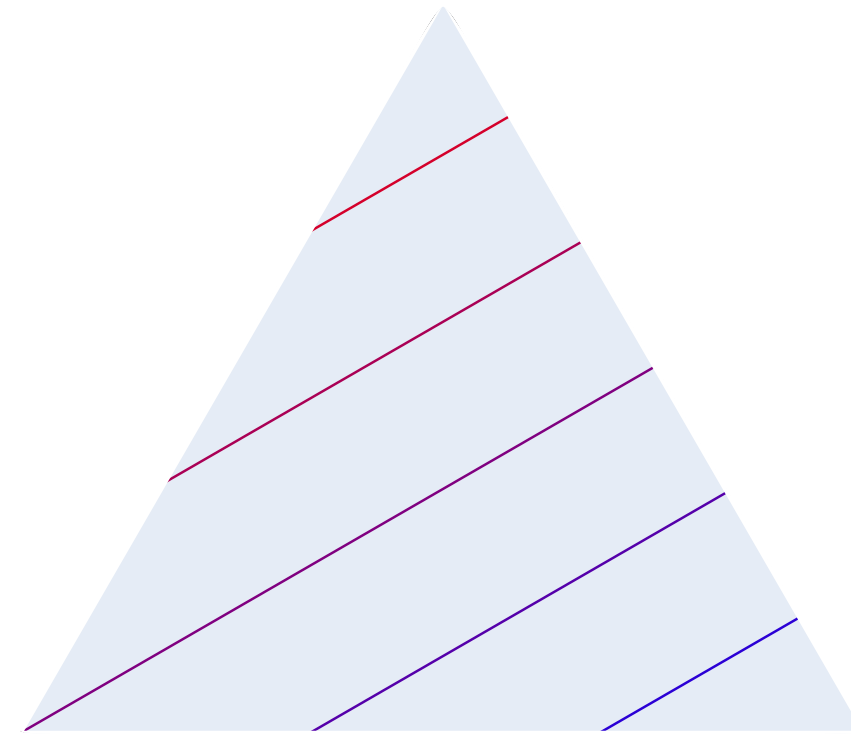
Summary and Open Problems

	<i>UTIL</i>	<i>CUT</i>	<i>NASH</i>	No rule!
efficiency	✓	-	✓	↯
strategyproofness	✓	✓	-	↯
contribution incentive-compatibility	-	✓	✓	
core fair share	-	-	✓	
group fair share (=in equilibrium)	-	✓	✓	
positive share	-	✓	✓	↯

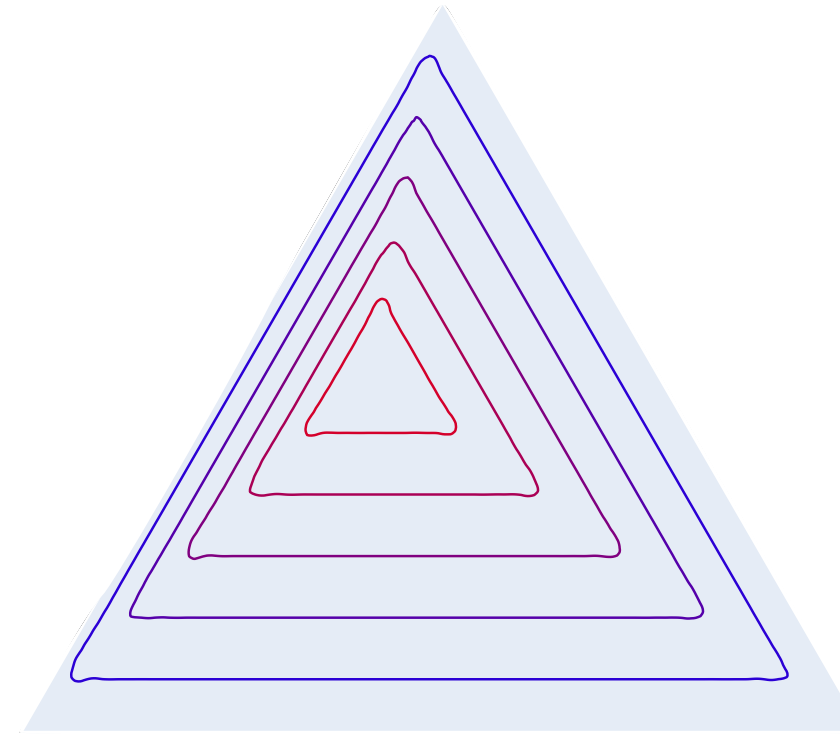
- ▶ Quantify **efficiency failures of *CUT*** and **strategyproofness failures of *NASH***
- ▶ Pending **axiomatic characterizations:**
 - ▶ *CUT* using contribution incentive-compatibility and strategyproofness
 - ▶ *NASH* using contribution incentive-compatibility and efficiency
 - ▶ *NASH* using core fair share



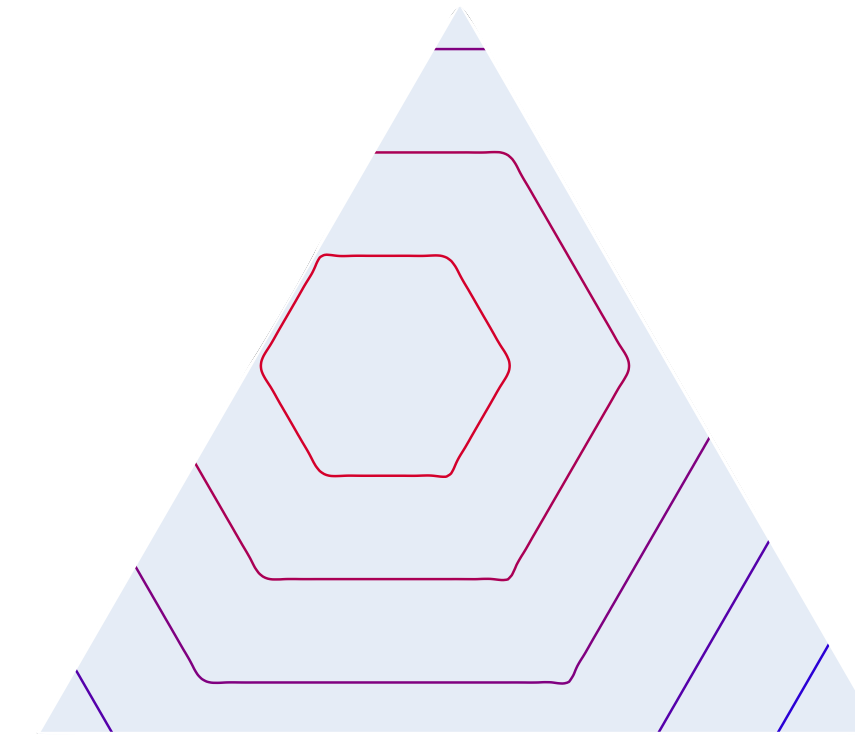
Potential Utility Functions



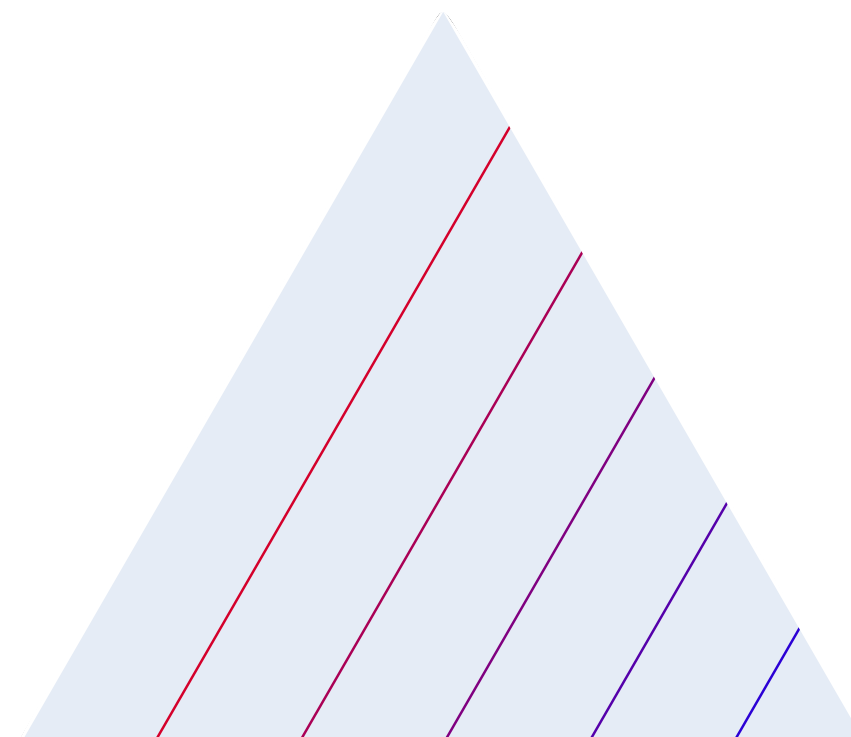
linear



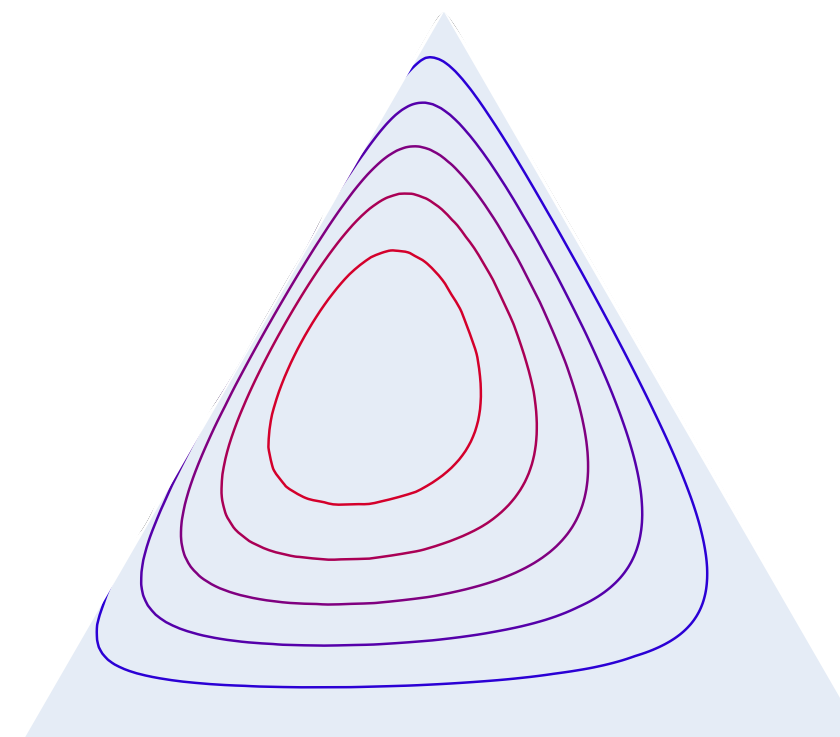
Leontief



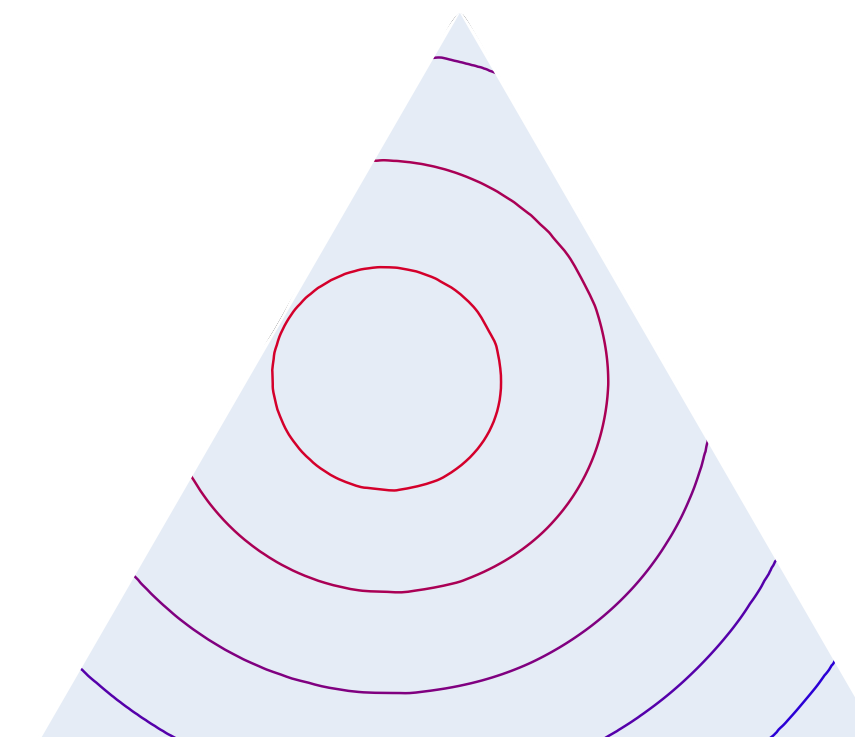
ℓ_1 disutilities



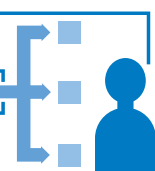
dichotomous

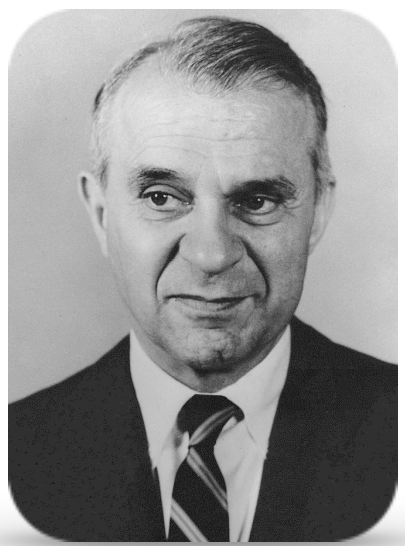


Cobb-Douglas



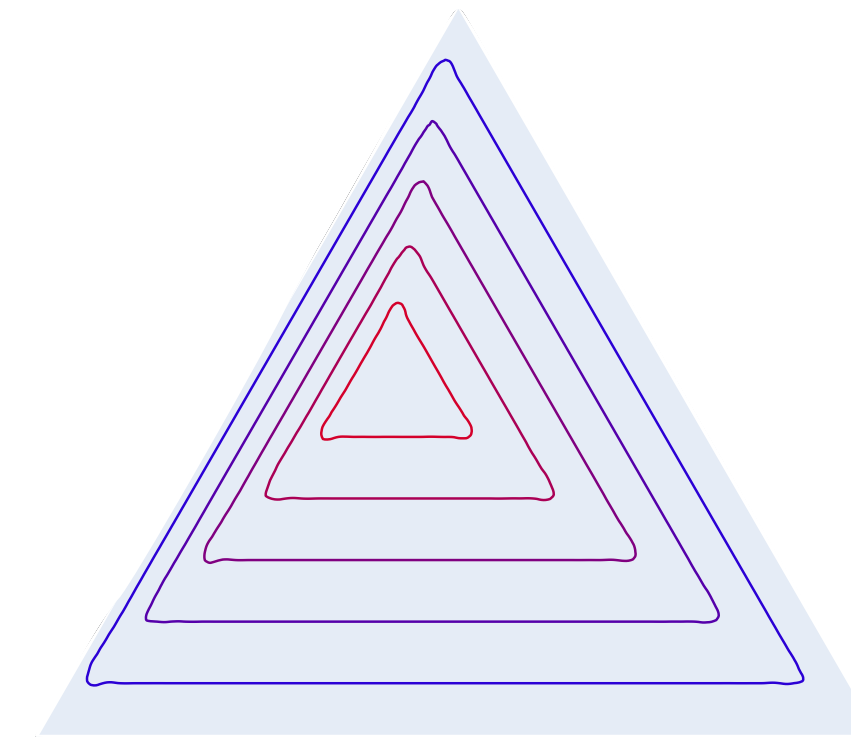
ℓ_2 disutilities





Wassily Leontief

Leontief Utilities



$$u_i(\delta) = \min_{x \in A: v_i(x) > 0} \frac{\delta(x)}{v_i(x)}$$

- $v_i(x) \geq 0$ for all $x \in A$ and $v_i(x) > 0$ for some $x \in A$.
- Charities are **complements** rather than substitutes.
- **Theorem (B. et al., 2023)**: Each utility profile admits a **unique equilibrium**. This distribution **maximizes Nash welfare** and thus is **efficient**.
- *EDR* (equilibrium distribution rule) returns the equilibrium distribution.
- Example:

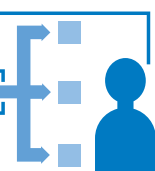
	a	b	c	C_i	u_i
δ_1	1	0.5		1.5	1
δ_2		0.5	1	1.5	1
δ	1	1	1	3	



- Healthcare example
 - Charity a supports patients with common disease
 - Each charity b_i supports patients with some rare disease.
- Unique equilibrium for linear utilities.

$v_i(b_i)=1$ $v_i(a)=2$

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	30											30
δ_2	30											30
δ_3	30											30
δ_4	30											30
δ_5	30											30
δ_6	30											30
δ_7	30											30
δ_8	30											30
δ_9	30											30
δ_{10}	30											30
δ	300											



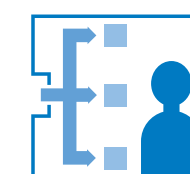
- Healthcare example
 - Charity a supports patients with common disease
 - Each charity b_i supports patients with some rare disease.
- **Uncoordinated rule for Leontief utilities**

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	20	10										30
δ_2	20		10									30
δ_3	20			10								30
δ_4	20				10							30
δ_5	20					10						30
δ_6	20						10					30
δ_7	20							10				30
δ_8	20								10			30
δ_9	20									10		30
δ_{10}	20										10	30
δ	200	10	10	10	10	10	10	10	10	10	10	



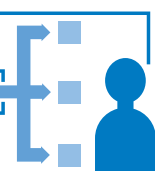
- Healthcare example
 - Charity a supports patients with common disease
 - Each charity b_i supports patients with some rare disease.
- **EDR** (unique equilibrium for Leontief utilities)

	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	C_i
δ_1	5	25										30
δ_2	5		25									30
δ_3	5			25								30
δ_4	5				25							30
δ_5	5					25						30
δ_6	5						25					30
δ_7	5							25				30
δ_8	5								25			30
δ_9	5									25		30
δ_{10}	5										25	30
δ	50	25	25	25	25	25	25	25	25	25	25	



Equilibrium Distribution Rule

- ▶ The tradeoffs present in the case of linear and dichotomous utilities vanish!
- ▶ **Theorem (B. et al., 2023):** *EDR* is (group-)strategyproof.
- ▶ **Theorem (\cdot):** The equilibrium is rational-valued and can be computed in polynomial time via convex programming and a separation oracle.
- ▶ **Theorem (\cdot):** *EDR* has nice monotonicity properties:
 - ▶ Agent increases contribution \Rightarrow funding of no charity decreases
 - ▶ Agent increases weight for charity \Rightarrow funding of charity does not decrease
 - ▶ For dichotomous utilities, both properties are violated by *NASH*.



Further Results

- ▶ **Theorem (B. et al., 2024):** *EDR* is the only rule that satisfies group-strategyproofness, core fair share, and continuity.
- ▶ For all $U \in \mathcal{U}^N$ and $i \in N$:
 - ▶ $u_i(f(U)) \geq \max_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ (Contribution incentive-compatibility)
 - ▶ $u_i(f(U)) > \min_{\delta_i^* \in \Delta(C_i)} u_i(f_{-i}(U) + \delta_i^*)$ (**Strict participation**)
 - ▶ $f_{-i}(U)$ distributes the amount $\sum_{j \in N \setminus \{i\}} C_j$ based on $(u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n)$.
- ▶ Strict participation is weaker than contribution incentive-compatibility.
- ▶ **Theorem (B. et al., 2023):** *EDR* satisfies strict participation.
- ▶ **Proposition:** No distribution rule satisfies contribution incentive-compatibility for Leontief utilities.



Proof

- ▶ **Proposition:** No distribution rule satisfies contribution incentive-compatibility.
- ▶ Proof by contradiction:

	a	b	c	d	C_i	u_i
δ_1					6	≥ 4
δ_2					6	≥ 3
δ	≥ 4	≥ 4	≥ 3	≥ 3		

$$u_1(\delta) \geq 4 \text{ and } u_2(\delta) \geq 3$$

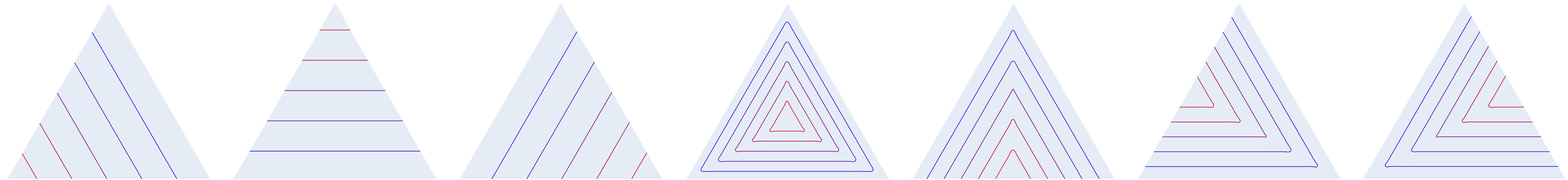
$$4 + 4 + 3 + 3 = 14 \geq C = 6 + 6 = 12 \quad \text{↯}$$

	a	b	c	d	C_1	u_1
δ_1	3	3			6	3

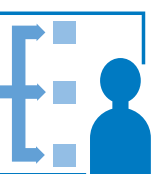
	a	b	c	d	C_2	u_2
δ_2		2	2	2	6	2



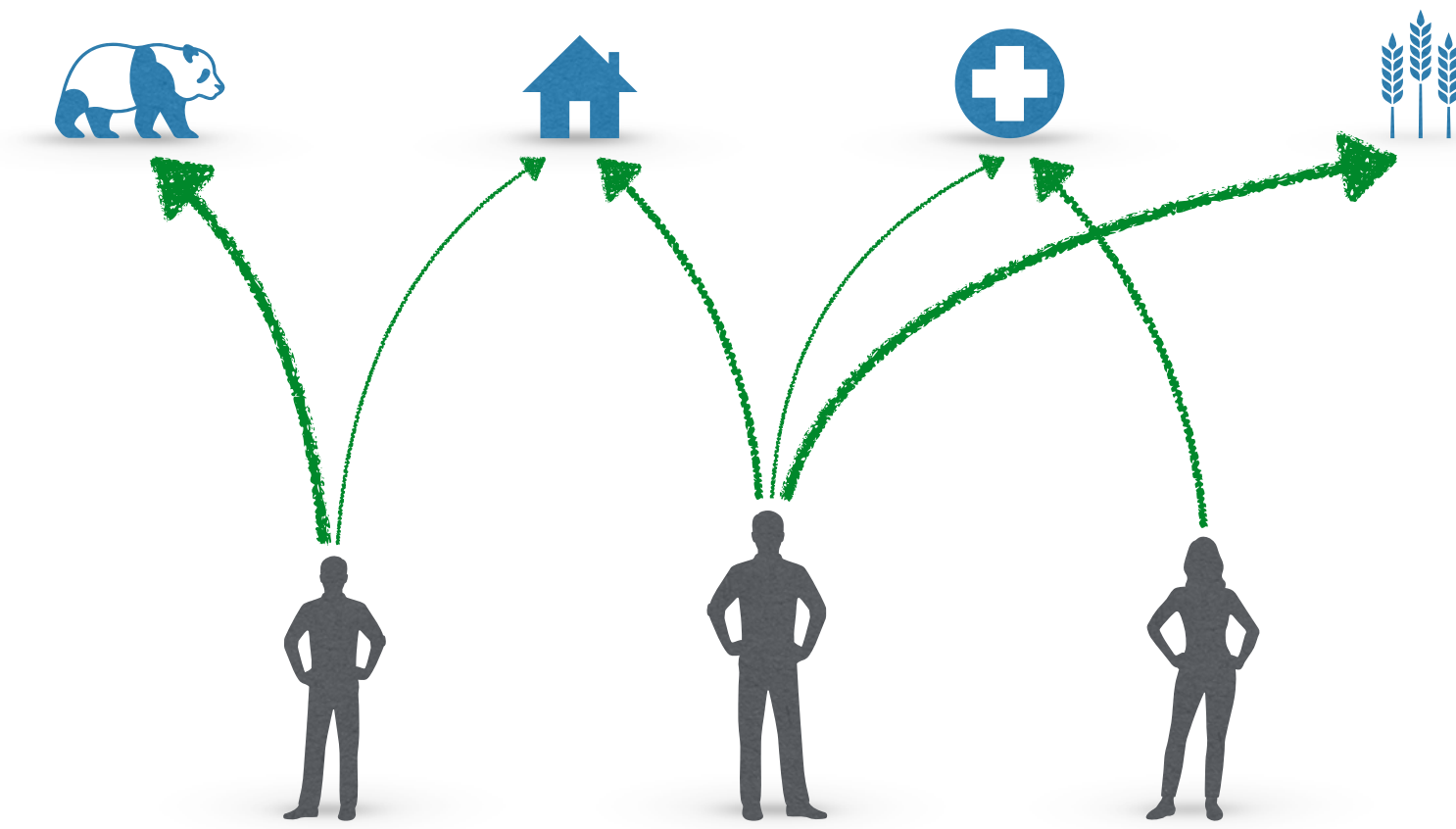
Binary Weights



- ▶ A Leontief utility function u_i has **binary weights** if $v_i(x) \in \{0,1\}$ for all $x \in A$.
- ▶ **Theorem (B. et al., 2023)**: For binary weights, *EDR* coincides with **egalitarian** rules:
 - Among all $\delta \in \Delta(C)$ with $\sum_{x \in A: v_i(x) > 0} \delta_i(x) = C_i$, *EDR* lexicographically maximizes both
 - ▶ $\min_{x \in A} \delta(x)$ (the minimal contribution to a charity), and
 - ▶ $\min_{i \in N} u_i(\delta)$ (the minimal utility of an agent).
- ▶ For binary weights, *EDR* can thus be computed via **linear programming**.



Spending Dynamics



- ▶ Each agent has set aside a, say, *monthly* budget for charitable activities.
- ▶ Agents become active in round-robin order.
- ▶ Each agent observes the accumulated distribution of the last $n - 1$ rounds and then distributes her own contribution **myopically optimal**.
- ▶ **Theorem (B. et al., 2023)**: The collective distribution of the last n rounds **converges to EDR**.
- ▶ Even with occasional changes to preferences and contributions, the relative overall distribution keeps converging towards the equilibrium distribution.

	a	b	c	d	C_i	u_i
δ_1	18	18			36	18
δ_2		6	24	24	54	24
δ_3			18		18	42
δ_1	21	15			36	21
δ_2		14	11	29	54	29
δ_3			18		18	29
δ_1	25	11			36	25
	\vdots	\vdots	\vdots	\vdots		
δ	27	27	27	27	108	



Thank you
for your contribution.

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ศูนย์ไตเทียม ศิริราช บ. น. ๒๒๒
ร.พ.ศิริราช ๒๒๒ ถนนศิริราช บ. น. ๒๒๒
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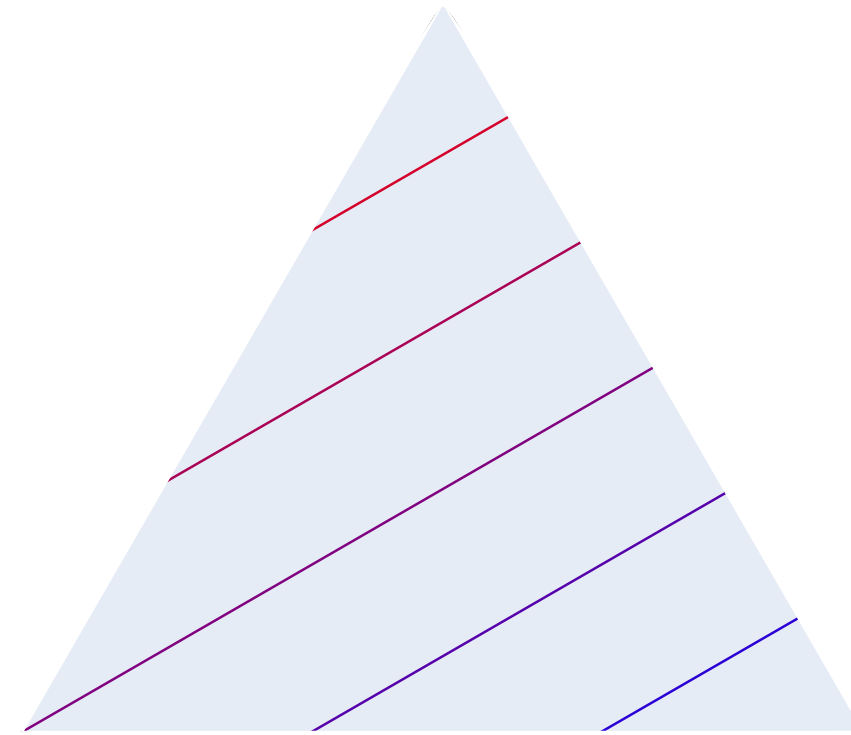
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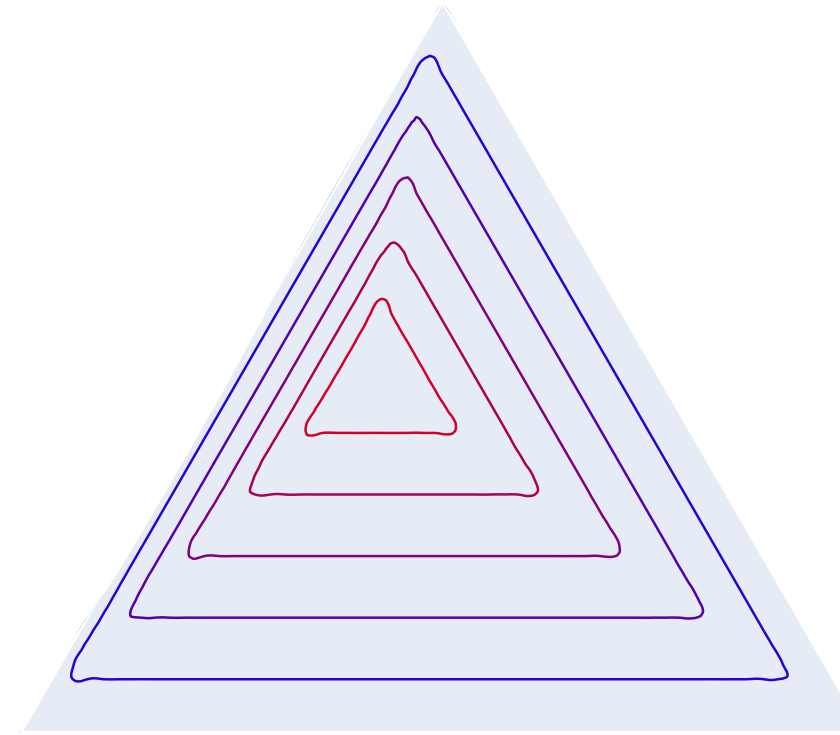
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unicef
for every child

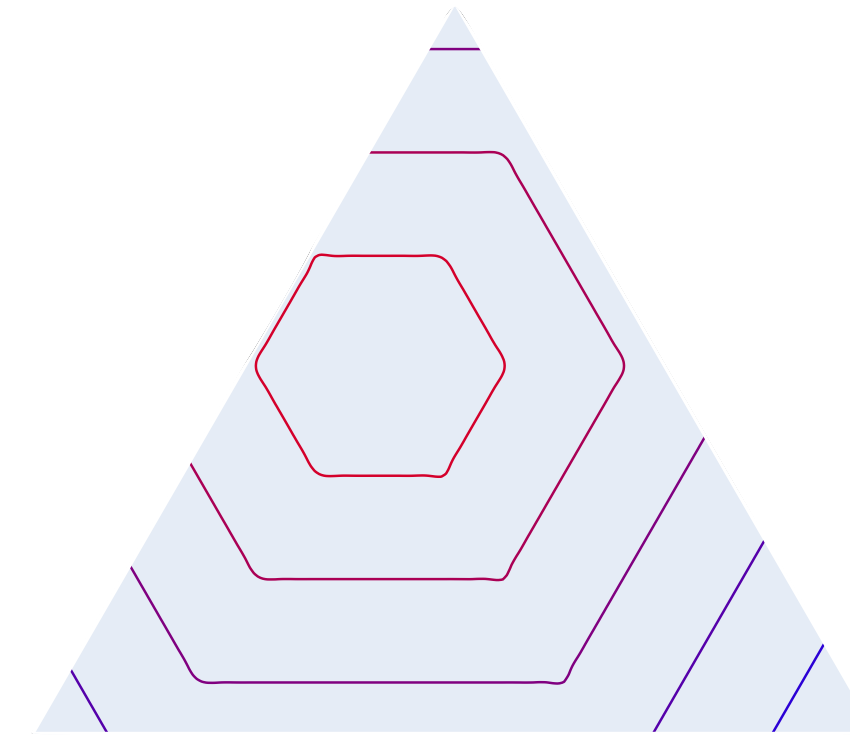
Potential Utility Functions



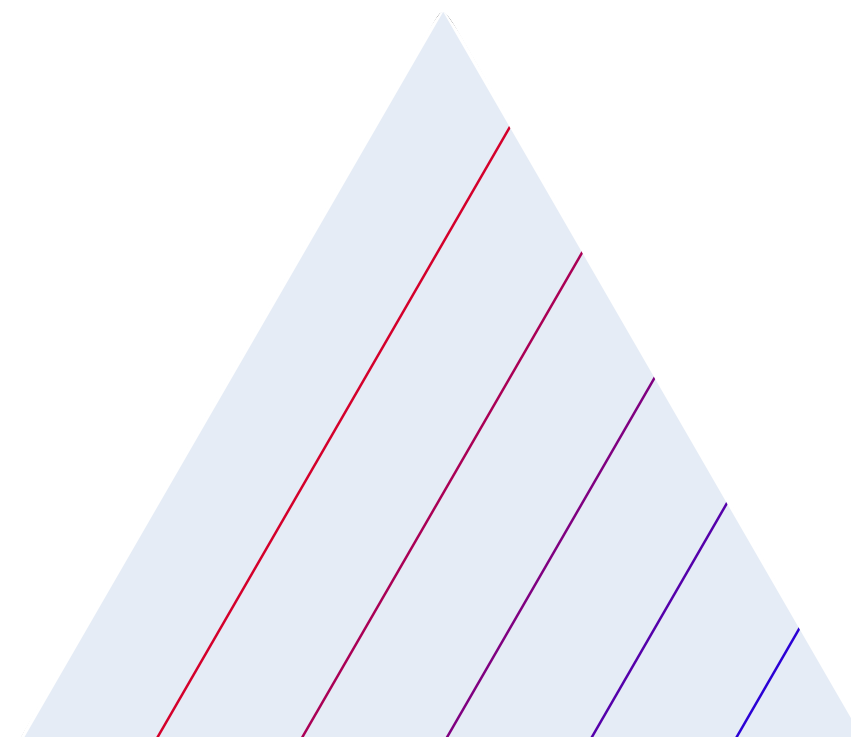
linear



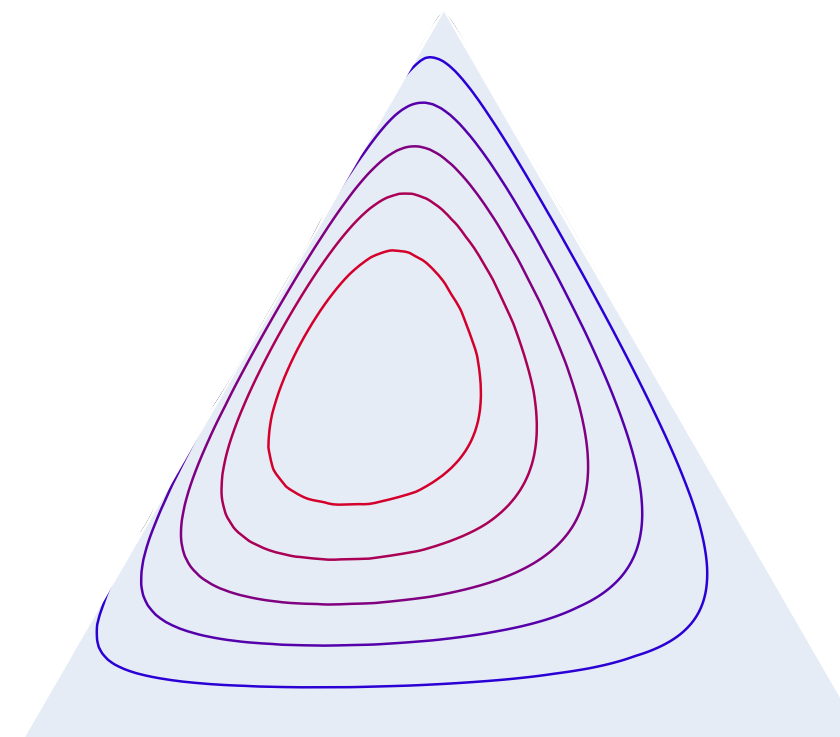
Leontief



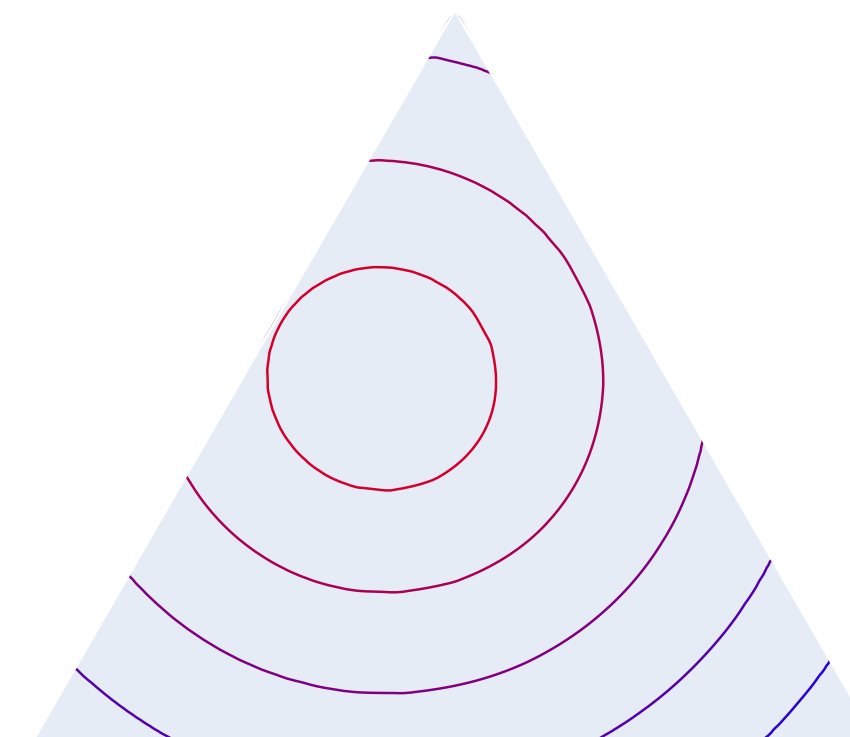
ℓ_1 disutilities



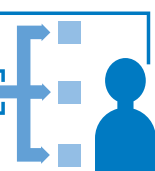
dichotomous

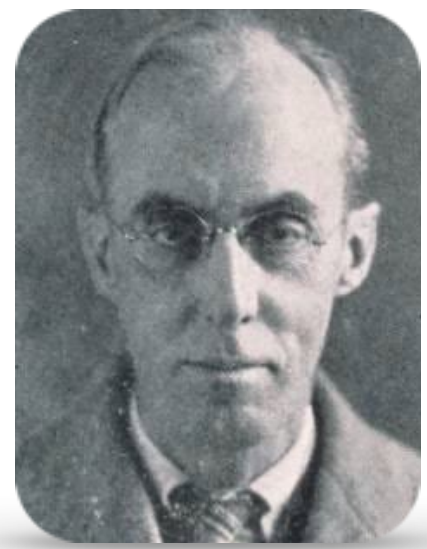


Cobb-Douglas



ℓ_2 disutilities



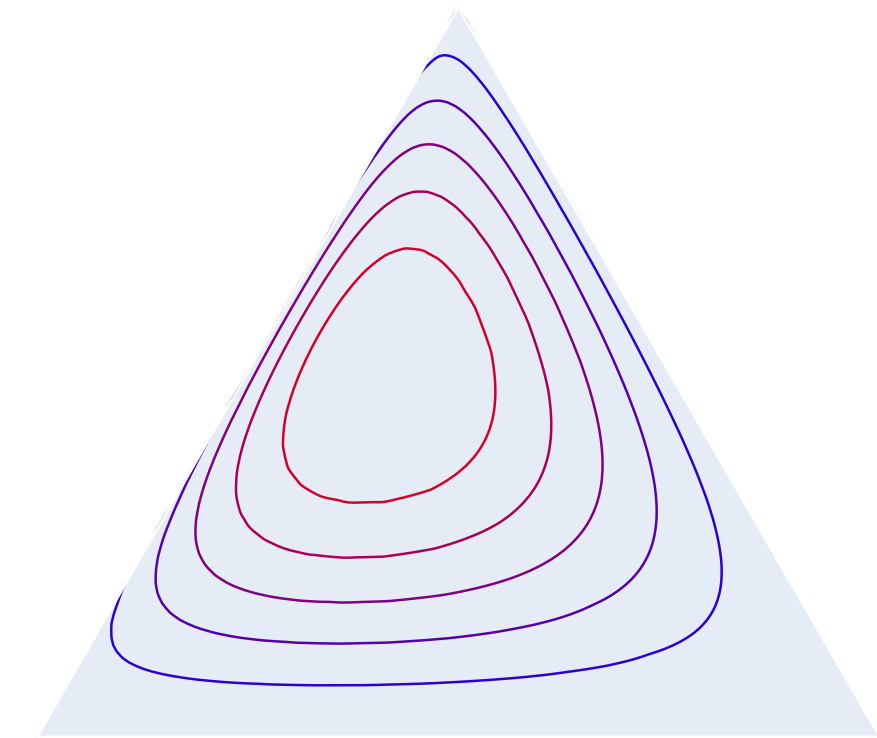


Charles Cobb



Paul Douglas

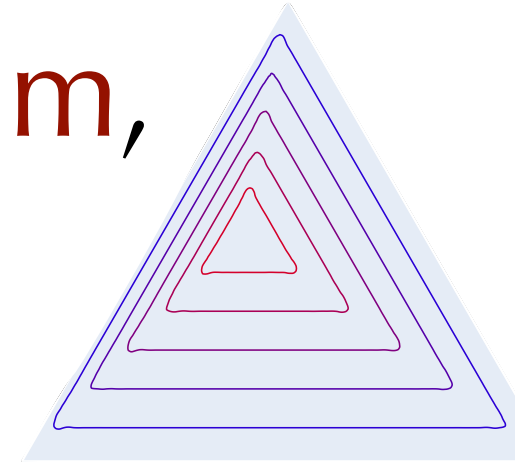
Cobb-Douglas Utilities



▶ $u_i(\delta) = \prod_{x \in A} \delta(x)^{v_i(x)}$ (Equivalently, $u_i(\delta) = \sum_{x \in A} v_i(x) \cdot \log \delta(x)$)

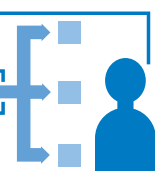
▶ $v_i(x) \geq 0$ for all $x \in A$ and $v_i(x) > 0$ for some $x \in A$.

▶ **Theorem (B. et al., 2023):** Each utility profile admits a **unique equilibrium**, which coincides with the equilibrium for Leontief utility functions using the same weights.



▶ Convergence of the **spending dynamics** also holds for Cobb-Douglas.

▶ However, **efficiency** and **strategyproofness** break down.



A Prisoners' Dilemma

- ▶ The equilibrium distribution can be inefficient for Cobb-Douglas utilities.
 - ▶ There is $\delta' \in \Delta(C)$ with $u_i(\delta') > u_i(\delta)$ for all $i \in N$.

equilibrium

	<i>a</i>	<i>b</i>	<i>c</i>	C_i	u_i
δ_1	4	2		6	16
δ_2		2	4	6	16
δ	4	4	4	12	

uncoordinated rule

	<i>a</i>	<i>b</i>	<i>c</i>	C_i	u_i
δ_1'	3	3		6	18
δ_2'		3	3	6	18
δ'	3	6	3	12	

- ▶ For Leontief utility functions, equilibrium distributions are always efficient!

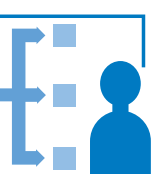
	(0,3,3)	(0,2,4)
(3,3,0)	(3,6,3)	(3,5,4)
(4,2,0)	(4,5,3)	(4,4,4)

Cobb-Douglas

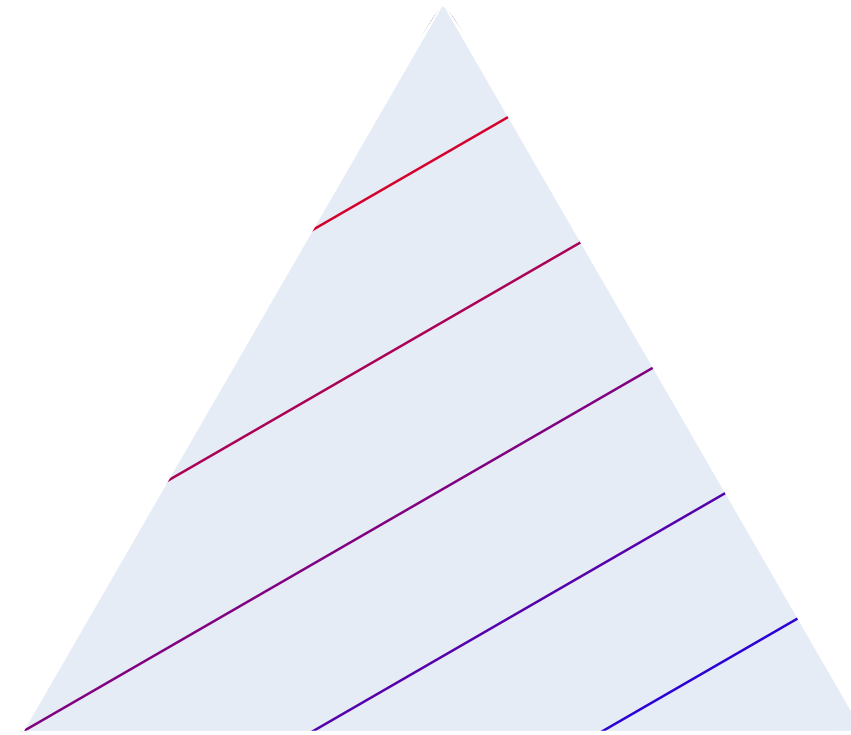
	(0,3,3)	(0,2,4)
(3,3,0)	18,18	15,20
(4,2,0)	20,15	16,16

Leontief

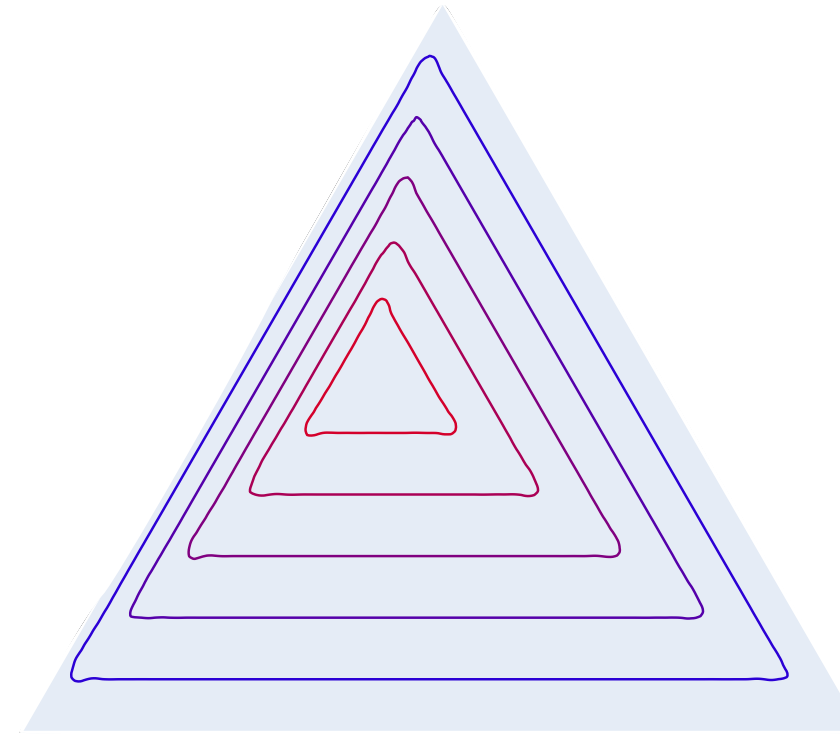
	(0,3,3)	(0,2,4)
(3,3,0)	3,3	3,4
(4,2,0)	4,3	4,4



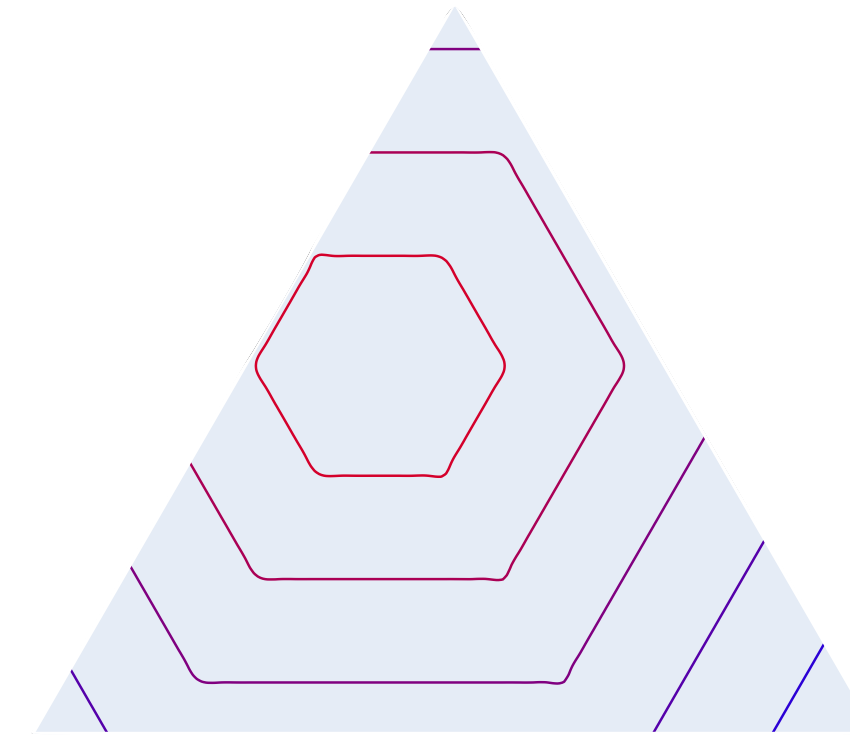
Potential Utility Functions



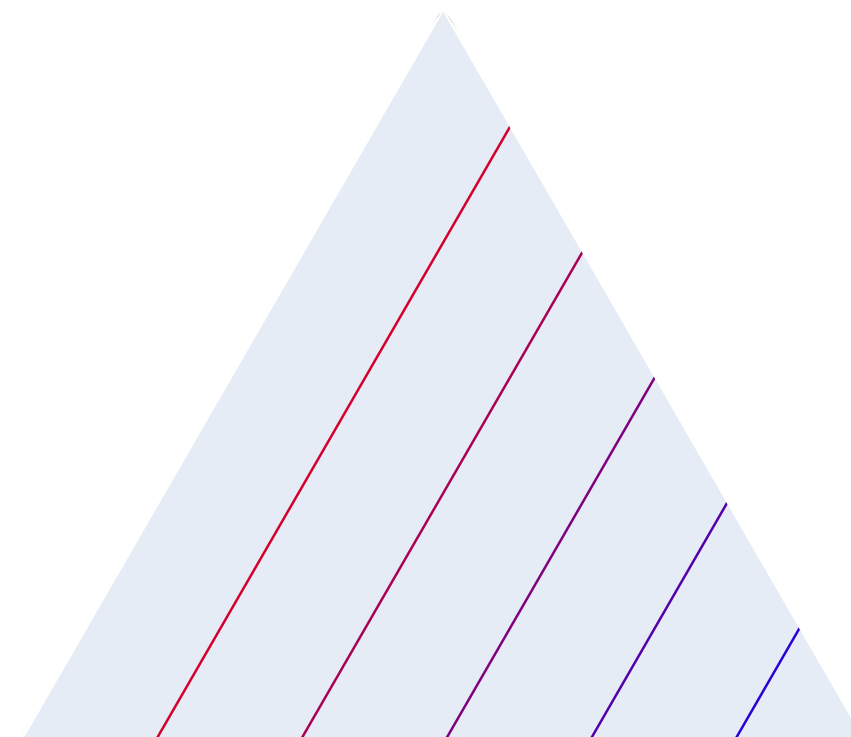
linear



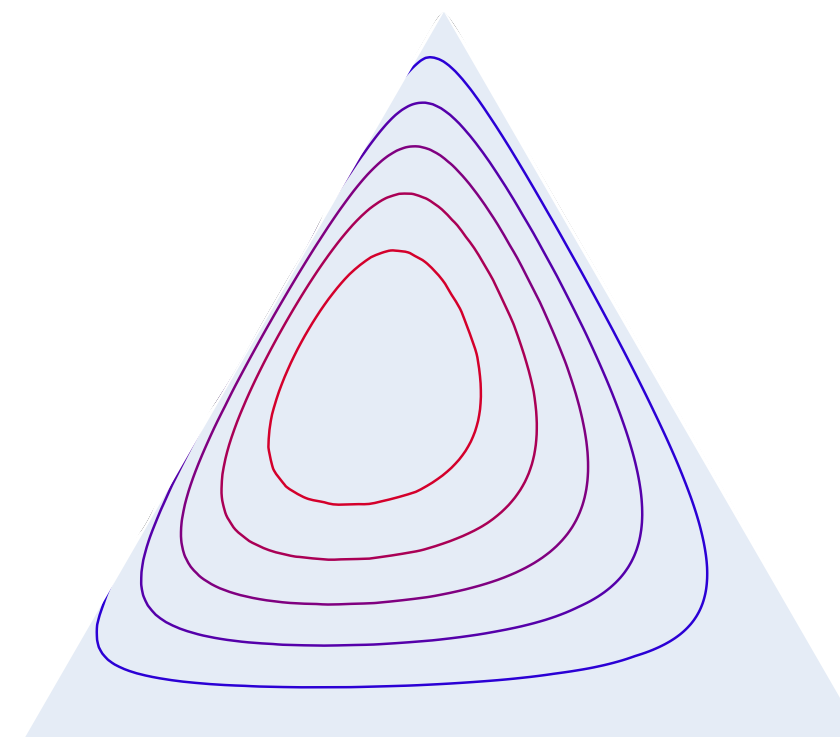
Leontief



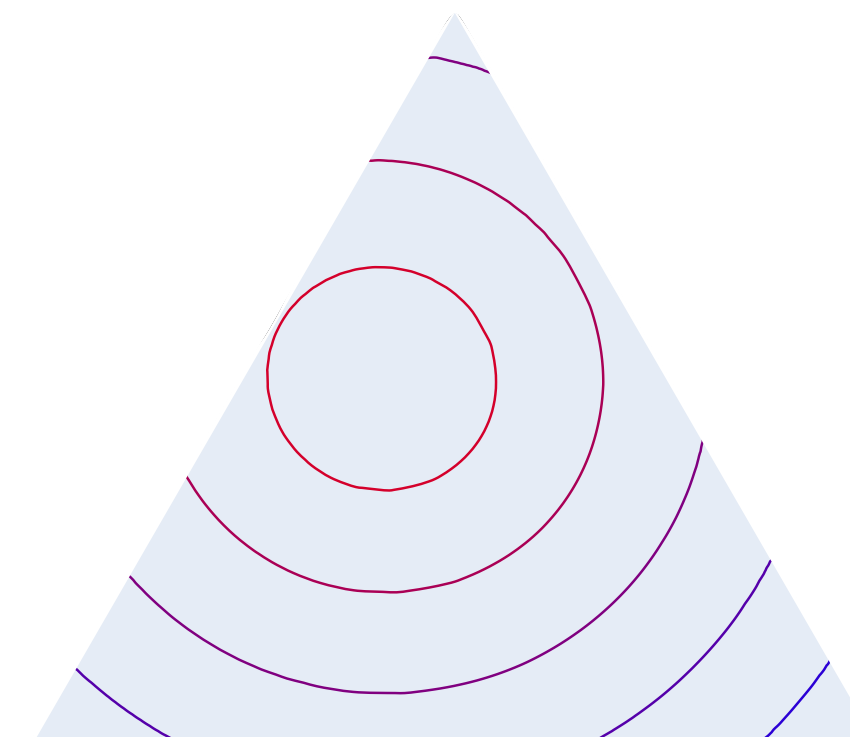
ℓ_1 disutilities



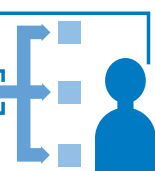
dichotomous



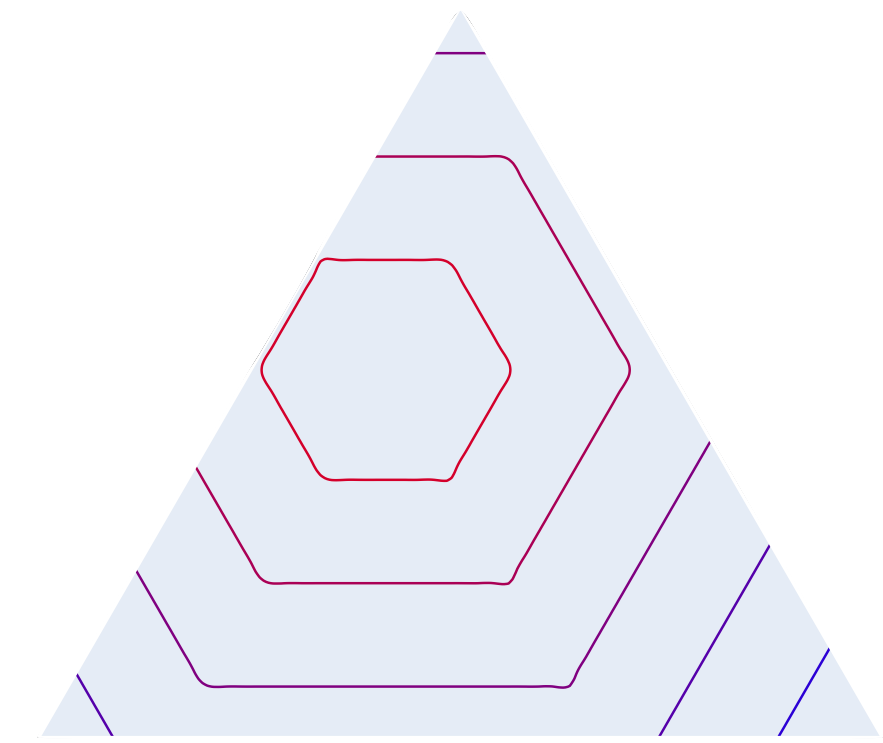
Cobb-Douglas



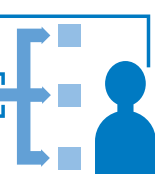
ℓ_2 disutilities



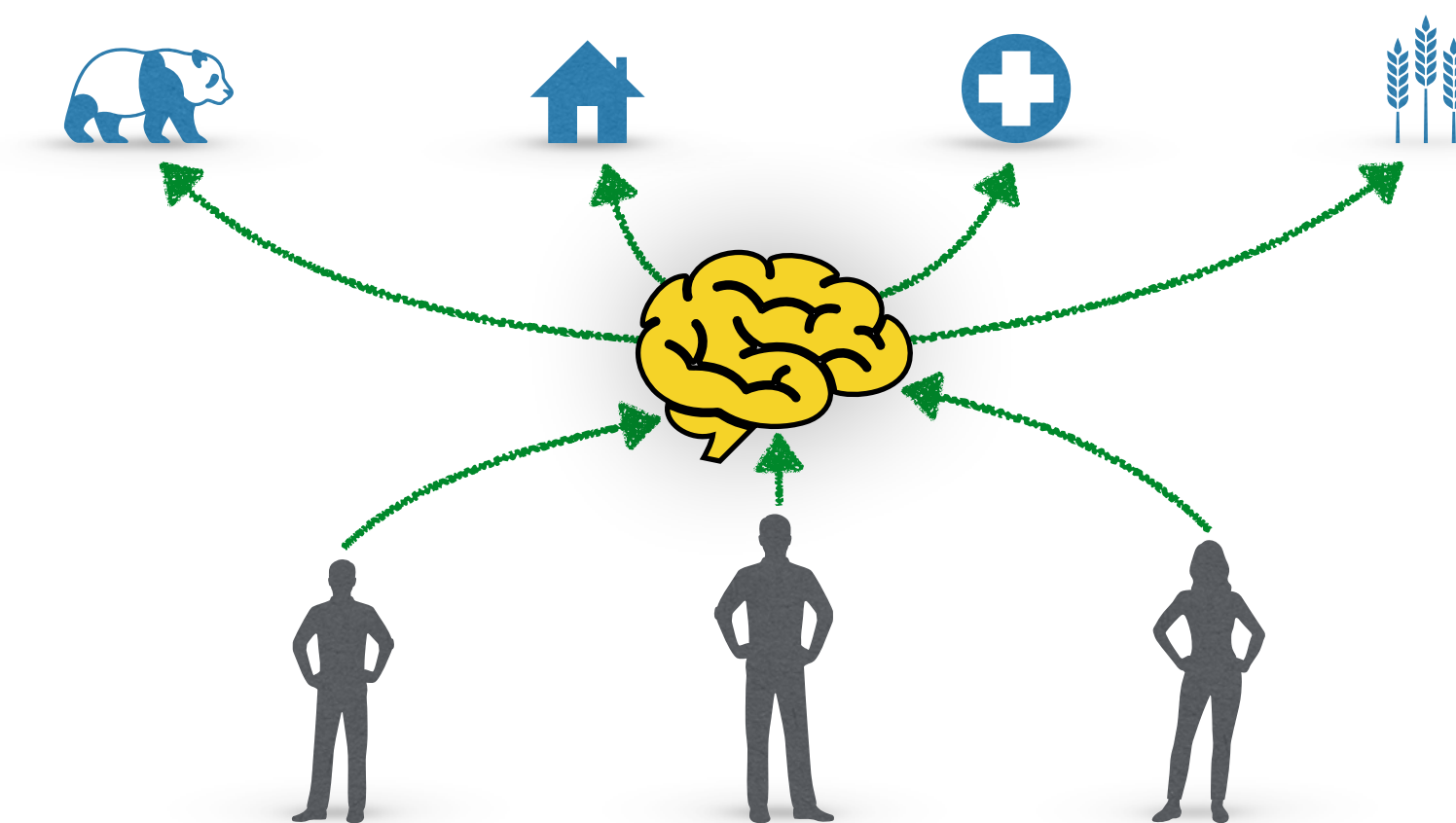
ℓ 1 Disutilities



- ▶ $u_i(\delta) = - \sum_{x \in A} |v_i(x) - \delta(x)|$
- ▶ $v_i(x) \geq 0$ for all $x \in A$ and $\sum_{x \in A} v_i(x) = C$.
- ▶ **Theorem (Linder et al., 2008):** The utilitarian rule (with careful tie-breaking) satisfies **efficiency** and **strategyproofness**.
- ▶ **Theorem (Freeman et al., 2021):** The “independent markets” rule satisfies **strategyproofness** and **proportionality** (a weakening of core fair share) but fails efficiency.
- ▶ **Theorem (B. et al., 2024):** No distribution rule satisfies **efficiency**, **strategyproofness**, and **proportionality**.



Summary



- ▶ Donor coordination can **increase the efficiency of charitable giving**.
- ▶ For **linear utilities**, results are mostly negative.
- ▶ For **dichotomous utilities**, desirable properties need to be traded off and *CUT* and *NASH* are attractive rules.
- ▶ For **Leontief utilities**, *EDR* satisfies virtually all desirable properties.
- ▶ Equilibrium existence, uniqueness, and convergence extend to **Cobb-Douglas utility functions**.
 - ▶ Efficiency and strategyproofness break down.
- ▶ Early mixed results for **ℓ_1 disutilities**.
 - ▶ Equilibrium distributions and spending dynamics for ℓ_1 are largely unexplored.



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