Superiority of Instantaneous Decisions in Thin Dynamic Matching Markets

COMSOC Video Seminar

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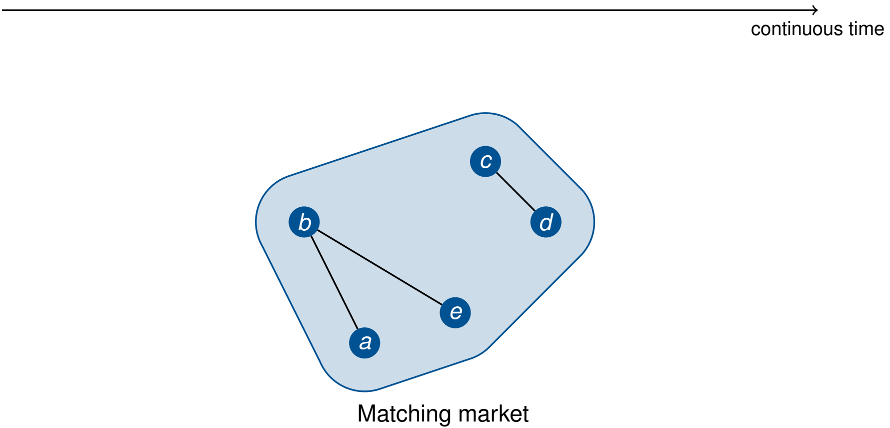
Motivation

- Markets of dynamically arriving agents seeking partner
- Abundance of applications
  - Labor markets
  - School choice
  - Dating platforms
  - Ride sharing
  - Kidney exchange
  - ...
- How to obtain matchings of good quality?

Dynamic Matching Markets



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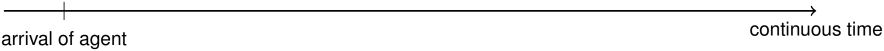
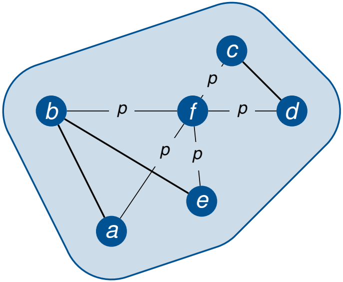
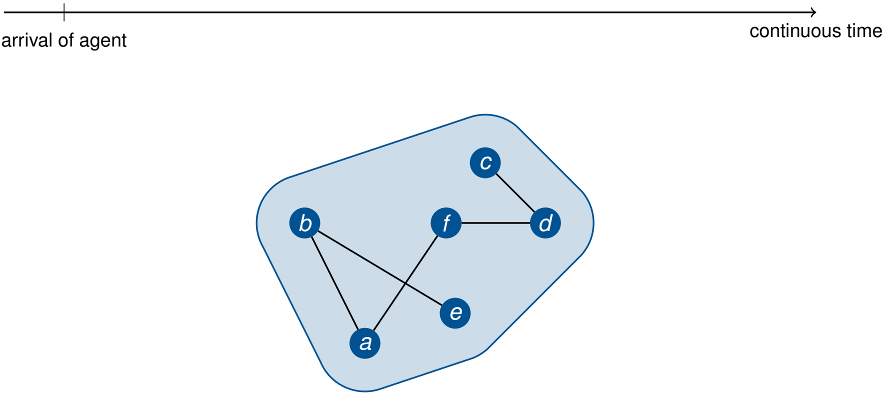




Matching market

Dynamic Matching Markets



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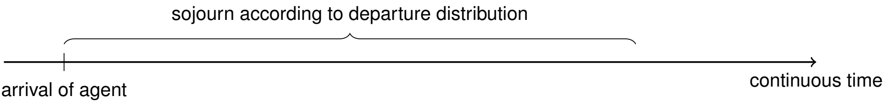

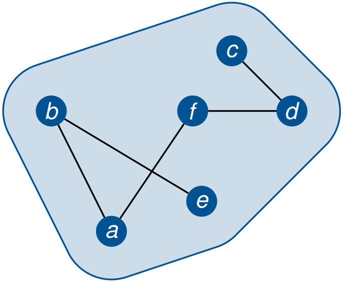

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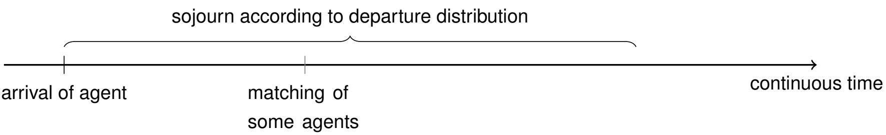

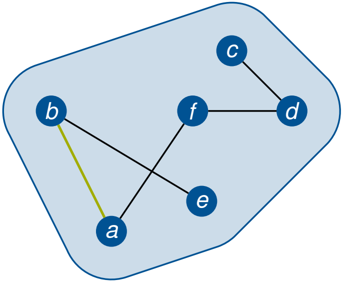

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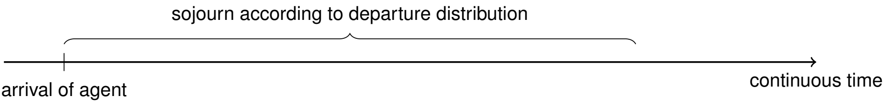

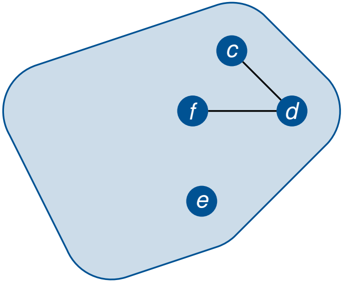

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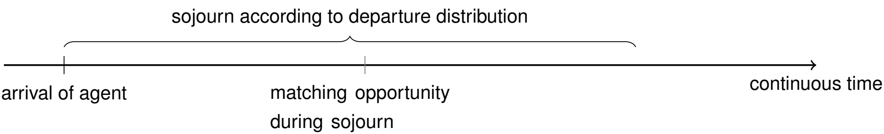

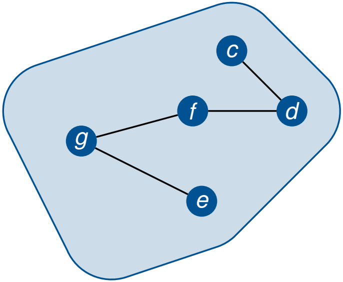

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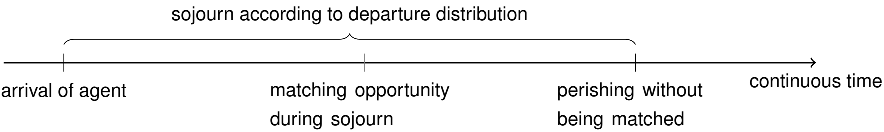

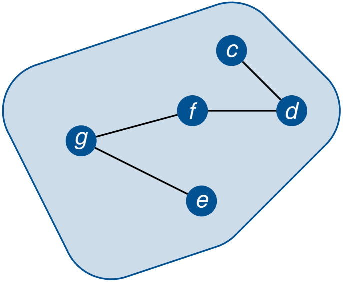

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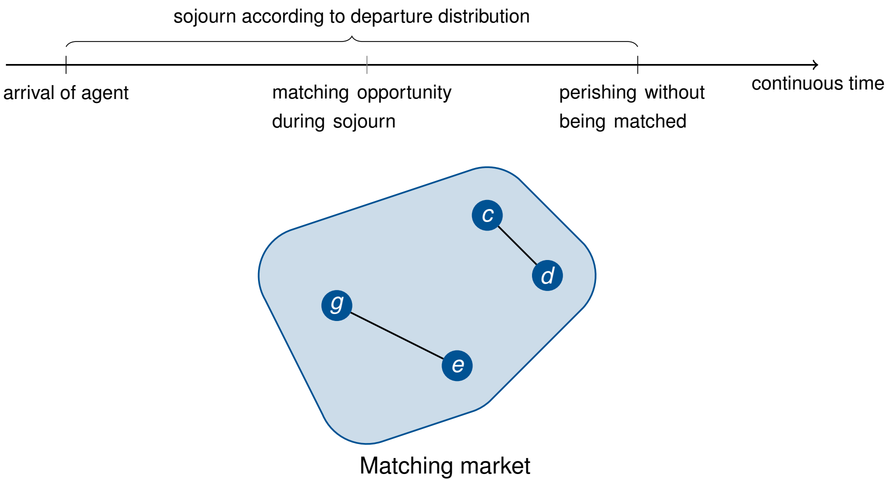
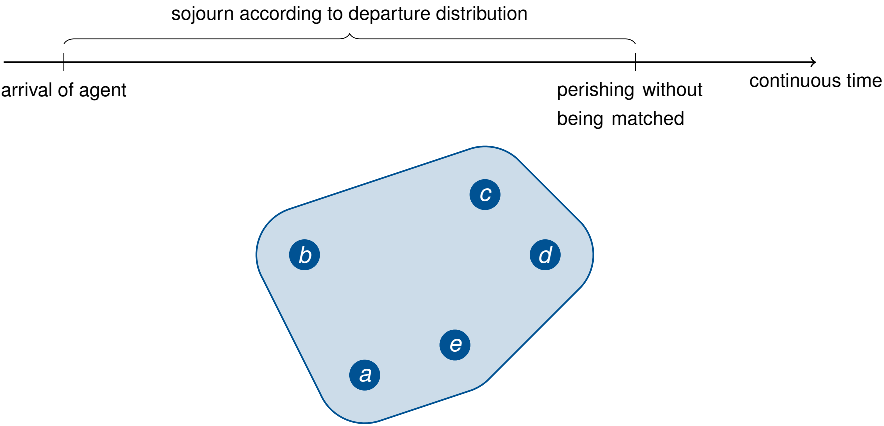


Model assumptions

- Arrival according to Poisson distribution with rate m
- Compatibility with respect to independent biased coin flips
- Departure with respect to some distribution
- How to match agents?
- How to measure performance?

Akbarpour et al., JPE 2020; Anderson et al., OR 2017

Matching Algorithms



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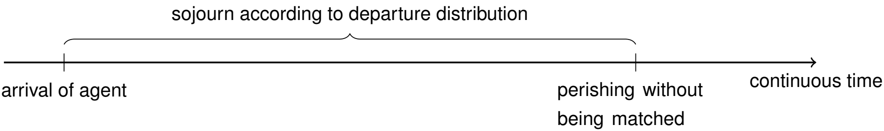

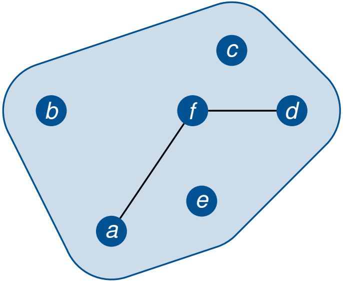

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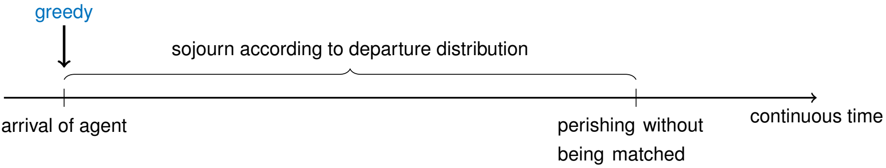

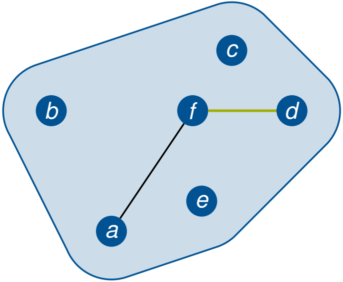

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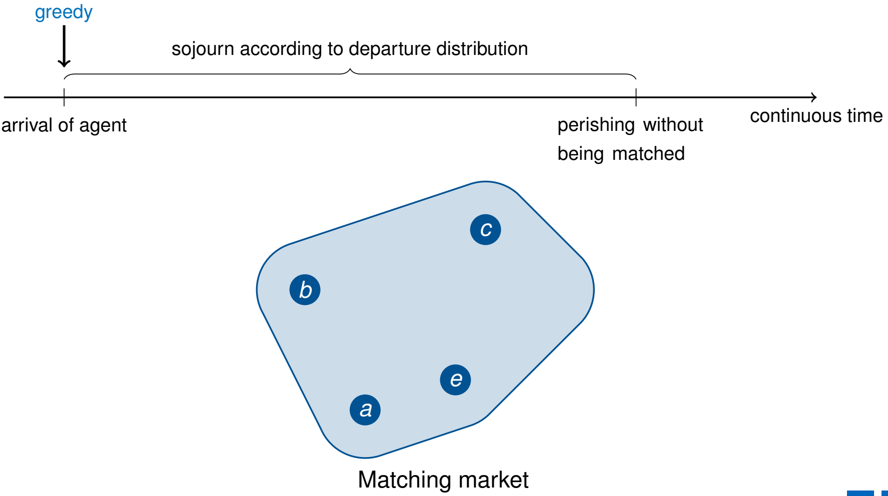

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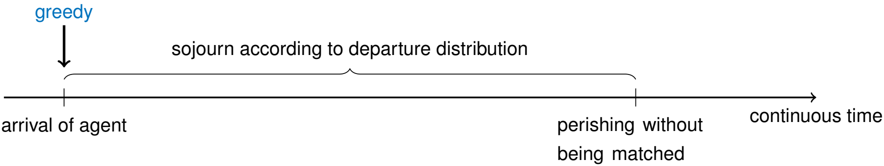

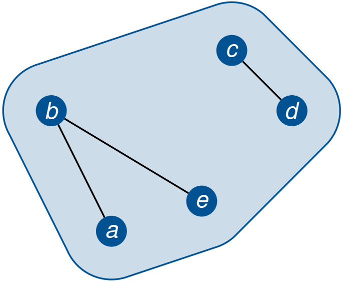

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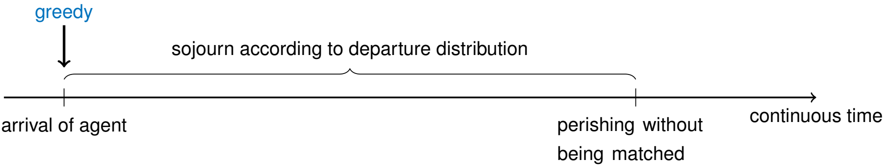

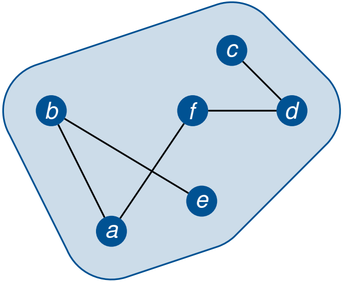

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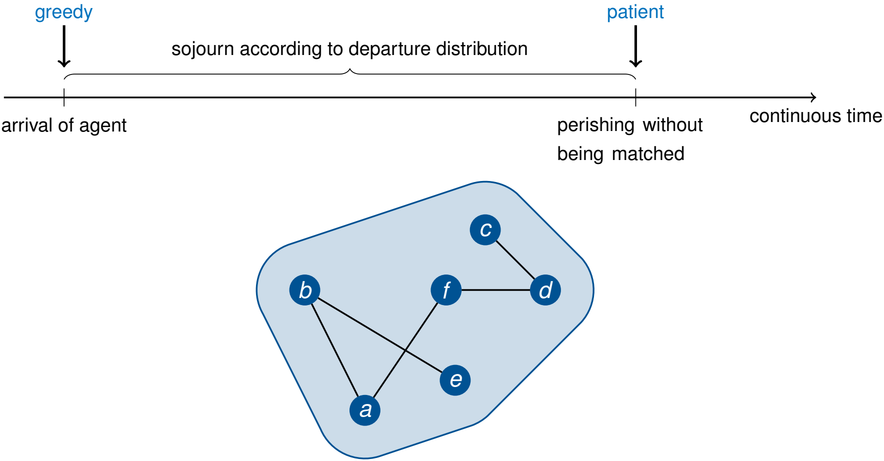

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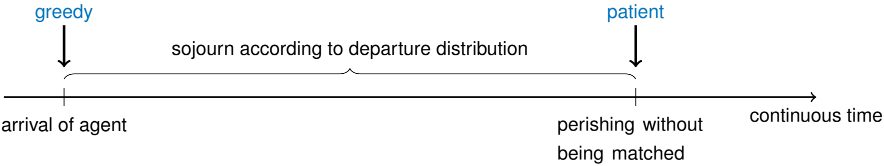

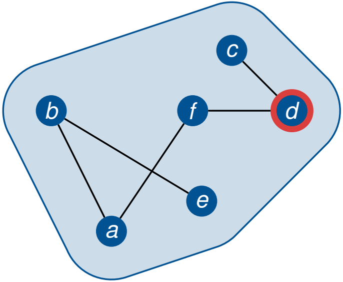

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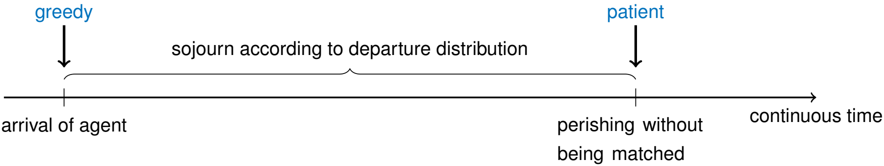

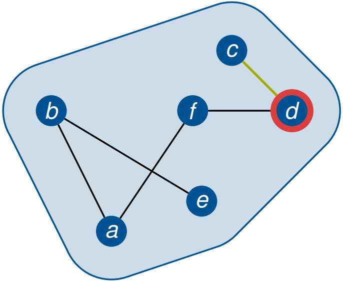

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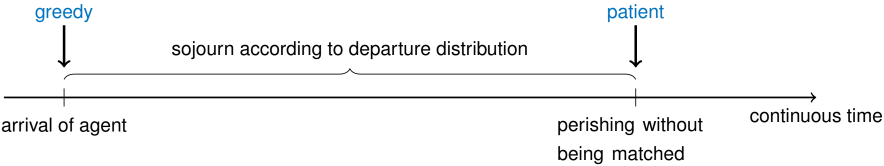

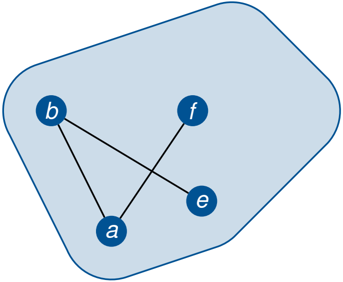

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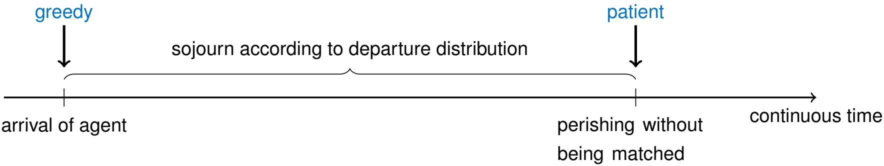

- Goal: minimize (asymptotic) loss, i.e., expected fraction of perishing agents

arrivals → $A_{[0,T]}$; matches → $\mathrm{ALG}(m,T)$; pool → Z_T; expected total arrivals → mT

$$\mathbf{L}_{\mathrm{ALG}} = \limsup_{m,T\to\infty} \frac{\mathbb{E}[|A_{[0,T]} - \mathrm{ALG}(m,T) - Z_T|]}{mT}$$

- Retain sparse market: density parameter $d = m \cdot p$

Discussion of Algorithms

- Greedy is very natural and frequently used in practice
- Greedy avoids long waiting times
- Patient algorithm needs information
- Other approaches: batching algorithms
- General paradigm in dynamic matching markets:
  Thick markets facilitate good performance of algorithms
- Promoted by a lot of recent work: Emek et al. (STOC 2016), Akbarpour et al. (JPE 2020), Baccara et al. (TE 2020), Loertscher et al. (JET 2020)
- Greedy type algorithms can perform well in thick market (Ünver, RES 2010; Ashlagi et al., OR 2019; Ashlagi et al., RES 2022)

Limitations of Thickness and Information

- Thick markets cause congestion (Roth, AER 2018)
- Bad in real-life data with respect to
  - Size of outcomes (Li and Netessine, MS 2020)
  - Quality of outcomes (Fong, 2020)
- Obtaining departure information can be costly or unethical (Reese et al., The Lancet 2015)
- Goal: The best of all worlds
  - No information
  - Thin market
  - Good quality of outcome
  - Low waiting times

Exponentially Distributed Departure Time

Theorem (Akbarpour et al., JPE 2020)

Assume that the departure time is exponentially distributed.
For $d \geq 2$, it holds that

$$\mathbf{L}_{\text{GDY}} \geq \frac{1}{2d+1},$$

$$\mathbf{L}_{\text{PAT}} \leq \frac{1}{2} e^{-d/2}.$$

Main Result

Theorem

Assume that departure times are distributed according to a probability measure μ with $\mu([0,1)) = 0$. For $d \geq 2$, it holds that

$$\mathbf{L}_{\text{GDY}} \leq e^{-\frac{d}{2\log(2)}}.$$

- Typical case: unit waiting times
- Formulation for arbitrary lower bound on maximum waiting time
- Close to optimal performance: $\mathbf{L}_{\text{ALG}} \geq e^{-2d}$
- High loss (e.g., of exponential distributed departure times) caused by instantaneous departures
- Small loss due to evenly distributed sojourn

Waiting Times

Proposition

Assume that the departure time is distributed according to an arbitrary probability measure μ. Then, for the total waiting time W,

$$\mathbb{E}_\mu[W] = \int_0^T \mathbb{E}_\mu[z_s]\, ds.$$

- Bound on waiting time of greedy: $\mathbb{E}_\mu[W] \leq \frac{6mT}{5d}$
- Optimal up to constant: $\mathbb{E}_\mu[W] \geq c\frac{mT}{d}$
- Very long waiting times under patient algorithm

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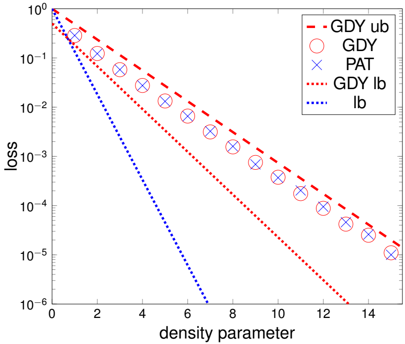
z_s — pool size at time s

Greedy and Patient under Unit Departure



- Simulations indicate identical loss
- Both guarantee exponentially small loss in theory
- Intuitive arguments for exact equivalence

Conclusion

Take home message

- Thin market setting where greedy performs close to optimal
- Circumvent congestion, information collection, and trade-off between quality and waiting

Future directions

- Analyze extension to other thin markets
- Search deep connection of greedy and patient under unit departure times

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