Computing Dominance-Based Solution Concepts

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Two common criticisms of Nash equilibrium are its dependence on very demanding epistemic assumptions and its computational intractability. We study the computational properties of less demanding set-valued solution concepts that are based on varying notions of dominance. These concepts are intuitively appealing, they always exist, and admit unique minimal solutions in important subclasses of games. Examples include Shapley's saddles, Harsanyi and Selten's primitive formations, Basu and Weibull's CURB sets, and Dutta and Laslier's minimal covering sets. We propose two generic algorithms for computing these concepts and investigate for which classes of games and which properties of the underlying dominance notion the algorithms are sound and efficient.

Categories and Subject Descriptors: F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Computations on discrete structures*; I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*; J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

General Terms: Algorithms, Economics, Theory

Additional Key Words and Phrases: Game Theory, Solution Concepts, Shapley's Saddles, CURB Sets

A D-set is defined as a minimal tuple of sets of actions that is internally and externally stable with respect to a so-called *dominance structure* D. Depending on D, a number of different solution concepts can be defined. We study the computational complexity of D-sets for the following dominance structures: pure and mixed strict (S, S^*) and very weak (V, V^*) dominance, Börgers dominance (B), covering (C_M), and deep covering (C_D).

Our results are summarized in the table below. For a given dominance structure D and a class of games (ordered by set inclusion), the table shows bounds on the *asymptotic number* of D-sets (unique, polynomial, or exponential). If a cell is highlighted in dark gray, the greedy algorithm finds all D-sets in the given class in polynomial time. If it is highlighted in light gray, the analogous statement holds for the sophisticated algorithm. If a cell spans several columns, the corresponding D-sets coincide within the respective class of games. C_D and C_M are only defined for symmetric matrix games.

<table>
<tr><th></th><th>S</th><th>B</th><th>S^*</th><th>C_D</th><th>C_M</th><th>V</th><th>V^*</th></tr>
<tr><td>normal-form games</td><td>poly</td><td>poly</td><td>poly</td><td></td><td></td><td rowspan="3">exp</td><td rowspan="4">exp</td></tr>
<tr><td>matrix games</td><td rowspan="4">unique</td><td rowspan="3">unique</td><td rowspan="3">unique</td><td></td><td></td></tr>
<tr><td>symmetric matrix games</td><td>unique</td><td>unique</td></tr>
<tr><td>confrontation games</td><td colspan="3">unique</td></tr>
<tr><td>tournament games</td><td colspan="2">unique</td><td colspan="4">unique</td></tr>
</table>

A full version of this paper is available at `http://dss.in.tum.de/files/brandt-research/dombased.pdf`.

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