Electricity Pricing and Minimum Make-Whole Payments in the Presence of Non-Convexities and Price-Sensitive Demand

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Introduction

Pricing on markets is one of the most fundamental topics in the economic and management sciences. Standard general equilibrium theory assumes divisible goods and convex preferences. However, most real-world markets trade indivisible goods and participants have non-convex preferences and complex constraints. This raises the question how prices can be computed in non-convex markets and which properties we can hope to achieve. While competitive equilibrium prices need to be non-linear and personalized in general (and might not exist at all in two-sided markets), that would convey little information other than that a bidder lost or won. Thus, anonymity (market prices being the same for all bidders without differentiation) and linearity (single price for each good) are important requirements for prices on many real-world markets (e.g., as baseline for derivatives).

Pricing in non-convex markets is arguably one of the central topics in market design, and one that has received significant attention in Information Systems over the years. For example, previous papers discuss pricing in ascending combinatorial auctions (Adomavicius et al. 2012) or combinatorial exchanges (Guo et al. 2012). This paper is in the tradition of prior market design literature in Information Systems (Bichler et al. 2010), and contributes to electricity spot markets which are central to discussions about climate change and a fair and just transition to a decarbonized society.

Electricity spot markets are composed of varying levels of demand (load) and matching levels of supply (generation). Ideally, supply and demand curves at each traded time slice determine prices in equilibrium. However, the bid languages on electricity markets today are highly specific. In U.S. markets, cost structures can be communicated with multi-part bids, usually consisting of start-up costs, no-load costs, and an offer curve. Generators can also express technical constraints such as minimum and maximum output levels or ramp rates. Demand-side bids comprise price-inelastic self-scheduling as well as price-sensitive bid curves. These bid languages cause non-convexities in the allocation problem (which yields the efficient dispatch). As the share of renewables grows, the operating cycles of conventional generators become shorter and the relevance of these non-convexities increases. Over the years, several pricing rules have been suggested aiming to mimic competitive equilibria in such non-convex markets (Liberopoulos and Andrianesis 2016). Pricing rules in practice compute anonymous and linear prices, but these prices do not constitute competitive equilibrium prices. Participants would prefer a different dispatch, prices alone are not *envy-free*, and the outcome would not be *stable*. Moreover, the generators often make a loss at the market prices, i.e. prices are not *individually rational*.

U.S. Independent System Operators (ISOs) use personalized side-payments to address this fact. Thereby, they effectively differentiate the linear and anonymous market prices from the payments of the market participants. Most ISOs pay *make-whole payments* to ensure individual rationality of all generators and stipulate penalties if a generator deviates from the optimal dispatch. In other words, they relax envy-freeness to only individual rationality requirements. We refer to such outcomes as having penalty-based stability. However, even the make-whole payments are a significant concern since they are not reflected in the public prices and distort investment signals. The Federal Energy Regulatory Commission (FERC) argues that "the use of side-payments can undermine the market's ability to send actionable price signals"[1] and that several ISO practices are unjust

[1] `https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation`

(O'Neill, Hytowitz, et al. 2019). Thus, a key challenge in ISO markets is to reduce side-payments. In a first contribution, we introduce an optimization model that, under the standard assumption of price-inelastic demand, computes prices that are individually rational, budget balanced, and clear the market with strict demand-supply equality at the efficient dispatch without requiring make-whole payments. However, this assumption of price-inelasticity is unlikely to hold in the future. Power systems are changing profoundly due to the expansion of Variable Energy Resources (VER). The characteristic variability and uncertainty of these VER require an integration of an active demand side and price-sensitive bids. In such two-sided markets, prices that are individually rational and clear the market at the efficient dispatch cannot always be budget balanced.

In a second contribution, we introduce an alternative pricing rule that minimizes make-whole payments while it still clears the market at the efficient dispatch. The pricing rule (PE-A) treats efficiency, individual rationality, and budget balance as first-order goals, and envy-freeness as a second-order goal. It can be computed in polynomial time and scales to large problem sizes. Importantly, we show that the prices, on average, do not increase compared to other established pricing rules in our experiments. The pricing rule is not restricted to electricity markets and can also be applied to other types of non-convex and two-sided markets.

In the following, we first discuss competitive equilibrium theory and alternative design desiderata for non-convex markets. We then introduce optimization models to compute respective prices. After briefly comparing against existing pricing rules, we provide results of experiments.

Competitive Equilibrium and Stability

In an auction market, there are K types of items, denoted by $k \in \mathscr{K} = \{1, \ldots, K\}$, buyers $i \in \mathscr{I} = \{1, \ldots, I\}$ and sellers $j \in \mathscr{J} = \{1, \ldots, J\}$. A bundle of interest to buyer i is described by a vector

$x_i \in \mathbb{Z}^K_{\geq 0}$. A bundle offered by seller j is described by a vector $y_j \in \mathbb{Z}^K_{\geq 0}$. Each buyer i (seller j) has a value function v_i (v_j) : $\mathbb{Z}^K_{\geq 0} \to \mathbb{R}_{\geq 0}$ over bundles of items x_i (y_j). An auctioneer wants to find an efficient allocation $(\mathbf{x}, \mathbf{y}) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ of items to bidders that maximizes *social welfare*, being defined as $\sum_{i \in \mathscr{I}} v_i(x_i) - \sum_{j \in \mathscr{J}} v_j(y_j)$, as well as linear and anonymous market prices $\lambda = \{\lambda(k)\}_{k \in \mathscr{K}} \in \mathbb{R}^K_{\geq 0}$. As discussed, linearity and anonymity are crucial on electricity and other real-world markets. The *demand correspondence* $D_i(\lambda)$ and $D_j(\lambda)$, resp., describe the set of bundles that maximize individual utility at prices λ, i.e., $D_i(\lambda) = \text{argmax}_{x \in \mathbb{Z}^K_{\geq 0}} \{v_i(x) - \langle \lambda, x \rangle\}$ and $D_j(\lambda) = \text{argmax}_{y \in \mathbb{Z}^K_{\geq 0}} \{\langle \lambda, y \rangle - v_j(y)\}$. If in an outcome (consisting of an allocation and prices) all bidders obtain a bundle from their demand correspondence, then the outcome is *envy-free* (EV). If we have EV and the market is *budget-balanced* (BB), we have a *competitive equilibrium* (CE). Moreover, if prices are linear and anonymous (LA), we refer to this as a Walrasian equilibrium.

Definition 1 (Walrasian (competitive) equilibrium, (WE))**.** *A price vector λ^* and a feasible allocation $(\mathbf{x}, \mathbf{y})$ form a Walrasian equilibrium if $\sum_{i \in \mathscr{I}} x_i = \sum_{j \in \mathscr{J}} y_j$, $x_i \in D_i(\lambda^*)$ for every buyer $i \in \mathscr{I}$, $y_j \in D_j(\lambda^*)$ for every seller $j \in \mathscr{J}$, and budget is balanced with $\sum_{i \in \mathscr{I}} \langle \lambda^*, x_i \rangle = \sum_{j \in \mathscr{J}} \langle \lambda^*, y_j \rangle$.*

In summary, a Walrasian equilibrium (WE) has the properties $BB \wedge EV \wedge LA$. Note that EV also implies individual rationality (IR) for all bidders and that BB and LA imply a strict demand-supply equivalence. The question is now under which conditions WE exist and whether they support efficient (welfare-maximizing) outcomes (EF). Bikhchandani and Mamer (1997) describe a multi-item market with arbitrary package bids. Their central theorem shows that there exist clearing prices for indivisible items if and only if the linear programming (LP) relaxation of the winner determination problem (WDP) has an integer solution. In this case, the dual variables $\lambda(k)$ constitute WE prices and the well-known first and second welfare theorems hold. Unfortunately, the LP relaxation of the WDP does not yield integer solutions in general, and thus we cannot expect WE to exist in general. In fact, it is well-known that WE only exist for restricted types of valuations for which the LP relaxation actually yields a feasible integer solution. For example, if all bidders' valuations are strong substitutes, this is a sufficient condition for WE to exist (Baldwin and Klemperer 2019). In practice, these conditions are rarely satisfied.

However, on regulated electricity markets, stability can also be enforced differently. ISOs stipulate penalties if a generator deviates from the efficient dispatch. Compared to a WE, they relax the EV condition and only ask for IR, implying new design desiderata for pricing on non-convex markets.

Definition 2 (Penalty-based stable, budget-balanced, and efficient outcome (PBE))**.** *Let* $\pi_i(\lambda) = v_i(x_i) - \langle \lambda, x_i \rangle$ *and* $\pi_j(\lambda) = \langle \lambda, y_j \rangle - v_j(y_j)$. *A linear and anonymous price vector* λ^* *and an efficient allocation* $(\mathbf{x}, \mathbf{y})$ *form a PBE if* $\pi_i(\lambda) \geq 0$, $\pi_j(\lambda) \geq 0$ *for every buyer* $i \in \mathscr{I}$ *and every seller* $j \in \mathscr{J}$, *and if the market is budget balanced with* $\sum_{i \in \mathscr{I}} \langle \lambda, x_i \rangle = \sum_{j \in \mathscr{J}} \langle \lambda, y_j \rangle$.

Proposition 1. *A combinatorial exchange can implement a PBE if the demand is price-inelastic and demand equals supply.*

Proof. We assume that the WDP is solved to optimality, providing an efficient allocation, $(\mathbf{x}, \mathbf{y}) = ((x_i)_{i \in \mathscr{I}}, (y_j)_{j \in \mathscr{J}})$ such that $\sum_{i \in \mathscr{I}} x_i = \sum_{j \in \mathscr{J}} y_j$. If all buyers are price-inelastic (thus for any price λ, $\pi_i(\lambda) \geq 0$ for all buyers i), we can choose linear and anonymous prices $\lambda^* = (\lambda^*(1), \ldots, \lambda^*(K))$ large enough such that $\pi_j(\lambda^*) \geq 0$ for all $j \in \mathscr{J}$, i.e. such that IR is satisfied for all generators. Finally the condition $\sum_{i \in \mathscr{I}} \langle \lambda^*, x_i \rangle = \sum_{j \in \mathscr{J}} \langle \lambda^*, y_j \rangle$ gives us budget balance. □

Since demand on electricity markets is not invariably price-inelastic (and thus PBEs do not exist in general), it is common to deviate from budget balance by providing make-whole payments that ensure individual rationality. As such personalized payments have an impact on the public market price signals, it is desirable to minimize them. We define the following design desiderata:

Definition 3 (Penalty-based stable and efficient outcome (PE))**.** *A linear and anonymous market price vector λ^*, personalized make-whole payments δ_i, δ_j, and an efficient allocation $(\mathbf{x}, \mathbf{y})$ form a PE if $\pi_i(\lambda) + \delta_i \geq 0$, $\pi_j(\lambda) + \delta_j \geq 0$ for every buyer $i \in \mathscr{I}$ and every seller $j \in \mathscr{J}$. For PE prices, we demand the total of the personalized make-whole payments to be minimal.*

Pricing Rules

For price computation, we want prices to be linear and anonymous and we enforce efficiency, i.e. social welfare maximization. We treat budget balance as first-order design goal and price-based stability as second-order goal. This means, we first aim for linear and anonymous prices minimizing the make-whole payments such that the price signal is not distorted due to large private and personalized payments. Such PBE or PE prices are not unique, and therefore we select those prices that minimize incentives to deviate.

Sets		**Decision Variables**	
$\mathscr{I} = \{1,...,I\}$	Buyers (index i)	$x \in \mathbb{R}^{IT}$	Buying quantities
$\mathscr{J} = \{1,...,J\}$	Generators (index j)	$d \in \{0,1\}^{rIT}$	Buy-side binary variables
$\mathscr{T} = \{1,...,T\}$	Time periods (index t)	$y \in \mathbb{R}^{JT}$	Selling quantities
$\mathscr{N} = \{1,...,N\}$	Nodes (index n)	$u \in \{0,1\}^{sJT}$	Sell-side binary variables
$\mathscr{L} = \{1,...,L\}$	Lines (index l)	$f \in \mathbb{R}^{LT}$	Line flows

Parameters			
$v \in \mathbb{R}^{IT}$	Buyer valuations	$e \in \mathbb{R}^{k}$	Buyer right-hand side
$c \in \mathbb{R}^{JT}$	Generator variable cost	$P \in \mathbb{R}^{NT \times LT}$	Inverse PTDF matrix
$h \in \mathbb{R}^{sJT}$	Generator fixed costs	$W \in \mathbb{R}^{NT \times JT}$	$\mathscr{J}$-to-$\mathscr{N}$-$\mathscr{T}$ mapping matrix
$A \in \mathbb{R}^{m \times JT}$	Generator constraint matrix I	$Z \in \mathbb{R}^{NT \times IT}$	$\mathscr{I}$-to-$\mathscr{N}$-$\mathscr{T}$ mapping matrix
$G \in \mathbb{R}^{m \times sJT}$	Generator constraint matrix II	$\overline{W} \in \mathbb{R}^{T \times JT}$	$\mathscr{J}$-to-$\mathscr{T}$ mapping matrix
$b \in \mathbb{R}^{m}$	Generator right-hand side	$\overline{Z} \in \mathbb{R}^{T \times IT}$	$\mathscr{I}$-to-$\mathscr{T}$ mapping matrix
$Q \in \mathbb{R}^{k \times IT}$	Buyer constraint matrix I	$\overline{F} \in \mathbb{R}^{LT}$	Upper flow limits
$R \in \mathbb{R}^{k \times rIT}$	Buyer constraint matrix II	$\underline{F} \in \mathbb{R}^{LT}$	Lower flow limits

Table 1: Notation for DCOPF

Unfortunately, these are computationally intractable problems if we want to compute them exactly. In lieu thereof, we choose the price vector that is closest to the dual variables of the LP relaxation of the allocation problem in a second step. If the allocation problem was a convex optimization problem, such dual prices would constitute a competitive equilibrium.

As an **allocation problem**, we assume a generic direct current optimal power flow (DCOPF) problem that is modeled as a mixed-integer program (MIP). In the abstract formulation, the set of traded goods $\mathscr{K}$ can now be described as the Cartesian product $\mathscr{N} \times \mathscr{T}$, where $\mathscr{N}$ represents a set of network nodes and $\mathscr{T}$ a set of time periods. Nodes are connected through a set of transmission lines $\mathscr{L}$. Table 1 provides an overview of the notation.

$$
\begin{aligned}
\max \quad & v'x - c'y - h'u && \text{Welfare maximization} && \text{(DCOPF)} \\
\text{s.t.} \quad & Ay + Gu \geq b && \text{Generator constraints} && (1) \\
& Qx + Rd \leq e && \text{Buyer constraints} && (2) \\
& Pf = Wy - Zx && \text{Power flows} && (3) \\
& \overline{W}y - \overline{Z}x = 0 && \text{Aggregate power balance} && (4) \\
& \underline{F} \leq f \leq \overline{F} && \text{Line flow limits} && (5) \\
& x \geq 0, y \geq 0, u \in \{0,1\}^{sJT}, d \in \{0,1\}^{rIT}, f \in \mathbb{R}^{LT} && && (6)
\end{aligned}
$$

For convenience, we define vector x_i to include only the buying quantities of buyer $i \in \mathscr{I}$ as non-zero components, i.e. $\sum_{i\in\mathscr{I}} x_i = x$. Similarly, we define the vectors d_i, y_j, u_j, as well as x_t, y_t, x_{it}, and y_{it}. The utility of buyer i (generator j) is then defined as $\pi_i(x_i,\lambda) = v'x_i - \lambda' Z x_i$ ($\pi_j(y_j,\lambda) = \lambda' W y_j - c' y_j - h' u_j$) with λ being the market price.

We first focus on the case of **pricing with price-inelastic demand** (i.e., $v'x$ can be removed from the objective function). Let (x^*, u^*, y^*, d^*, f^*) denote the optimal solution to this problem. As the buyers are price-inelastic, there will always be prices $\lambda^{PBE} \in \mathbb{R}^{NT}_{\geq 0}$ such that no generator incurs losses (see Proposition 1). PBE-P is thus always feasible. It aims to minimize prices such that at the efficient dispatch no generator makes a loss at any time (first constraint), and that there are no negative congestion revenues (second constraint). The latter prevents that prices are set low at demand-intensive nodes and high at supply-intensive nodes, implying missing money only due to nodal price discrepancies. The third constraint describes another MIP where participants compute their payoff-maximizing dispatch at the prices. The difference between the objective function value of the lower-level program and the profit in the upper level should be minimized.

$$\begin{aligned} \min_{\lambda,\pi,\gamma} \quad & \sum_{j\in\mathscr{J}} \gamma_j && \text{(PBE-P)} \\ \text{s.t.} \quad & \\ & \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 && \forall j \in \mathscr{J}, t \in \mathscr{T} \\ & \lambda' Z x^* - \lambda' W y^* \geq 0 \\ & \pi_j - (\lambda' W y^*_j - c' y^*_j - h' u^*_j) \leq \gamma_j && \forall j \in \mathscr{J} \\ & \pi_j = \max_{y,u} (\lambda' W y_j - c' y_j - h' u_j) \quad \text{s.t. } (1),(6) && \forall j \in \mathscr{J} \\ & \lambda \in \mathbb{R}^{NT}_{\geq 0}, \pi \in \mathbb{R}^{J}_{\geq 0}, \gamma \in \mathbb{R}^{J} \end{aligned}$$

Solving bilevel MIPs is Σ_2^p-hard in general (Jeroslow 1985), and there is no hope that realistic problem sizes of PBE-P could be solved in practice. Therefore, instead of PBE-P we solve PBE-A, omitting the lower-level optimization and instead minimizing the difference to so-called ELMP prices. Extended Locational Marginal Pricing (ELMP), as discussed below, aims at minimizing lost opportunity costs, and if a market is convex, it actually does. With an L_1 norm in the objective, the problem can be modeled as a linear program, which can be solved in polynomial time.

$$\begin{aligned} \min_{\lambda} \quad & \|\lambda - \lambda^{ELMP}\|_1 && \text{(PBE-A)} \\ \text{s.t.} \quad & \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 \qquad \forall j \in \mathscr{J}, t \in \mathscr{T} \\ & \lambda' Z x^* - \lambda' W y^* \geq 0 \\ & \lambda \in \mathbb{R}^{NT}_{\geq 0} \end{aligned}$$

We now consider **pricing with price-sensitive demand**, i.e. buyers submit valuations v and thus a PBE does not always exist. We therefore sacrifice budget balance (BB) but still ensure EF and IR.

As a result, market prices are still linear and anonymous (LA), but individual payments are not. We define the following problem to compute minimal make-whole payments $\delta^{\mathscr{I}*}$ and $\delta^{\mathscr{J}*}$:

$$
\begin{aligned}
\min_{\lambda,\delta^{\mathscr{I}},\delta^{\mathscr{J}}} \quad & \|\delta^{\mathscr{I}}\|_1 + \|\delta^{\mathscr{J}}\|_1 && \text{(PE-a)}\\
\text{s.t.} \quad & v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathscr{I}}_{it} \geq 0 && \forall i \in \mathscr{I}, t \in \mathscr{T}\\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} + \delta^{\mathscr{J}}_{jt} \geq 0 && \forall j \in \mathscr{J}, t \in \mathscr{T}\\
& \lambda' Z x^* - \lambda' W y^* \geq 0\\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \delta^{\mathscr{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathscr{J}} \in \mathbb{R}^{JT}_{\geq 0}
\end{aligned}
$$

Again, the resulting price vectors are not unique, and we could formulate a bilevel integer program PE-P aiming to minimize prices, satisfy IR, and in the lower-level programs minimize the incentives to deviate from the efficient dispatch. Similar to the case with price-inelastic demand, we can then replace the bilevel integer program by a tractable linear program that minimizes the distance to ELMP. We refer to this rule as PE-A.

$$
\begin{aligned}
\min_{\lambda,\pi,\gamma,\delta^{\mathscr{I}},\delta^{\mathscr{J}}} \quad & \sum_{i\in\mathscr{I}} \gamma_i + \sum_{j\in\mathscr{J}} \gamma_j && \text{(PE-P)}\\
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathscr{I}}_{it} \geq 0 && \forall i \in \mathscr{I}, t \in \mathscr{T}\\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} + \delta^{\mathscr{J}}_{jt} \geq 0 && \forall j \in \mathscr{J}, t \in \mathscr{T}\\
& \lambda' Z x^* - \lambda' W y^* \geq 0\\
& \delta^{\mathscr{I}} = \delta^{\mathscr{I}*}\\
& \delta^{\mathscr{J}} = \delta^{\mathscr{J}*}\\
& \pi_i - (v'x^*_i - \lambda' Z x^*_i) \leq \gamma_i && \forall i \in \mathscr{I}\\
& \pi_i = \max_{x,d}(v'x_i - \lambda' Z x_i) \quad \text{s.t. } (2),(6) && \forall i \in \mathscr{I}\\
& \pi_j - (\lambda' W y^*_j - c' y^*_j - h' u^*_j) \leq \gamma_j && \forall j \in \mathscr{J}\\
& \pi_j = \max_{y,u}(\lambda' W y_j - c' y_j - h' u_j) \quad \text{s.t. } (1),(6) && \forall j \in \mathscr{J}\\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \pi \in \mathbb{R}^{I+J}_{\geq 0}, \gamma \in \mathbb{R}^{I+J}, \delta^{\mathscr{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathscr{J}} \in \mathbb{R}^{JT}_{\geq 0}
\end{aligned}
$$

There is a significant literature on **existing pricing rules** for electricity spot markets, and a detailed discussion of all proposals is beyond the scope of this paper. An excellent overview of various pricing rules with price-inelastic demand is provided by Liberopoulos and Andrianesis (2016). The widely established *Integer Programming (IP) pricing* (O'Neill, Sotkiewicz, et al. 2005) fixes integer variables to their optimal values, resulting in a linear program. The duals of the nodal balance constraints eventually provide linear and anonymous market prices. Budget balance is violated due to the need for make-whole payments, and the prices do not constitute a competitive equilibrium. The above mentioned *ELMP* relaxes binary variables to continuous variables and takes the duals of the relaxed problem as market prices. Again, there are individual make-whole payments, and stability of the solution is enforced via penalties. ELMP represents an approximation of Convex Hull Pricing (CHP) which indeed minimizes lost opportunity costs, but is too computationally expensive to be implemented in the field. Recently, O'Neill, Hytowitz, et al. (2019) suggested *AIC pricing* to address biased market prices resulting from make-whole payments. AIC pricing implements IP pricing as a first stage. In a second step, it relaxes the integer variables of generators that make a loss for the actual AIC pricing run and adjusts their objective function coefficients to reflect the average costs. The approach does not consider make-whole payments for buyers, but proposes a price differentiation via Ramsey-Boiteux-like pricing. In contrast, PE-A minimizes make-whole payments for both sides of the market in a single optimization and does not involve any price-differentiation. We argue that if make-whole payments are low (as verified in our experiments), there is no need to resort to discriminatory prices because the market price includes "almost" all information about supply and demand. For our experiments, we will ignore price differentiation in AIC, but instead compute make-whole payments to allow for better comparisons.

Numerical Experiments

For some **illustrative examples**, we go from simple to more complex environments. We start with a *simple convex setting*, consisting of two generators G1 and G2 and two buyers B1 and B2 at a single node and over three time periods. G1 offers up to 15 MW for $5/MW, and G2 offers up to 20 MW for $3/MW. B1 (B2) schedules price-inelastic demand of 4 MW (3 MW), 6 MW (6 MW), and 10 MW (12 MW), resp., for hour 1, 2, and 3. The optimal solution, as shown in Tables 2 and 3, is to let G2 satisfy the entire demand in the first two periods, while G1 satisfies only the residual demand of 2 MW in excess of the maximum load of G2 in the third period. IP, ELMP, AIC, and PBE-A prices are identically set and constitute a WE, PBE, and PE.

Next, we introduce *non-convexities* for the generators, i.e. G1 (G2) has a minimum load of 2 MW (10 MW) per period and no-load costs of $8 ($10) that occur as fixed costs when committed. It is also assumed that G1 requires a minimum runtime of three periods. The optimal dispatch in Table 5 now involves G1 satisfying the entire demand in $t = 1$ and running at a minimum load in the remaining periods, while G2 satisfies the residual demand. As seen in Table 4, neither pricing rule yields a WE. IP, ELMP, and AIC result in individual losses, at least for some of the hours, and thus fail to produce a PBE. PBE-A avoids any make-whole payments and yields a PBE.

We now introduce *price-sensitive demand*. We assume that half of the demand is retained as price-inelastic. For the remaining half, B1 bids $10/MW and B2 bids $2/MW in each period, resp. Due to the price-sensitive demand, we now use PE-A instead of PBE-A. Under the welfare-optimal dispatch (see Table 7), no PBE is possible. In order to satisfy the price-inelastic demand, both generators need to produce at least at their minimum loads, and make-whole payments thus become inevitable. As Table 6 shows, PE-A achieves the lowest aggregate make-whole payments

($10.50 to G1, $6.50 to B2), which are as close to budget balance as possible.

[$/MW]	IP	ELMP	AIC	PBE-A
t=1	3.00	3.00	3.00	3.00
t=2	3.00	3.00	3.00	3.00
t=3	5.00	5.00	5.00	5.00
MWP	0.00	0.00	0.00	0.00

Table 2: Base Case: Prices

[MW]	G1	G2	B1	B2
t=1	0	7	4	3
t=2	0	12	6	6
t=3	2	20	10	12

Table 3: Base Case: Dispatch

[$/MW]	IP	ELMP	AIC	PBE-A
t=1	5.00	3.50	6.14	6.14
t=2	3.00	3.50	3.00	9.00
t=3	5.00	6.10	9.00	9.00
MWP	38.00	40.29	22.00	0.00

Table 4: Non-Convexities: Prices

[MW]	G1	G2	B1	B2
t=1	7	0	4	3
t=2	2	10	6	6
t=3	2	20	10	12

Table 5: Non-Convexities: Dispatch

[$/MW]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	6.45	6.45
t=2	2.00	3.50	4.00	4.17
t=3	3.00	3.50	3.71	8.58
MWP	64.00	50.75	26.57	17.00

Table 6: Price-Sensitive Demand: Prices

[MW]	G1	G2	B1	B2
t=1	5.5	0	4	1.5
t=2	2	10	6	6
t=3	2	14	10	6

Table 7: Price-Sensitive Demand: Dispatch

Finally, we conduct tests on the **IEEE RTS System** introduced by Grigg et al. (1999). In accordance with Zoltowska (2016), we select the single area, 24-node topology for a representative winter day. For data on generation cost or demand valuation we rely on the cases studies of Garcia-Bertrand et al. (2006) and Zoltowska (2016) on this system. Our base setting includes 32 generators with minimum/maximum loads, minimum runtimes, no-load costs and an offer curve representing variable costs. First, the demand of the 17 consumers is assumed to be *price-inelastic*. Table 8 reports statistics on prices, make-whole payments (MWP) as well as the penalties necessary to avoid generators to deviate from the efficient dispatch. A PBE is only achieved by PBE-A. All other pricing rules require make-whole payments to ensure individual rationality, with classical IP pricing causing the biggest violation of budget balance. AIC prices have price peaks that allow for overall profitability for the generators, but when individual periodic losses are still compensated, make-whole payments still occur during low-price periods. In contrast, ELMP produces a smooth price profile with little volatility and low lost opportunity costs (as reflected by the sum of make-whole payments and penalties). PBE-A adjusts this price profile only slightly in order to ensure a PBE, mainly by increasing prices at the nodes where most of the otherwise unprofitable generators are situated. With *price-sensitive* demand, as reported in Table 9, it is not possible to achieve a PBE in this environment. Similar to the price-inelastic case, IP pricing diverges most from budget balance, with make-whole payments amounting to 2.5% of the total incurred generation costs. ELMP prices are higher on average, resulting in less make-whole payments for the generators. PE-A requires make-whole payments of only \$112.41 while penalties are only marginally increased. Only IP prices are on average lower than PE-A prices, and the total lost opportunity costs of PE-A are minimal among the pricing rules under consideration. PE-A prices are minimal in make-whole payments, closest to stability, and imply low and smooth price profiles.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	22.32	8.81	35,749.28	0.00	0.00	0.00	4.63%	1.39s
ELMP	22.62	6.40	6,193.37	0.00	2,460.20	0.00	0.80%	1.39s
AIC	29.62	16.92	26,114.72	0.00	46,095.51	0.00	3.38%	2.68s
PBE-A	23.35	7.53	0.00	0.00	17,966.49	0.00	0.00%	1.46s

Table 8: IEEE RTS Statistics with Price-Inelastic Demand

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.63	5.03	14,272.57	0.33	0.00	0.42	2.52%	1.57s
ELMP	21.02	5.35	490.39	781.43	257.43	803.04	0.22%	1.55s
AIC	21.27	6.36	11,048.85	945.38	0.00	0.00	2.12%	2.96s
PE-A	20.85	5.23	0.00	112.43	929.52	804.66	0.02%	1.66s

Table 9: IEEE RTS Statistics with Price-Sensitive Demand

Conclusions

In current electricity markets, there is a significant discussion about out-of-market make-whole payments paid by the ISOs to some of the generators. These payments can be significant and distort market price signals. We show that with the standard assumption of price-inelastic demand and demand-supply equivalence no make-whole payments are necessary. However, with the advent of VER, demand response becomes of central importance, and ISOs need to incorporate price-sensitive demand. In such markets zero make-whole payments are impossible in general. Based on this insight, we introduce the PE-A pricing rule that minimizes make-whole payments, and compare it to several existing payment rules. Rather than trying to mimic competitive equilibrium prices based on linear relaxations of the underlying non-convex allocation problem, we set budget balance as first-order and envy-freeness as second-order design goal and optimize these objectives directly. Our experiments show that high side-payments can either be avoided or reduced substantially. Prices under PE-A do not increase on average compared to established pricing rules, and the changes in the overall payments of market participants are very small. The new pricing rules are based on optimization problems that can be solved in polynomial time and whose principles are easy to understand and communicate. It does not depend on the specifics of the underlying allocation problem and can be applied to other non-convex markets as well.

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