Coordinating Charitable Donations

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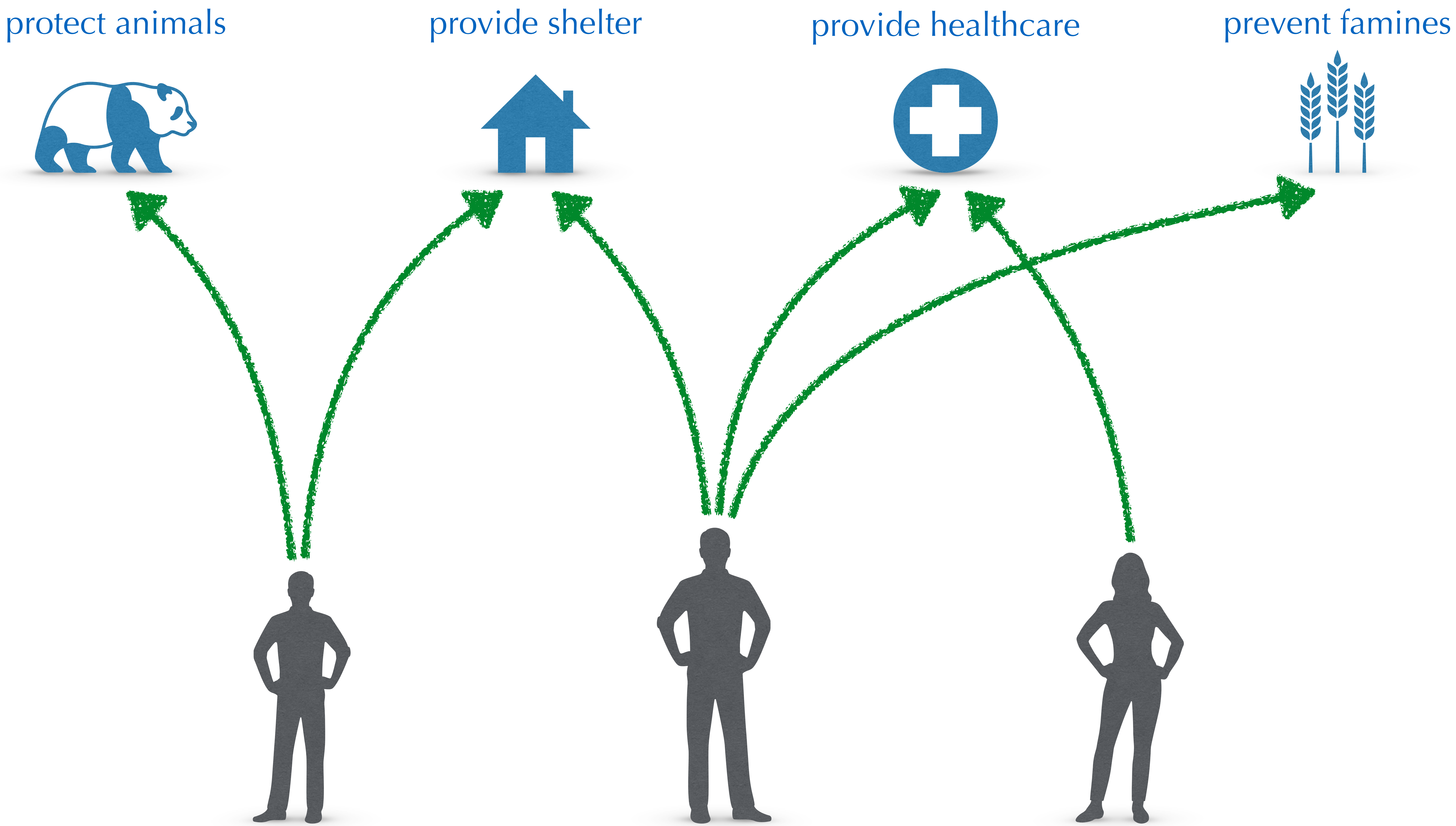
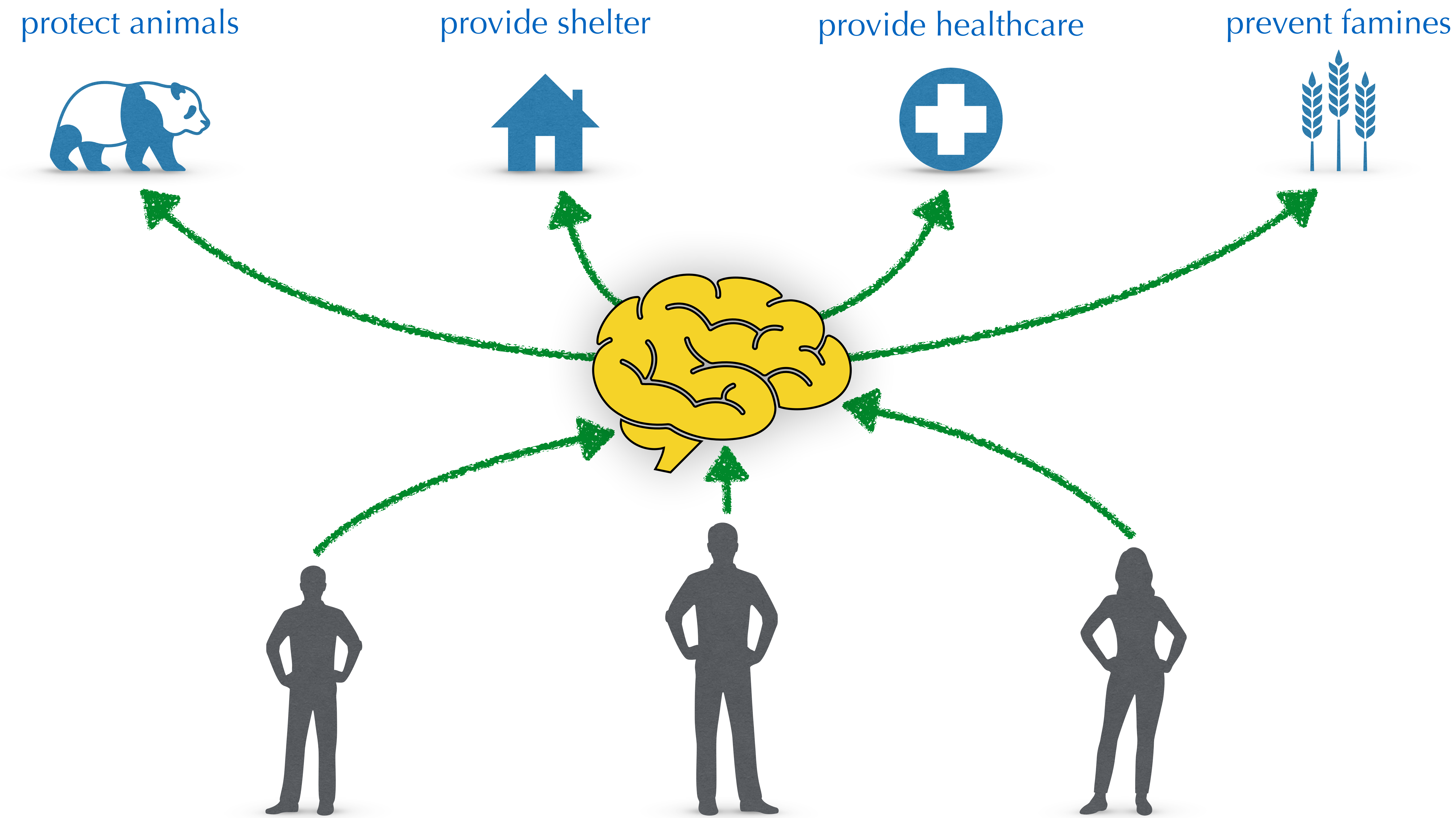
DSS
Decision Sciences & Systems


protect animals
provide shelter
provide healthcare
prevent famines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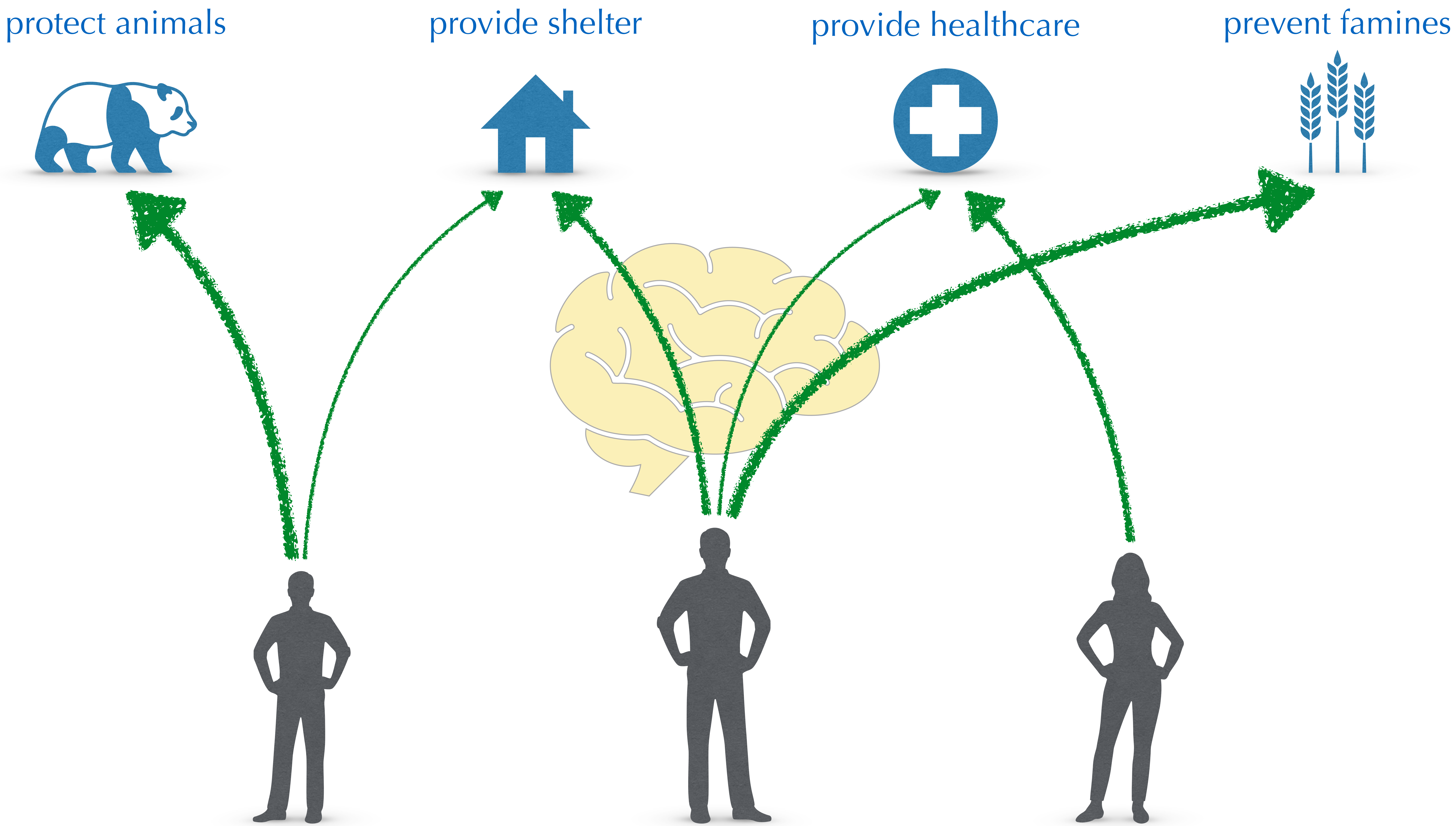
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Potential applications

- Cinque per mille
  - Italian citizens can select one of over 70k non-profit organizations.
  - Revenue Agency will divert 0.5% of citizen's income tax to this organization.
  - 2022: €510m
- AmazonSmile
  - Customers can select one of over 1m non-profit organizations.
  - Amazon donates 0.5% of customer's purchase price to this organization.
  - 2013–2023: $400m
- Employee charity matching programs
  - Microsoft (2022): $250m to 32k organizations
  - Apple (2011–2022): $880m to 44k organizations
- Private charity by groups of donors

The Model

- Each agent $i \in N$ contributes amount $C_i > 0$ of a divisible and homogeneous resource to a common pool.
  - $C = \sum_{i \in N} C_i$ is called the endowment.
  - The resource could, for example, be money or time.
- Let A be a set of potential recipients of the contributions, called charities.
- A distribution $\delta \in [0,C]^A$ is a function with $\sum_{x \in A} \delta(x) = C$.
  - The set of all distributions is denoted by $\Delta(C)$.
- Agent i receives utility $u_i(\delta)$ from δ.
- A distribution rule f maps a utility profile $(u_i)_{i \in N}$ to a distribution δ.

Related Models

- Private provision of public goods (e.g., Bergstrom, Blume, and Varian, 1986)
  - agents distribute their wealth between a private and a public good
  - no preferences over different public goods
- Participatory budgeting (e.g., Cabannes, 2004)
  - typically fixed costs for projects, which are either fully funded or not at all
  - exogenous endowment
- Probabilistic social choice (e.g., Gibbard, 1977)
  - typically ordinal preferences
  - exogenous fixed "endowment"
- Fair mixing (e.g., Bogomolnaia, Moulin, and Stong, 2005)
  - dichotomous preferences
  - exogenous fixed "endowment"

Linear Utilities

- $u_i(\delta) = \sum_{x \in A} \delta(x) \cdot v_i(x)$
  - Charities are substitutes with constant marginal rates of substitution.
- Example:

$v_i(x) = 1$ $v_i(x) = 0$

uncoordinated

	a	b	c	C_i	u_i
δ_1	0.5	0.5		1	**1.5**
δ_2		0.5	0.5	1	**1.5**
δ	0.5	1	0.5	2	

coordinated

	a	b	c	C_i	u_i
δ_1		1		1	**2**
δ_2		1		1	**2**
δ		2		2	

- A decomposition of a distribution δ is a vector of distributions $(\delta_i)_{i \in N}$ such that $\sum_{i \in N} \delta_i(x) = \delta(x)$ for all $x \in A$ and $\sum_{x \in A} \delta_i(x) = C_i$ for all $i \in N$.

Three Axioms

- f only returns efficient distributions δ.
  - There is no $\delta' \in \Delta(C)$ with $u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and $u_i(\delta') > u_i(\delta)$ for some $i \in N$.
  - Example: *Utilitarian rule* (entire endowment is spent on welfare-maximizing charities)
- f is strategyproof if $u_i(f(u_1, \ldots, u_{|N|})) \geq u_i(f(u_1, \ldots, u_i', \ldots, u_{|N|}))$ for all $i \in N$ and u_i'.
  - Example: *Dictatorial rules* (entire endowment is spent on top charities of fixed agent)
- f only returns equilibrium distributions δ.
  - δ admits a decomposition $(\delta_i)_{i \in N}$ such that $u_i(\delta) \geq u_i(\delta - \delta_i + \delta_i')$ for all $i \in N$ and $\delta_i' \in \Delta(C_i)$.
  - indispensable when rules merely issue *recommendations* how to distribute one's contribution
  - Example: *Uncoordinated rule* (each agent divides her contribution on her top charities)
  - **Proposition (Brandl et al., 2022)**: δ is in equilibrium iff the cumulative contribution of any coalition is distributed on charities that are most preferred by at least one coalition member.

Limitations under Linear Utilities

- Linear utility functions entail severe tradeoffs.
  - **Theorem (Hylland, 1980)**: Only *dictatorial rules* are strategyproof and efficient.
  - **Proposition**: Unique equilibrium distributions can be inefficient.

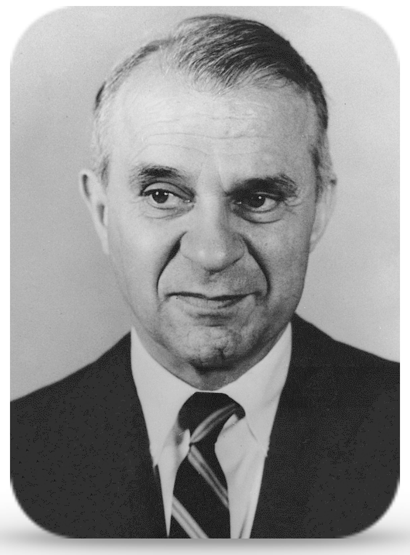
$v_i(x) = 1.5$

	a	b	c	C_i	u_i
δ_1	1			1	**1.5**
δ_2			1	1	**1.5**
δ	1		1		

	a	b	c	C_i	u_i
δ_1		1		1	**2**
δ_2		1		1	**2**
δ		2			

  - **Theorem (Gibbard, 1977)**: When agents have unique top charities and identical contributions, only the *uncoordinated rule* is anonymous, unanimous, and strategyproof.
- Interesting possibilities emerge for dichotomous utility functions.
  - e.g., *conditional utilitarian rule*, *Nash product rule*, all three axioms remain incompatible.

Leontief Utilities



Wassily Leontief

- $$u_i(\delta) = \min_{x \in A:\, v_i(x)>0} \frac{\delta(x)}{v_i(x)}$$
  - $v_i(x) \geq 0$ for all $x \in A$ and $v_i(x) > 0$ for some $x \in A$.
- Charities are complements rather than substitutes.
- **Theorem**: Each utility profile admits a unique equilibrium distribution. This distribution maximizes Nash welfare and thus is efficient.
- *EDR* (equilibrium distribution rule) returns the equilibrium distribution.
- Example:

	a	b	c	C_i	u_i
δ_1	1	0.5		1.5	**1**
δ_2		0.5	1	1.5	**1**
δ	1	1	1	3	

Equilibrium Distribution Rule

- The tradeoffs present in the case of linear utilities vanish!
- **Theorem**: *EDR* is (group-)strategyproof.
- **Theorem**: Agents are strictly better off by increasing their contribution.
  - This property is violated by the utilitarian rule for linear utilities.
- **Theorem**: The contribution to a charity weakly increases when
  - agents increase their valuation for the charity, or
  - when agents increase their contributions.
  - Both properties are violated by the Nash product rule for linear utilities.
- *EDR* can be computed via convex programming.

Best-Response Spending Dynamics

- Consider a multi-round setting with a constant flow of donations.
  - For example, each agent has set aside a *monthly* budget for charitable activities.
- Agents become active in round-robin order.
- Agents can observe the accumulated distribution of the last $n - 1$ rounds and then distribute their own contribution myopically optimal.
- **Theorem**: The accumulated distribution of the last n rounds converges to the equilibrium distribution.
- No central authority required, no direct revelation of preferences.
- Even with occasional changes to preferences and contributions, the relative overall distribution keeps converging towards the equilibrium distribution.

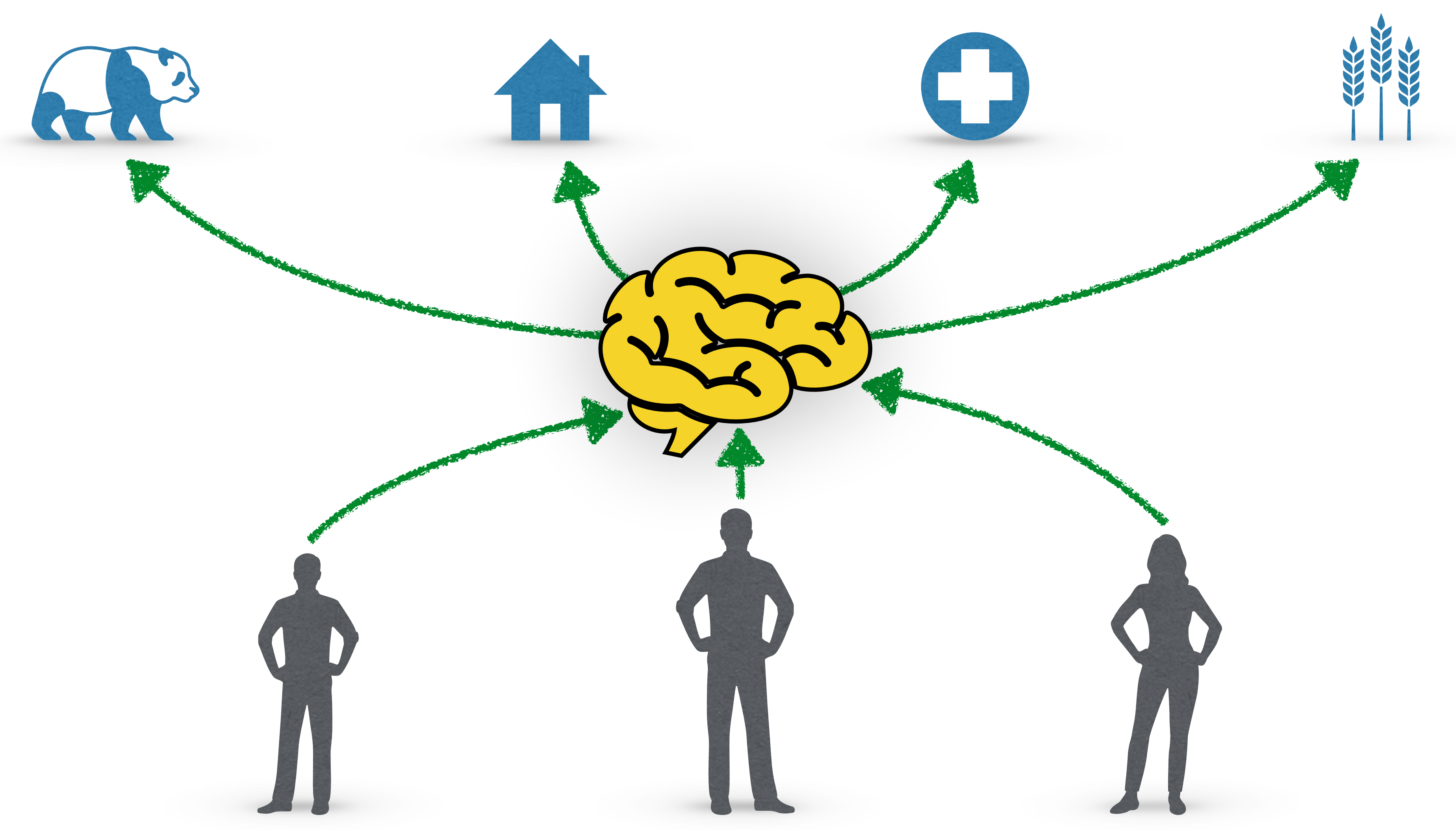
Binary Weights

- A Leontief utility function u_i has binary weights if $v_i(x) \in \{0,1\}$ for all $x \in A$.
- **Theorem**: For binary weights, *EDR* coincides with egalitarian rules:

  Among all $\delta \in \Delta(C)$ which can be decomposed such that $\sum_{x \in A:\, v_i(x)>0} \delta_i(x) = C_i$,

  EDR lexicographically maximizes both
  - $\min_{x \in A} \delta(x)$ (the minimal contribution to a charity), and
  - $\min_{i \in N} u_i(\delta)$ (the minimal utility of an agent).
- **Theorem**: For binary weights, *EDR* can be computed via linear programming.
  - It is open whether *EDR* can be computed via linear programming for non-binary weights. Equilibrium distributions are always rational-valued.

Conclusion and Outlook



- Donor coordination can increase the efficiency of charitable giving.
- For linear utilities, desirable properties need to be traded off.
- For Leontief utilities, *EDR* satisfies virtually all desirable properties.
- Equilibrium existence, uniqueness, and convergence extend to additively separable, strictly concave utility functions.
    - Efficiency and strategyproofness break down.