Pricing in Non-Convex Markets: How to Price Electricity in the Presence of Demand Response

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A Walrasian competitive equilibrium defines a set of linear and anonymous prices where no coalition of market participants wants to deviate. Walrasian prices do not exist in non-convex markets in general, with electricity markets as important real-world example. However, the availability of linear and anonymous prices is important for derivatives markets and as a signal for scarcity. Prior literature on electricity markets assumed price-inelastic demand and introduced numerous heuristics to compute linear and anonymous prices on electricity markets. At these prices market participants often make a loss. As a result, market operators provide out-of-market side-payments (so-called make-whole payments) to cover these losses. Make-whole payments dilute public price signals and are a significant concern in electricity markets. Besides, demand-side flexibility becomes increasingly important with growing levels of renewable energy sources. Demand response implies that different flexibility options come at different prices, and the proportion of price-sensitive demand that actively bids on power exchanges will further increase. We show that with price-inelastic demand there are simple pricing schemes that are individually rational (participants do not make a loss), clear the market, support the efficient solution and do not require make-whole payments. With the advent of demand-side bids budget balanced prices cannot exist anymore, and we propose a pricing rule that minimizes make-whole payments. We describe design desiderata that different pricing schemes satisfy and report results of experiments that evaluate the level of subsidies required for linear and anonymous prices on electricity spot markets with price-sensitive demand.

Key words: Electricity market design, demand flexibility, non-convexities, pricing

1. Introduction

In many parts of the world, electricity markets have developed from monopolies to competitive wholesale markets. For example, European countries and large parts of the U.S. liberalized their electricity markets in the 1990s. Short-term electricity procurement is now carried out via power exchanges in these jurisdictions. These power exchanges determine central price signals for over-the-counter trades and futures markets (Shah and Chatterjee 2020). Typically, on day-ahead markets hourly products for the next day are traded. After the day-ahead markets, the market operators use real-time markets in the U.S. (or intraday markets in Europe) to deal with changes in supply and demand that are closer to the actual dispatch. We will distinguish these types of *electricity spot markets* from *futures markets* where participants can hedge against longer-term price risks.

Spot markets are significant in size. In 2020, European coupled day-ahead markets alone cleared 1,530 TWh in 27 countries with average prices between 30 and 40 EUR/MWh (NEMO Committee 2021). Similarly, the cost of serving load amounted to $8.9 billion the Californian market, covering 26,000 circuit miles, roughly 1,000 power plants, a population of 30 million, and about 9,700 pricing nodes (California ISO 2018, 2021).

With climate change and a transition to renewable energy sources (RES) such as wind and solar power, we move to an economy with many thousands of small generators and a more price-sensitive demand side that actively bids in electricity markets and offers flexibility to cope with variability in the supply (IRENA 2019, Hytowitz et al. 2020). Changes on electricity markets are not only relevant for market operators, but they impact generators, industrial and retail consumers alike. These changes in the market have led to renewed interest in the design of electricity markets. While many aspects of electricity market design are similar to other markets, a few features stand out. First, demand and supply need to be balanced at all times to guarantee a stable electricity grid. Second, electricity markets are characterized by non-convex preferences. For example, electricity suppliers often incur fixed costs for starting up and running their generators. On the demand side, industrial customers typically need a certain volume of electricity over consecutive hours to finish production or maintain energy-intensive services. Such consumption profiles can sometimes be shifted over time, but the profiles themselves must not be altered. These non-convexities in the preferences typically lead to non-convex optimization problems that need to be solved in order to determine the efficient or welfare-maximizing dispatch and prices. We will use the term *non-convex markets* in what follows. Non-convexities have received some attention in electricity market design, but due to increasing price-sensitive bid-in demand on spot markets, new approaches are required.

1.1. Competitive Equilibrium

Early in the research on markets, general equilibrium theory studied demand, supply, and prices for multiple goods or objects on markets. The Arrow-Debreu model shows that under convex preferences, perfect competition, and demand independence, there must be a set of competitive equilibrium prices (Arrow and Debreu 1954, McKenzie 1959, Gale 1963, Kaneko 1976). The results derived from the Arrow-Debreu model led to the well-known welfare theorems, representing important arguments for markets to be used as efficient or welfare-maximizing means to allocate scarce resources such as electricity. The first theorem states that any Walrasian equilibrium leads to a Pareto efficient allocation of resources. The second theorem states that any efficient allocation can be attained by a Walrasian equilibrium under the Arrow-Debreu model assumptions (Mas-Colell et al. 1995). Walrasian equilibrium prices are such that there is a single price for each product (i.e., prices for a package are *linear*) and this is the same price for all participants (i.e., *anonymous* prices with no price differentiation).

However, standard general equilibrium theory assumes divisible goods and convex preferences. Most real-world markets such as those for electricity, transportation, radio frequency spectrum, or environmental access rights are traded as indivisible goods and participants have non-convex preferences and complex constraints. Such markets have led to substantial interest in the question when Walrasian equilibria exist. Unfortunately, in markets with indivisible goods, it is well-known that only very restricted types of valuations (e.g., substitutes valuations) allow for convex allocation problems and Walrasian equilibria (Kim 1986, Bikhchandani and Mamer 1997, Leme 2017, Baldwin and Klemperer 2019).

This raises the question how prices can be computed in the presence of non-convex preferences for indivisible goods and which properties we can hope to achieve compared to Walrasian equilibria. Established market design desiderata are *efficiency* (i.e., maximization of welfare or gains from trade), *individual rationality* (i.e., participants should not make a loss), *budget balance* (i.e., the market operator should not make a loss or a gain), and *envy-freeness* (i.e., participants would not want a different allocation at the prices). These axioms are not only central to economic theory (Mas-Colell et al. 1995), but are widely adopted and natural design desiderata for practical market design. If the allocation problem is convex, duality theory and dual prices in convex optimization provides a principled way to determine competitive equilibrium prices that satisfy these desiderata (Bichler et al. 2020). In non-convex markets, it is well known that competitive equilibrium prices might need to be non-linear and personalized and even such prices might not exist (Bichler and Waldherr 2017). Thus, in a combinatorial auction or a combinatorial exchange that allows for supply and demand bids on packages of items, each bidder might need to have a different price for the same package (personalized prices), and each package price could differ from the sum of the item prices in this package (non-linear prices). As a simple example, consider a single supplier with an (indivisible) sell bid of 2 MWh for $30, while there is one buyer asking for 1 MWh for at most $10, and another buyer asking for 1 MWh for $28. Linear and anonymous market prices could not be higher than $10/MWh and as such there would be no trade and no gains from trade. With price differentiation, trade would be possible. However, non-linear and personalized prices would convey little information other than that a bidder lost or won. Besides, if prices should serve as a baseline for derivatives as is the case for options or futures, this is hardly possible with non-linear prices that differ among participants. In other words, anonymity and linearity are important requirements for prices on electricity markets but also other domains (Bichler et al. 2018).

1.2. Pricing on Electricity Spot Markets

Electricity spot markets are composed of varying levels of "demand" (load) and matching levels of "supply" (generation). Market participants submit supply and demand bids according to a certain *bid language* that determines the form of the allocation problem (which yields the efficient dispatch) and the pricing rule. As introduced earlier, generators on electricity markets often exhibit start-up or no-load costs, economies of scale (often implemented via piecewise-linear cost functions), or minimum-generation requirements. Bid languages are designed to incorporate these aspects, which necessarily translate into non-convex allocation problems (Herrero et al. 2020). In 2005, the Pennsylvania, Jersey, Maryland Power Pool (PJM) introduced mixed integer programming (MIP) in order to address these non-convexities and to determine the efficient allocation or dispatch (O'Neill et al. 2020). Since 2018, all Independent System Operators (ISOs) in the U.S, use MIPs to compute the efficient dispatch instead of the Lagrangian relaxation that was used before. Dual prices as they are available for convex optimization problems are not available in such markets, which led to a fundamental question: How can market prices per hour be computed in such non-convex markets?

One approach followed by European day-ahead markets is to sacrifice efficiency. The EUPHEMIA algorithm that is used to clear European day-ahead markets first solves a welfare maximization allocation problem as a mixed-integer program and then iteratively tries to find linear and anonymous prices that clear the market. If such prices cannot be found, additional constraints are added to the welfare maximization problem (Committee et al. 2020). However, it is unclear how much of the gains from trade are sacrificed this way. Furthermore, this approach inevitably leads to paradoxically rejected bids (Meeus et al. 2009). In particular, there are generators with an ask price that is less than the market price, yet they will not be dispatched. Such prices are also not *envy-free* and hence not a Walrasian equilibrium. We will not further discuss this approach in our paper and focus on market designs as in the U.S. that implement the efficient outcome.

Over the years, several pricing rules have been suggested aiming to mimic competitive equilibrium prices on such MIP-based electricity markets (Liberopoulos and Andrianesis 2016). Locational marginal pricing (LMP) rules of many ISOs are based on IP pricing (aka. Integer Pricing), where the allocation problem is solved to optimality, the integer variables are fixed, and the prices are then derived from the dual variables of the demand-supply constraint of the resulting (convex) linear program (O'Neill et al. 2005). IP pricing computes anonymous and linear prices, but these prices do not constitute competitive equilibrium prices. Some generators might not maximize their individual profits and want to deviate, i.e., switch to a different dispatch at those prices, and IP prices are thus not *envy-free*. The latter is central to the definition of a competitive equilibrium and it leads to stability of the outcome. Importantly, besides a lack of stability, the generators often make a loss at the IP prices, i.e., prices are not even *individually rational*. Pricing in U.S. ISO markets has changed in an attempt to reduce the weight of uplift, and to internalize all operational costs into market prices as far as possible (Herrero et al. 2020). Some ISOs switched from IP pricing to Extended LMPs (ELMPs) in the recent years, which are based on the dual variables from the demand-supply constraint in the LP relaxation of the underlying MIP. However, similar issues arise. As a consequence, U.S.

ISOs continue to search for improvements via new formulations for ELMP.[1]

ISOs use personalized side-payments to address the fact that the public market prices from IP pricing or ELMP are neither envy-free nor individually rational. This effectively differentiates the linear and anonymous market prices from the payments of the market participants, which are then non-linear and personalized. These external side-payments could be so high that no generator would want to change its dispatch and envy-freeness is achieved (*lost opportunity cost payments*). Lost opportunity costs payments may be very large if the market contains non-convexities, and these payments could even go to generators that were not scheduled (Eldridge et al. 2019). Electricity markets are highly regulated markets and as such there are alternative means to enforce stability other than high lost opportunity cost payments. Actually, most ISOs only pay *make-whole payments* to ensure individual rationality of all generators and stipulate penalties that a generator has to pay if it indeed deviates from the optimal dispatch. In other words, they relax envy-freeness to only individual rationality requirements. We refer to such outcomes as having *penalty-based stability*.

However, even the make-whole payments are a significant concern (Hytowitz et al. 2020). The U.S. Federal Energy Regulatory Commission (FERC) regulates the U.S. wholesale power markets to promote just competition. In 2018, the FERC found that the practices of several ISOs were unjust and ordered them to change their pricing because prices did not accurately reflect the cost of serving load (O'Neill et al. 2019). Make-whole payments are not reflected in the public price signals, and they lead to biased investment signals. This also constitutes a problem for futures markets, where spot market prices serve as the key reference. In addition, the FERC has released several orders and notices about pricing, which argue that "the use of side-payments can undermine the market's ability to send actionable price signals."[2] Similarly, O'Neill et al. (2019) state that "the make-whole payments are not transparent to other market participants and are allocated too broadly to provide correct price incentives for market participants to make efficient entry and exit decisions as well as efficient investments in facilities and equipment." In summary, a challenge in U.S. ISO markets is to reduce side-payments, which are a clear sign of inefficient pricing, while still ensuring individual rationality of all market participants.

In a *first contribution*, we introduce an optimization model which always computes prices that are individually rational, budget balanced, and clear the market at the efficient dispatch without make-whole payments under the assumptions of price-inelastic demand and strict demand-supply equality. These assumptions are standard in the electricity market literature (Liberopoulos and Andrianesis 2016).

[1] https://www.misoenergy.org/stakeholder-engagement/stakeholder-feedback/msc-elmp-iii-whitepaper-20190117/

[2] https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation

1.3. Price-Sensitive vs. Price-Inelastic Demand

While the academic literature on electricity market pricing almost exclusively relies on the assumption of price-inelastic demand, this assumption is unlikely to hold in the future (Herrero et al. 2020). Power systems are changing profoundly due to the introduction of large volumes of RES. The largest proportion of RES capacity are Variable Energy Resources (VER) such as solar and wind power. The characteristic variability and uncertainty of these VER require an integration of demand flexibility (Reihani et al. 2016). Demand response is the most immediately available way of increasing demand flexibility and the cheaper option compared to storage technologies (EU 2016). For example, industrial processes for the production of pulp and paper are able to provide demand response with a duration of up to three hours without any notice time (EU 2016). Still, this flexibility comes at a cost and bidders want lower prices if they provide more flexibility. As indicated, it is expected that in the future we will see a much increased amount of price-responsive demand (Hytowitz et al. 2020). The recent FERC order 2222 from 2020 also aims at an active demand side to bid in wholesale markets. However, such price-sensitive bids for flexible demand make market design more challenging. First, new bid formats lead to additional non-convexities which even increase the make-whole payments needed with currently used pricing rules. Second, prices that are individually rational and clear the market at the efficient dispatch cannot always be budget balanced anymore, as we will show.

In a *second contribution*, we introduce alternative pricing rules that minimize make-whole payments while they still clear the market at the efficient dispatch with price-sensitive demand. The pricing rules introduced in this paper are based on a mathematical program which differs significantly from IP pricing, ELMP, or other proposals in the literature. Similar to existing pricing rules on ISO markets, it treats efficiency and individual rationality as first-order goals (i.e., enforces these directly in the model), while budget balance and envy-freeness are treated as second-order goals. However, in contrast to existing literature, we prioritize budget balance over envy-freeness in a lexicographical way. The violation of budget balance and the resulting side-payments have led to concerns by regulators and market participants, as we discussed earlier. Envy-freeness should lead to stability of the outcome in markets as participants do not have an incentive to deviate. In highly regulated and transparent electricity markets, stability can be achieved by imposing penalties, which is already common on ISO markets today. Participants cannot easily deviate from the efficient dispatch determined by the market operator, and the level of penalties (that generators would only have to pay if they deviated from the efficient dispatch) is much less of a concern than high personalized side-payments by the market operator that are not reflected in the market prices.

The new pricing rule (PE-A) that we propose can be computed in polynomial time and scales to large problem sizes. Importantly, we show that the prices, on average, do not increase compared to other established pricing rules in our experiments and the impact on the payments of individual market participants are small. However, PE-A avoids large make-whole payments as they occur with IP pricing, even with price-sensitive demand. In Section 5.2 we analyze our proposed pricing schemes based on a widely-used benchmark data set: the IEEE RTS benchmark market consisting of 24 nodes, 24 hours, 32 generators (with non-convex cost functions), and 17 consumers. The average make-whole payments for PE-A pricing only amount to 0-0.15% of the total costs in all treatments. In contrast, for IP pricing or ELMP the make-whole payments were 4-5% on average for all generators. Actually, for some generators the make-whole payments could be more than 10% of their payment with IP pricing. Such high make-whole payments can be avoided with PE-A and we achieve almost budget balanced outcomes in all experiments. We also compare PE-A with a simple implementation of Average Incremental Cost (AIC) pricing, a pricing rule that was recently proposed to address high side-payments on electricity spot markets. PE-A is not restricted to specifics of the allocation problem on electricity markets and can also be applied to other types of non-convex and two-sided markets.

1.4. Positioning in the Literature

This paper draws on different streams in the literature. The fundamental problem of pricing in multi-object markets is central to micro-economic theory and the management sciences. The fact that non-convex preferences lead to problems in equilibrium theory is known for a long time (see for example Farrell (1959)). Several contributions such as the well-known Shapley-Folkman-Starr lemma (Starr 1969) suggest that nearly competitive equilibria are possible if the market grows large. A number of more recent articles suggest that Walrasian prices can be approximated in (very) large markets and that such markets are approximately incentive-compatible (Azevedo et al. 2013, Azevedo and Budish 2019).

Electricity markets are already very large with hundreds of participants, but the non-convexities still matter. The question how actual pricing rules for such non-convex electricity markets should be designed has led to a number of heuristics such as IP pricing and ELMP in the operations research and power engineering literature (O'Neill et al. 2005, 2016, Liberopoulos and Andrianesis 2016, Eldridge et al. 2019, O'Neill et al. 2019). We will revisit this literature in Section 4.4. These heuristics typically aim to approximate a competitive equilibrium and relax budget balance and envy-freeness.

The Information Systems literature has made numerous contributions to market design in general and to pricing in non-convex markets more specifically. Some of the work deals with pricing in combinatorial auctions (Xia et al. 2004, Adomavicius and Gupta 2005, Adomavicius et al. 2012, Guo et al. 2012, Petrakis et al. 2013, Bichler et al. 2013, Adomavicius et al. 2020), while other articles deal with combinatorial exchanges (Guo et al. 2012, Bichler et al. 2018). The design of energy markets has also received attention in Information Systems more recently (Ketter et al. 2016, Valogianni and Ketter 2016, Koolen et al. 2018). This paper combines these two strands suggesting a new approach to pricing in electricity markets that substantially reduces or even eliminates the need for side-payments. Our approach can well be relevant to other non-convex markets such as those used in transportation (Caplice and Sheffi 2003, Garrido 2007) or for the trading fishery access rights (Bichler et al. 2019).

1.5. Organization of the Paper

The rest of the paper is structured as follows: in Section 2, we provide a short introduction to electricity market design. In Section 3, we discuss competitive equilibrium theory and show when anonymous and linear prices are possible on budget-balanced electricity markets. Section 4 introduces optimization models to compute prices in environments with price-inelastic and price-sensitive demand. We briefly characterize existing proposals for electricity prices before we provide results of experiments in Section 5. Section 6 provides a summary and conclusions.

2. Bid Languages and Demand-Side Flexibility

Let us provide a brief overview of electricity market design and the role of demand response for future market designs. The pricing rules that can be employed on a market depend on the underlying allocation problem, which again depends on the types of bids or the bid language available on a market. The bid languages on electricity markets today are specific and aim at reflecting the underlying cost functions of generators and – in part – valuation functions of the demand side. They allow the participants to communicate their valuations or cost structures effectively. The market operator then solves the allocation problem and determines a schedule of generation and prices (Cramton 2017). Day-ahead markets are complemented by intraday (Europe) or real-time (U.S.) markets. These markets modify the day-ahead schedule to determine the actual physical dispatch. Especially in European countries, the day-ahead market is considered to be the main reference market, while in the U.S. it mostly possesses the notion of a forward market for the real-time market that determines the dispatch (Antonopoulos et al. 2020).

Bid languages allow for the expression of the underlying costs in order to enable efficient outcomes (Cramton 2003). For instance, generators typically incur certain fixed costs for starting up and running a generator, as well as variable electricity production costs. Moreover, the operation is often subject to technical conditions, e.g., referring to minimum runtimes or ramping constraints. On the demand side, market participants might want to express certain flexibility options, and this will become much more of an issue in the future with increasing levels of RES. Let us briefly summarize the state-of-the-practice.

In European markets, aside from regular bids for individual hours of the day, the bid language allows for *block bids*. The latter represent a set of individual bids that can be executed only in total or not at all (Committee et al. 2020). Cost structures are communicated as single-part offers, requiring market participants to aggregate various cost components into a single parameter. It is explicitly refrained from the communication of multiple cost components in order to promote decentralized decision-making on the part of the market participants (Herrero et al. 2020). Most European markets allow for price-sensitive bids on the demand side, although between 2010 and 2015 an estimated 82-89% of the bids were not price-sensitive (EU 2016). In Europe, the market is cleared with (zonal) linear and anonymous prices without any side-payments which leads to efficiency losses in the dispatch (Meeus et al. 2009). Overall, the bid language permits a less detailed expression of cost functions than, for instance, bid languages used in the U.S.

Market participants in the U.S. are generally permitted to indicate their costs in a more granular way than in European markets (Madani et al. 2018). Cost structures can be communicated with *multi-part bids*, usually consisting of start-up costs, no-load costs as well as an offer curve. Furthermore, generators can express technical constraints such as minimum up and down times, minimum and maximum output levels, ramp rates, or start-up times. This allows generators to express their cost characteristics very effectively (Cramton 2017). So-called self-schedules are pure quantity bids specifying an amount of energy that needs to be dispatched regardless of price levels or cost structures. Demand-side bids comprise price-inelastic self-scheduling as well as price-sensitive bid curves (Cramton 2017).

As an example of an ISO bid language, PJM allows for fixed-demand bids and price-sensitive bids on the demand side. A fixed-demand bid or self-schedule is price-inelastic and defines a level of energy to be purchased at any price over a particular hour at a location or node. In contrast, price-sensitive bids specify a defined level of energy, a location and a price, above which the demand bid is zero. More than 90% of the bids in the PJM market were fixed-demand bids in 2019, and only a very small proportion is price-sensitive at this point (Monitoring Analytics 2019). This explains why most proposals for pricing rules in the literature assume only price-inelastic demand. However, this will change with increasing levels of demand response, which specifies flexible bids to be executed but only up to a certain price.

U.S. ISO markets aim to find a welfare-maximizing dispatch based on bids, yet in contrast to European markets, they first determine the efficient dispatch before they compute prices. While European markets compute prices for large price zones, the prices on U.S. electricity markets are computed per node in the electricity grid. The nodal system aims to consider physical grid constraints in the optimization. In nodal markets, bids and offers, resource constraints, network constraints, transmission losses and certain ancillary service requirements are co-optimized (Cramton 2017). As a result, the electricity price reflects the marginal cost of supplying electricity at a specific node in the network (assuming the underlying problem was convex). Locational marginal prices have also been suggested for European markets (Purchala 2018, Ashour Novirdoust et al. 2021). For the remainder of this paper, we discuss markets as they are operated by ISOs in the United States, in Australia, in South American markets, and many other parts of the world.

As indicated, demand-side bidding is central to accommodate the volatile nature of VER in the future. ISOs in the U.S. have already taken steps to accommodate demand-side flexibility and price-sensitive bids.

For example, MISO is undergoing reforms[3] to better incorporate Demand Responsive Resources (DRRs) into the price formation (in both day-ahead and real-time markets). The above mentioned FERC Order 2222 promotes participation of the demand side, in particular distributed energy resources, and storage in wholesale electricity markets. There is significant potential for industrial demand flexibility, but industry will only invest in flexibility options if it comes with lower electricity prices (EU 2016). Therefore, the increase of price-sensitive bids in wholesale electricity markets is to be expected in the future.

A number of proposals have been made for the demand side to express flexibility (Liu et al. 2015, Ottesen and Tomasgard 2015, Ottesen et al. 2016). Flexibility extensions of a bid language on the demand side can include shiftable volumes (asking to meet a certain volume within a certain time frame), shiftable profiles (allowing to shift a pre-determined demand profile over time), or adjustable demand (involving extensible or curtailable demand). Such flexibility options in the bid language would be a powerful way to address the intermittent nature of VER, but they lead to substantial non-convexities due to additional integer variables in the allocation problem. For example, thermal power plants have ramping constraints that make the production available in one period dependent on the production in the preceding and following periods. The introduction of renewable energy sources leads to an increased use of the of thermal units, and ramping constraints are expected to be binding more frequently (Herrero et al. 2020). Ignoring such constraints in the day-ahead schedule can significantly degrade the efficiency of the dispatch. Thus, one cannot expect the non-convexities on electricity markets to vanish, especially in a future with large proportions of renewable energy sources. Such demand flexibility and price-sensitive demand have ample consequences on the properties of prices that we can compute, as we will show.

3. Competitive Equilibrium

In this section, we introduce necessary notation, summarize existing theory on pricing in non-convex markets, and discuss design desiderata for electricity markets.

3.1. Notation and Economic Environment

In the auction market, there are K types of items (goods; hours and locations in a day-ahead market), denoted by $k \in \mathcal{K} = \{1, \ldots, K\}$, buyers $i \in \mathcal{I} = \{1, \ldots, I\}$ and sellers $j \in \mathcal{J} = \{1, \ldots, J\}$. In the multi-unit case, we have multiple *homogeneous* units (e.g., the minimum bid increment) for each of the *heterogeneous* K items $k \in \mathcal{K}$. A bundle of interest to buyer i (seller j) is described by a vector $x_i \in \mathcal{X}$ ($y_j \in \mathcal{Y}$) where $\mathcal{X}$ ($\mathcal{Y}$) is a compact subset of $\mathbb{Z}^K_{\geq 0}$. Each buyer i (seller j) has a monotonously increasing (decreasing) value function v_i: $\mathcal{X} \to \mathbb{R}_{\geq 0}$ (v_j: $\mathcal{Y} \to \mathbb{R}_{\geq 0}$) over bundles of items or objects x_i (y_j).

An auctioneer wants to find an allocation of items to bidders. The auctioneer aims for *allocative efficiency*. This means the auctioneer wants to maximize *social welfare* which is the gains from trade for

[3] https://www.misoenergy.org/stakeholder-engagement/issue-tracking/update-to-demand-response-deployment-tools/

all participants (the buyers and sellers). The goal of the auctioneer is to find an efficient allocation $(\mathbf{x}, \mathbf{y}) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ and linear and anonymous market clearing prices $\lambda = \{\lambda(k)\}_{k \in \mathcal{K}} \in \mathbb{R}^K_{\geq 0}$. The *linearity of prices* refers to the property that individual prices are set for each item $k \in \mathcal{K}$; the price for a bundle x_i is then simply the sum of the prices of its components, i.e., it is given by the dot product $\lambda' x_i$. *Anonymity* means that the resulting prices λ are the same for all bidders and there is no price differentiation. Competitive equilibrium prices might also be non-linear and personalized, but linearity and anonymity are crucial on electricity and other real-world markets as we discussed earlier. We assume, buyer i's (direct) utility from bundle x_i is given by $\pi_i(x_i, \lambda) = v_i(x_i) - \lambda' x_i$, and seller j's utility from bundle y_j is given by $\pi_j(y_j, \lambda) = \lambda' y_j - v_j(y_j)$. Such utility functions are linear in price and referred to as quasilinear utility functions. All market participants are assumed to be price-takers, meaning that they cannot influence the market prices on their own. The social welfare can now be defined as $\sum_{i \in \mathcal{I}} v_i(x_i) - \sum_{j \in \mathcal{J}} v_j(y_j)$, as prices cancel when the utilities of market participants are added.

With linear and anonymous prices $\lambda = (\lambda(1), \ldots, \lambda(k), \ldots, \lambda(K))$, the *indirect utility function* is defined as

$$u_i(\lambda) = \max_{x \in \mathcal{X}} \{v_i(x) - \lambda' x\} \quad \text{and} \quad u_j(\lambda) = \max_{y \in \mathcal{Y}} \{\lambda' y - v_j(y)\}.$$

The indirect utility function is widely used in economics and returns the maximal utility that bidder i can obtain at prices λ. The *demand correspondence* $D_i(\lambda)$ and $D_j(\lambda)$, resp. describe the set of bundles that maximize the indirect utility function at prices λ, i.e.,

$$D_i(\lambda) = \arg\max_{x \in \mathcal{X}} \{v_i(x) - \lambda' x\} \quad \text{and} \quad D_j(\lambda) = \arg\max_{y \in \mathcal{Y}} \{\lambda' y - v_j(y)\}.$$

3.2. Competitive Equilibrium

If in an outcome (consisting of an allocation and prices) all bidders are allocated a bundle from their demand correspondence, then the outcome is *envy-free* (EV). No bidder would want to get another bundle, as a bidder cannot increase her utility at these prices. If we have EV and the market is *budget-balanced* (BB), we have a *competitive equilibrium* (CE). If competitive equilibrium prices are linear and anonymous (LA), we also refer to this as a Walrasian equilibrium.

Definition 1 (Walrasian (competitive) equilibrium, (WE)). A price vector λ^* and a feasible allocation $(\mathbf{x}, \mathbf{y})$ form a *Walrasian equilibrium* if $\sum_{i \in \mathcal{I}} x_i = \sum_{j \in \mathcal{J}} y_j$, $x_i \in D_i(\lambda^*)$ for every buyer $i \in \mathcal{I}$, $y_j \in D_j(\lambda^*)$ for every seller $j \in \mathcal{J}$, and budget is balanced with $\sum_{i \in \mathcal{I}} \lambda^{*'} x_i = \sum_{j \in \mathcal{J}} \lambda^{*'} y_j$.

The BB condition implies that an unallocated item has a price of zero. Note that getting a bundle from the demand correspondence implies individual rationality (IR), because if bidders would make a loss with a bundle it would never be in their demand correspondence. However, EV is a much stronger condition than IR. In summary, a Walrasian equilibrium (WE) has the properties $BB \wedge EV \wedge LA$. Later we will distinguish between linear and anonymous (LA) prices and linear and anonymous payments (LAP). For now, we assume that prices coincide with the payments.

The question is now under which conditions Walrasian equilibria exist and whether they support efficient (welfare-maximizing) outcomes (EF). To study these questions in a market with quasilinear utilities and independent private values, we use the following mathematical optimization problem describing a (combinatorial) exchange, which allows for arbitrary package bids. This bid language does not impose any restrictions on the types of valuations or cost functions and can be seen as the most general form of non-convex markets. As a matter of fact, the most prominent element of bid languages used in European day-ahead markets are block bids, i.e., package bids on adjacent time slots, and they can be easily captured in the following optimization problem. Electricity markets in the USA stipulate different bid languages to reduce the number of bids that participants need to submit, but they can be seen as a specific type of combinatorial exchange.

Let $\mathcal{X}_i \subseteq \mathbb{Z}^K_{\geq 0}$ denote all bundles for which buyer i submitted a bid, and $\mathcal{Y}_j \subseteq \mathbb{Z}^K_{\geq 0}$ denote all bundles for which seller j submitted an ask. For simplicity, we make the natural assumption that every bidder submits a bid with value 0 for the empty bundle. Let $z_i(x) \in \{0,1\}$ be a binary decision variable denoting whether buyer i wins bundle $x \in \mathcal{X}_i$, and $z_j(y) \in \{0,1\}$ be a binary decision variable denoting whether seller j wins bundle $y \in \mathcal{Y}_j$. The parameters $x(k)$ and $y(k)$ describe how many units a buyer wants or a seller provides of item k in a bundle. The allocation or *winner determination problem* (WDP) can then be written as an integer program as follows:

$$
\begin{aligned}
\max \quad & \sum_{i \in \mathcal{I}} \sum_{x \in \mathcal{X}_i} v_i(x) z_i(x) - \sum_{j \in \mathcal{J}} \sum_{y \in \mathcal{Y}_j} v_j(y) z_j(y) && && \text{(WDP)} \\
\text{s.t.} \quad & && && \\
& \sum_{x \in \mathcal{X}_i} z_i(x) \leq 1 && \forall i \in \mathcal{I} && (\pi_i) \\
& \sum_{y \in \mathcal{Y}_j} z_j(y) \leq 1 && \forall j \in \mathcal{J} && (\pi_j) \\
& \sum_{i \in \mathcal{I}} \sum_{x \in \mathcal{X}_i} x(k)\, z_i(x) \leq \sum_{j \in \mathcal{J}} \sum_{y \in \mathcal{Y}_j} y(k) z_j(y) && \forall k \in \mathcal{K} && (\lambda(k)) \\
& z_i(x) \in \{0,1\} && \forall i \in \mathcal{I}, \forall x \in \mathcal{X}_i && \\
& z_j(y) \in \{0,1\} && \forall j \in \mathcal{J}, \forall y \in \mathcal{Y}_j &&
\end{aligned}
$$

The WDP determines an allocation of bundles maximizing gains from trade, i.e., an efficient outcome. It assumes that participants specify a package bid for each possible package of interest, but they can only win at most one. This is also referred to as an XOR bid language. While such a bid language is fully expressive, it requires exponentially many bids and is impractical for most applications. This is why electricity markets specify compact bid languages assuming some knowledge of the cost functions of generators. Bikhchandani and Mamer (1997) describe a multi-item, single-unit market. Their central theorem shows that there exist clearing prices for the indivisible single-unit problem if and only if the LP relaxation of WDP has an integer solution. In this case, the dual variables $\lambda(k)$ constitute Walrasian equilibrium (WE) prices, and the dual variables π_i and π_j determine the surplus of buyer i and seller j, respectively. The result can be proven via the strong duality theorem and the complementary slackness conditions in linear programming. As was already noted by Bikhchandani and Mamer (1997), the result for multi-item, *multi*-unit markets also directly follows from their result, by considering each of the multiple units as separate items. As a result, the welfare theorems hold in the quasilinear model:

Theorem 1 **(First and second welfare theorem)**. *Let* $(\mathbf{x}, \mathbf{y})$ *be an equilibrium allocation induced by a Walrasian equilibrium price vector* λ*, then* $(\mathbf{x}, \mathbf{y})$ *yields the optimal social welfare. Conversely, if* $(\mathbf{x}, \mathbf{y})$ *is a Pareto efficient allocation, then it can be supported by a Walrasian price vector* λ *so that* $(\lambda, \mathbf{x}, \mathbf{y})$ *forms a Walrasian equilibrium.*

Unfortunately, the LP relaxation of WDP does not yield integer solutions in general, and thus we cannot expect WE to exist in general. In fact, it is well-known that WE only exist for restricted types of valuations for which the LP relaxation actually yields a feasible integer solution. For example, if all bidders' valuations are strong substitutes, this is a sufficient condition for WE to exist (Bikhchandani and Mamer 1997, Leme 2017, Baldwin and Klemperer 2019, Bichler et al. 2020). In practice, these conditions are rarely satisfied. In particular, non-convex cost functions on electricity markets lead to non-convex allocation problems that do not satisfy conditions for WE.

Competitive equilibrium prices do not need to be linear and anonymous. Bikhchandani and Ostroy (2002) show that for combinatorial auctions with arbitrary valuations, competitive equilibrium prices need to be personalized and non-linear. Such prices convey little information other than a particular package was winning or losing. In fact, for combinatorial exchanges with multiple buyers and sellers, there can even be situations where no competitive equilibrium exists (Bichler and Waldherr 2017). As discussed earlier, linear and anonymous prices on day-ahead electricity markets are an important baseline for forward markets and they serve as investment signals. Therefore, we need to relax some of the design desiderata of Walrasian equilibria.

3.3. Penalty-Based Stability

We discussed that prices should be linear and anonymous (LA), they should support the efficient allocation (EF), and neither the participants (IR) nor the auctioneer (BB) should make a loss. If envy-freeness (EV) was additionally satisfied, prices would support a Walrasian equilibrium. The welfare theorems (Theorem

1) suggest that all of these axioms are satisfied in convex markets. With general preferences in non-convex markets, however, this is impossible to achieve (Bikhchandani and Ostroy 2002, Bichler and Waldherr 2017). Market operators might not want to relax EF and IR as welfare should be maximized and no participants should incur losses from submitting bids. Current pricing schemes such as IP pricing and ELMP sacrifice BB and EV, but the side-payments that arise from the violation of BB have led to controversy as outlined in the introduction.

EV describes price-based stability where at the prices no participant would want to deviate. In highly regulated and transparent markets such as electricity markets, stability can also be enforced without prices. As a matter of fact, U.S. ISOs such as ERCOT, MISO, NYISO, or CAISO enforce stability of the outcome via penalties in case a generator deviates from the efficient dispatch (O'Neill et al. 2020). As compared to WE, they relax the EV condition and only ask for IR. In what follows, we will show that with price-inelastic demand and a strict demand-supply equivalence, we can always find prices that satisfy $IR \wedge BB \wedge LA \wedge EF$. While we focus on an electricity market example, the insights are relevant to all types of non-convex markets. Let us introduce a simplified example of a single hour traded on an electricity market to illustrate which properties we can hope to achieve with linear and anonymous prices. From now on, we require strict demand-supply equivalence.

Example 1. Suppose we have three generators G1, G2, and G3 (the sellers on electricity markets). G1 produces 10 MWh and asks for \$500 (\$50/MWh). G2 produces 20 MWh and asks for \$300 (\$15/MWh). Finally, G3 produces 30 MWh and asks for \$700 (\$23.3/MWh). A buyer needs exactly 30 MWh in this hour and can either purchase from G1 and G2 or from G3, where buying from G3 is the efficient dispatch. Bids are indivisible. There are several options for the ISO:

1. The ISO could select the efficient dispatch, but set the price just below \$15/MWh, the ask of G2. The efficient dispatch with G3 is selected, but G3 makes a loss. In order to achieve IR, the market maker can pay G3 $\$700 - 30\text{MWh} * \$15/\text{MWh} = \$250$ as a *make whole payment*. These side-payments are commonly used in U.S. electricity markets, but they violate BB.[4]
2. The ISO could select the efficient dispatch and set the price at the ask of G3, i.e., \$23.3/MWh. At this price it would be attractive for G2 to produce, and her ask is "paradoxically rejected." It is common on U.S. electricity markets to define a penalty for G2 in case she does. This penalty would be at the difference of her ask and the market price. In our example, this penalty for G2 would be $20\text{MWh} * \$23.3/\text{MWh} - \$300 = \$166.67$. The market satisfies EF, IR, and BB, but not EV, as G2 does not maximize her payoff at the prices. As such, it is efficient but not a WE.

[4] Much of the literature on pricing in electricity markets and their current implementations suggests that the results are a CE. As introduced earlier, a CE requires envy-freeness and budget balance, but budget balance is not satisfied here.

3. The ISO could pick the inefficient dispatch with generators G1 and G2 and set the price at \$50/MWh. No side-payments by an ISO are needed, but there is a welfare loss of \$100. This alternative is implemented on European day-ahead markets. G3 is paradoxically rejected.

If we use penalties to enforce stability of the outcome, we can define new design desiderata for pricing on non-convex markets.

Definition 2 (Penalty-based stable, budget-balanced, and efficient outcome (PBE)). A linear and anonymous price vector λ^* and an efficient allocation $(\mathbf{x}, \mathbf{y})$ form a penalty-based stable and efficient outcome if $\pi_i(\lambda) \geq 0$, $\pi_j(\lambda) \geq 0$ for every buyer $i \in \mathcal{I}$ and every seller $j \in \mathcal{J}$, if the market is budget balanced with $\sum_{i \in \mathcal{I}} \lambda^{*'} x_i = \sum_{j \in \mathcal{J}} \lambda^{*'} y_j$.

Note that budget balance and linear and anonymous prices in non-convex markets imply a strict demand-supply equivalence. If buyers and sellers have the very same anonymous and linear price vector (λ) and buyers buy less than what the sellers sell, then make-whole payments are required and BB is violated. To see this, assume that a seller sells a package of 2 MWh and a buyer is interested in only 1 MWh. We have a non-convexity arising from the indivisible package bid of the seller which does not allow us to price one of the two MWh in the seller's package at zero. Even if the buyer has a higher value for 1 MWh than what the seller asks for the package, we cannot achieve budget balance with a single price λ. Thus, the auctioneer needs to compensate the seller for the second MWh. But even if we have strict demand-supply equivalence, a PBE might not be possible as the following example shows.

Example 2. Suppose there are generators G1 and G2 both asking for \$30 for 3 MWh. Buyer B1 wants to buy 4 MWh for \$20 in total, and buyer B2 is price-inelastic with a demand of 2 MWh. With an ask price of \$10/MWh, the two generators ask for \$60 in total. However, as the market price cannot be higher than \$5/MWh, which is what B1 is willing to pay, the buyers will only pay \$30 for the 6 MWh in total. The ISO would need to pay a total of \$30 of make-whole payments to the two generators to facilitate the efficient trade at a price of \$5/MWh. The ISO could also set a different market price, but at any price it is inevitable to compensate the losses of some of the market participants.

The efficient trade would only be possible if the bids of the demand-side are all higher than the average ask price or all buyers are price-inelastic. As indicated, the latter is the standard assumption in the literature on electricity market design.

Definition 3. Buyer $i \in \mathcal{I}$ is *price-inelastic* if for any bundle $x \in \mathcal{X}$, $v_i(x) - \lambda' x \geq 0$ for all $\lambda \in \mathbb{R}^K_{\geq 0}$. Such a condition implies that for any price vector λ, $\pi_i(\lambda) \geq 0$.

Proposition 1. *A combinatorial exchange can implement a PBE, if the demand is price-inelastic and demand equals supply.*

Proof: We assume that we can solve the WDP to optimality, providing an efficient allocation (EF), $(\mathbf{x},\mathbf{y}) = ((x_i)_{i\in\mathcal{I}}, (y_j)_{j\in\mathcal{J}})$ such that $\sum_{i\in\mathcal{I}} x_i = \sum_{j\in\mathcal{J}} y_j$. Furthermore, if we assume that all the buyers are price-inelastic (thus for any price λ, $\pi_i(\lambda) \geq 0$ for all buyers i), we can choose a linear and anonymous price vector $\lambda^* = (\lambda^*(1), \ldots, \lambda^*(K))$ large enough such that $\pi_j(\lambda^*) \geq 0$ for all $j \in \mathcal{J}$. For example, one can set $\lambda^*(k)$ as the highest average cost for item k, such that IR is satisfied for all generators. Finally the condition $\sum_{i\in\mathcal{I}} \lambda^{*'} x_i = \sum_{j\in\mathcal{J}} \lambda^{*'} y_j$ gives us budget balance (BB). As a result, this combinatorial exchange can implement a *PBE*. Q.E.D.

With price-inelastic demand and strict demand-supply equivalence, we can increase the linear and anonymous price until we obtain IR for the generators. The same would hold true if some buyers are price-sensitive but all their bids are higher than the average cost of the sellers. Since these conditions are rarely met on electricity markets, it is common to deviate from budget balance (BB) by providing make-whole payments that ensure individual rationality of the generators. We want these make-whole payments to be minimal, because such personalized payments are not reflected in the public market prices. Let us now define a penalty-based stable and efficient outcome (PE):

Definition 4 (Penalty-based stable and efficient outcome (PE)). A linear and anonymous market price vector λ^*, personalized make-whole payments δ_i, δ_j, and an efficient allocation $(\mathbf{x},\mathbf{y})$ form a penalty-based stable and efficient outcome if $\pi_i(\lambda^*) + \delta_i \geq 0$, $\pi_j(\lambda^*) + \delta_j \geq 0$ for every buyer $i \in \mathcal{I}$ and every seller $j \in \mathcal{J}$. For PE prices, we demand the total of the personalized make-whole payments to be minimal.

Here the make-whole payments compensate (aggregate) losses that result from the allocated bundle. As we will see, there can be different notions of make-whole payments on electricity markets, such as compensating item-level losses. Next, we will introduce optimization problems to compute PBE whenever it exists or PE otherwise.

4. Pricing Rules

For price computation, we want prices to be linear and anonymous and we enforce efficiency, while other design goals can be relaxed. We treat BB as first-order design goal and price-based stability as second-order goal. This means, we first aim for linear and anonymous prices eliminating or minimizing the make-whole payments that the ISO needs to pay such that prices better reflect the value of electricity as compared to pricing schemes where the price signal is significantly distorted due to large private and personalized make-whole payments.

PBE (and also PE) prices are not unique. Therefore, we select those prices that minimize incentives to deviate. Unfortunately, these are computationally intractable problems if we want to compute them exactly, as we will show. In lieu thereof, we choose the price vector that is closest to the dual variables of the LP relaxation of the allocation problem in a second step. If the allocation problem was a convex optimization problem, such dual prices would constitute a competitive equilibrium, i.e., a stable solution that satisfies EV.

Before we get to price computation, let us introduce an abstract version of the central allocation problem on electricity markets. Then, we introduce optimization models to compute prices on markets with price-inelastic and such with price-sensitive demand to compute PBE or PE prices, resp. Finally, we compare these pricing rules with other approaches in the literature.

4.1. Allocation Problem

In the last section, we have discussed combinatorial exchanges with package bidding as they do not restrict the types of preferences that a participant might have. Combinatorial exchanges with package bids are impractical for electricity markets because they would require bidders to submit an exponential set of bids. Rather, electricity markets use compact bid languages (Goetzendorff et al. 2015) that only require generators to specify a small number of parameters describing their underlying cost functions and technical constraints as well as buyers to specify their bid curves.

Unit commitment (UC) problems represent our starting point. Operational constraints on thermal generation units such as ramping limits and minimum up/down times require those units to be committed in advance of when they are needed, typically via day-ahead unit commitment. Unit commitment models determine the optimal scheduling of a given set of power suppliers in order to meet electricity demand. Such models minimize total system costs subject to market clearing conditions (supply meets demand) and technical power plant constraints (Stott et al. 2009). Unit commitment models are generation scheduling models, determining the output of each generator. We use the term security constrained unit commitment model (SCUC) if it additionally includes network characteristics and constraints (van den Bergh et al. 2014).

The SCUC problem can be formulated as a mixed-integer non-linear problem. The non-linearity comes from the fact that transmission lines are typically high-voltage alternating current (AC). An AC optimal power-flow model (ACOPF) provides a non-linear system which describes the energy flow through each transmission line accurately, and is theoretically the best approach to solve the SCUC (Carpentier 1985). The ACOPF is non-linear, non-convex and an NP-hard mixed-integer optimization problem (Zohrizadeh et al. 2020). Although there are various approaches to global optimization, an exact solution to the ACOPF can be considered intractable for realistic networks (Watson et al. 2015). This led to significant research into convex relaxations of the problem (Zohrizadeh et al. 2020). The linear relaxation is also referred to as the direct current (DC) optimal power-flow model (DCOPF), and versions of this are widely used among U.S. ISOs to compute the efficient dispatch and prices (Eldridge et al. 2017).

In our paper, we focus on pricing and thus assume a generic DCOPF model that is modeled as a mixed-integer linear program (MIP). Appendix A provides an overview of the notation. In the abstract formulation, buyers and generators / sellers are again denoted by the sets $\mathcal{I}$ and $\mathcal{J}$, respectively. The set of traded goods $\mathcal{K}$ can now be described as the Cartesian product $\mathcal{N} \times \mathcal{T}$, where $\mathcal{N}$ represents a set of network nodes and $\mathcal{T}$ a set of time periods. Nodes are connected through a set of transmission lines $\mathcal{L}$. The objective of DCOPF aims at maximizing welfare, taking into account buyers' valuations (v) and generators' variable and fixed costs (c and h, respectively). Both buyers (2) and generators (1) can specify constraint matrices A, G and Q, R, respectively, in order to communicate their preferences and feasible bundles. DC power flows are determined in (3) (with P as inverse matrix of the power transfer distribution factors and W and Z as mappings of buyers and sellers to their respective nodes), with a requirement of aggregate balance (4) and a consideration of line flow limits (5). The decision variables include buying (x) and selling (y) quantities, the associated binary variables (d and u), as well as power flows (f). As indicated by the integer multipliers r and s, the binary variables d and u can account for several categories such as start-up and commitment variables for generators.

$$\max_{x,y,u,d,f} \quad v'x - c'y - h'u \tag{DCOPF}$$

s.t.

$$Ay + Gu \geq b \tag{1}$$

$$Qx + Rd \leq e \tag{2}$$

$$Pf = Wy - Zx \tag{3}$$

$$\overline{W}y - \overline{Z}x = 0 \tag{4}$$

$$\underline{F} \leq f \leq \overline{F} \tag{5}$$

$$x \geq 0 \tag{6}$$

$$y \geq 0 \tag{7}$$

$$u \in \{0,1\}^{sJT} \tag{8}$$

$$d \in \{0,1\}^{rIT} \tag{9}$$

$$f \in \mathbb{R}^{LT} \tag{10}$$

For convenience, we define vector x_i to include only the buying quantities of buyer $i \in \mathcal{I}$ as non-zero components, i.e., $\sum_{i \in \mathcal{I}} x_i = x$. Similarly, we define the vectors d_i, y_j, and u_j for buyers $i \in \mathcal{I}$ and generators $j \in \mathcal{J}$, respectively. The utility of buyer i is then defined as $\pi_i(x_i, \lambda) = v'x_i - \lambda' Z x_i$ with λ being the market price vector. The utility of generator j is $\pi_j(y_j, \lambda) = \lambda' W y_j - c'y_j - h'u_j$. Similarly, x_t and y_t are the vectors containing the buying and selling quantities of all buyers and generators resp. in period $t \in \mathcal{T}$ as non-zero components. The vectors x_{it} and y_{jt} consequently only include one non-zero component, namely the particular quantity of buyer i and generator j resp. in period t.

The DCOPF model does not allow for non-linear costs or non-linear AC power flows. We also abstract from transmission network elements such as transformers, shunts, or auxiliary services. However, the

DCOPF formulation provides the overall structure of a MIP used for unit commitment problems, that allows us to perform a meaningful analysis of different pricing rules in our experiments in Section 5.

4.2. PBE Pricing with Price-Inelastic Demand

We first focus on the case of price-inelastic demand. This complies with the traditional notion of electricity as a basic and indispensable necessity. If demand x has no attached valuations, $v'x$ can be removed from the objective function of DCOPF, and the generators' cost shall be minimized. Demand flexibility can be taken into account (by constraint 2), as long as buyers are price-inelastic. Let x^*, u^*, y^*, d^*, and f^* denote the optimal solution to this modified problem, which is efficient with demand-supply equivalence. As the buyers are price-inelastic, there will always be a price profile $\lambda^{PBE} \in \mathbb{R}^{NT}_{\geq 0}$ over locations and time periods such that no generator incurs losses (see Proposition 1).

The following bilevel integer program PBE-P computes prices such that at the efficient dispatch no generator makes a loss at any time (first constraint), and that there are no negative congestion revenues (second constraint). The latter prevents that nodal prices are set low at demand-intensive nodes and high at supply-intensive nodes, implying missing money only due to nodal price discrepancies. In the first constraint, individual rationality is based on *hourly* losses incurred by the respective market participant. Even if a loss in a certain hour is offset by a higher gain in the subsequent hour, the loss is compensated by a make-whole payment. The third constraint makes sure that incentives for generators to deviate from the efficient solution are minimal. π_j describes the payoff that a generator $j \in \mathcal{J}$ would have at the prices λ, if she could choose her dispatch such that it maximizes her payoff. The latter is computed in the lower level optimization (fourth constraint). This model would lead to prices that are IR and BB and it would minimize the gains by deviating from the efficient solution by an individual. In a Walrasian equilibrium of a convex economy, also coalitions of market participates cannot deviate. We do not consider such blocking coalitions in this model.

$$
\begin{aligned}
\min_{\lambda,\pi,\gamma} \quad & \sum_{j \in \mathcal{J}} \gamma_j && \text{(PBE-P)} \\
\text{s.t.} \quad & && \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 && \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \geq 0 && \\
& \pi_j - (\lambda' W y^*_j - c' y^*_j - h' u^*_j) \leq \gamma_j && \forall j \in \mathcal{J} \\
& \pi_j = \max_{y,u} (\lambda' W y_j - c' y_j - h' u_j) \quad \text{s.t. } (1), (7), (8) && \forall j \in \mathcal{J} \\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \pi \in \mathbb{R}^{J}_{\geq 0}, \gamma \in \mathbb{R}^{J} &&
\end{aligned}
$$

Solving bilevel mixed integer programming problems is Σ_2^p-hard (Jeroslow 1985), a complexity class that is clearly intractable. While this is no proof that the specific problem PBE-P is in this complexity class, realistic problem sizes of PBE-P are very large and the problems need to be solved in due time. For example, the time to compute allocation and pricing in European day-ahead markets is only 17 minutes (NEMO Committee 2021).[5]

Given the associated practical complexity of solving PBE-P in the required time frames, we suggest an alternative based on Extended Locational Marginal Pricing (ELMP), which will be described in Section 4.4.2 in greater detail. In essence, ELMP is a tractable heuristic where binary variables of the DCOPF (constraints 8 and 9) are relaxed to continuous variables and prices are retrieved from the duals of the nodal demand-supply constraints. ELMP aims at minimizing lost opportunity costs, and if a market is convex it actually does. Therefore, instead of PBE-P we solve PBE-A. The latter omits the lower-level optimization and instead minimizes the difference between the price vector that satisfies the individual rationality constraints (λ) and the ELMP prices (λ^{ELMP}). Note that computing ELMP is a linear program, and therefore λ^{ELMP} can be computed effectively with state-of-the-art linear programming solvers (see numerical results in Section 5.2).

$$
\begin{aligned}
\min_{\lambda} \quad & \|\lambda - \lambda^{ELMP}\|_1 && \text{(PBE-A)} \\
\text{s.t.} \quad & \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \geq 0 \\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}
\end{aligned}
$$

Here we use $\|\lambda - \lambda^{ELMP}\|_1$ in an attempt to minimize incentives to deviate from the efficient dispatch. With an L_1 norm in the objective, PBE-A can also be modeled as a linear program which can be solved in polynomial time. One can also minimize the squared Euclidean norm, which makes this a quadratic problem which might lead to less variation in the components of the price vector. Wolfe's combinatorial algorithm is widely used to solve such problems. Even though this algorithm does not run in polynomial time in the worst case (De Loera et al. 2020), it is very effective in practice and can serve as an alternative.

4.3. PE Pricing with Price-Sensitive Demand

We now assume price-sensitive demand, i.e., some or all of the buyers submit valuations v, as represented by the DCOPF. As we have shown, a PBE does not always exist for DCOPF. We can sacrifice budget balance (BB) but still ensure EF and IR. As a result, market prices are still linear and anonymous (LA), but individual payments are not (no LAP). Let $x^*, y^*, u^*, d^* f^*$ be the optimal solution to DCOPF. We define the following problem to compute the minimal make-whole payments associated to a price vector λ.

$$
\min_{\lambda, \delta^{\mathcal{I}}, \delta^{\mathcal{J}}} \quad \|\delta^{\mathcal{I}}\|_1 + \|\delta^{\mathcal{J}}\|_1 \qquad \text{(PE-}\alpha\text{)}
$$

[5] Prior to 08/07/2021, the allowed computation time was only 12 minutes.

$$
\begin{aligned}
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \ge 0 && \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} + \delta^{\mathcal{J}}_{jt} \ge 0 && \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \ge 0 \\
& \lambda \in \mathbb{R}^{NT}_{\ge 0}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\ge 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\ge 0}
\end{aligned}
$$

The variables $\delta^{\mathcal{I}}$ and $\delta^{\mathcal{J}}$ represent the required make-whole payments to buyers $\mathcal{I}$ and generators $\mathcal{J}$. Note that we again consider *hourly* losses for the calculation of make-whole payments which slightly extends the requirements for make-whole payments compared to Definition 4 (which only asks for no aggregate losses over all hours). The optimal make-whole payments from PE-α are given as $\delta^{\mathcal{I}*}$ and $\delta^{\mathcal{J}*}$. Again, the resulting price vectors are not unique, and we could formulate a bilevel integer program aiming to satisfy individual rationality, and to minimize incentives to deviate to another dispatch at these prices.

$$
\begin{aligned}
\min_{\lambda,\pi,\gamma,\delta^{\mathcal{I}},\delta^{\mathcal{J}}} \quad & \sum_{i\in\mathcal{I}} \gamma_i + \sum_{j\in\mathcal{J}} \gamma_j && \text{(PE-P)} \\
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \ge 0 && \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} + \delta^{\mathcal{J}}_{jt} \ge 0 && \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \ge 0 \\
& \delta^{\mathcal{I}} = \delta^{\mathcal{I}*} \\
& \delta^{\mathcal{J}} = \delta^{\mathcal{J}*} \\
& \pi_i - (v'x^*_i - \lambda' Z x^*_i) \le \gamma_i && \forall i \in \mathcal{I} \\
& \pi_i = \max_{x,d} (v'x_i - \lambda' Z x_i) \quad \text{s.t. } (2), (6), (9) && \forall i \in \mathcal{I} \\
& \pi_j - (\lambda' W y^*_j - c' y^*_j - h' u^*_j) \le \gamma_j && \forall j \in \mathcal{J} \\
& \pi_j = \max_{y,u} (\lambda' W y_j - c' y_j - h' u_j) \quad \text{s.t. } (1), (7), (8) && \forall j \in \mathcal{J} \\
& \lambda \in \mathbb{R}^{NT}_{\ge 0}, \pi \in \mathbb{R}^{I+J}_{\ge 0}, \gamma \in \mathbb{R}^{I+J}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\ge 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\ge 0}
\end{aligned}
$$

Similar to our discussion on the case with price-inelastic demand, we replace the bilevel integer program by a tractable linear program that minimizes the distance to ELMP prices, subject to having the minimal make-whole payments given by PE-α.

$$
\begin{aligned}
\min_{\lambda,\delta^{\mathcal{I}},\delta^{\mathcal{J}}} \quad & \|\lambda - \lambda^{ELMP}\|_1 && \text{(PE-A)} \\
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \ge 0 && \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \ge 0 && \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \ge 0 \\
& \delta^{\mathcal{I}} = \delta^{\mathcal{I}*}
\end{aligned}
$$

$$\delta^{\mathcal{J}} = \delta^{\mathcal{J}*}$$
$$\lambda \in \mathbb{R}^{NT}_{\geq 0}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\geq 0}$$

4.4. Comparison to Existing Pricing Rules

There is a significant literature on pricing rules for electricity spot markets, and a detailed discussion of all proposals is beyond the scope of this paper. An excellent and up-to-date overview of pricing in electricity markets is provided by Liberopoulos and Andrianesis (2016). Note that the literature in their paper is entirely based on the assumption of price-inelastic demand.

In our discussion, we focus on Integer Programming (IP) pricing and Extended Locational Marginal Pricing (ELMP) since they are used by U.S. ISOs in practice. Furthermore, we consider Average Incremental Cost (AIC) pricing, a recent proposal that also addresses the problem of significant make-whole payments. As introduced earlier, relevant criteria are efficiency (EF), individual rationality (IR), budget balance (BB), linear and anonymous prices (LA), and linear and anonymous payments (LAP). Note that IP pricing, ELMP and AIC satisfy EF and IR, which are widely considered essential on electricity markets. With price inelastic demand, PBE-A provides a straightforward way to *guarantee BB and LAP*. In case of price-sensitive demand, PE-A is the only pricing rule that *minimizes make-whole payments* under linear and anonymous prices. Let us now provide a brief description of IP, ELMP, and AIC pricing.

4.4.1. IP Pricing IP pricing (O'Neill et al. 2005) was an early and widely adopted proposal for pricing on electricity markets. First, the efficient dispatch is computed via DCOPF. Then the integer variables are fixed to their optimal values resulting in a linear program. The duals of the nodal balance constraints eventually provide linear and anonymous market prices, while the duals associated to constraints with integer variables determine individual uplift payments. O'Neill et al. (2005) originally describe a problem with price-inelastic demand. IP pricing can, however, also be adapted to settings with price-sensitive demand (Madani et al. 2018). It was also extended to multi-period, multi-nodal markets in many U.S. ISOs, including CAISO, PJM, or SPP. In practice, the uplift payments are restricted to be non-negative make-whole payments. Thus, market participants can retain their profits, and only individual losses are compensated by make-whole payments to ensure individual rationality. Budget balance is violated due to the make-whole payments, and the prices do not constitute a competitive equilibrium.

4.4.2. ELMP Pricing ELMP relaxes binary variables of the DCOPF to continuous variables and takes the duals of the relaxed problem as market prices. MISO introduced ELMP in 2011, but similar approaches were implemented by ISO-NE (O'Neill et al. 2019). Similar to IP pricing, there are individual make-whole payments, and stability of the solution is enforced via penalties. Lost opportunity costs (LOCs) describe the forgone profit from the most profitable alternative level of electricity production at the prices. In total, the make-whole payments and the required penalties yield the LOCs of a generator. ELMP pricing represents an approximation of Convex Hull Pricing (CHP), as introduced by Gribik et al. (2007). CHP computes prices that indeed minimize LOCs, but it is computationally expensive and thus it has not been implemented in the field (Schiro et al. 2016). However, for simple problem formulations ELMP and CHP prices are equivalent (Hua and Baldick 2017). Evidence by MISO suggests that lost opportunity costs can be reduced by ELMP pricing compared to IP pricing, yet the general economic properties of ELMP remain unclear (Schiro et al. 2016).

4.4.3. AIC Pricing In a series of essays, O'Neill et al. (2019) challenge established pricing rules on electricity markets and criticize that the resulting make-whole payments lead to biased market prices. They suggest AIC pricing, which implements IP pricing as a first stage. In a second step, the AIC price computation relaxes the integer variables of generators that make a loss for the actual AIC pricing run and adjusts their objective function coefficients to reflect the average costs, i.e., it distributes the fixed costs of a generator over the quantity allocated to the generator. In a stylized market with only a single period this would eliminate the make-whole payments of the generators. In a market with multiple periods, O'Neill et al. (2020) suggests an iterative process comprising several pricing runs to achieve budget balance. The approach does not consider make-whole payments for the demand side, but proposes price differentiation among buyers via Ramsey-Boiteux-like pricing.

AIC pricing provides an innovative new approach to electricity market pricing. Similar to PBE-A or PE-A, the goal is to eliminate or minimize make-whole payments. But there are also differences. First, PE-A minimizes make-whole payments for both sides of the market in a single optimization. Second, unlike AIC pricing, PE-A does not involve price differentiation on the demand side, but sticks to linear and anonymous prices. Price-differentiation among buyers can be a very useful tool to deal with the non-convexities in a market. However, it is also challenging. First, personalized prices lead to some level of intransparency in the market compared to an anonymous linear price for all market participants. Again, not all information is contained in the public price signal. Second, there is a difference between differential and anonymous prices in terms of manipulability. Uniform multi-unit auctions and the Walrasian mechanism are known to be strategy-proof in the large (Azevedo and Budish 2019). This means, with many participants truth-telling is approximately optimal and the impact of a single participant on the price becomes negligible with many participants. This is no longer the case if the payments of a participant are personalized. A pay-as-bid pricing scheme is manipulable and bidders will not reveal their true preferences. The only exception is the Vickrey-Clarke-Groves payment rule which is the unique payment rule that is dominant-strategy incentive-compatible (Green and Laffont 1979).

In our experiments we show that the make-whole payments necessary to achieve linear and anonymous prices are negligible even with price-sensitive demand. We argue that if make-whole payments are so low, there is no need to restrict to discriminatory prices for each buyer or many anonymous but non-linear prices (say for different volumes of electricity demanded), because the market price includes “almost” all information about supply and demand.

For our experiments in Section 5, we will ignore price differentiation in AIC, but instead compute make-whole payments to allow for a comparison to other pricing rules. Besides, we only consider a single AIC pricing run, and not multiple iterations.

4.4.4. Alternative Proposals Various other pricing rules have been suggested in the past two decades. Some, such as Direct Minimum Uplift (DMU) pricing, refrain from linear and anonymous prices and are thus beyond the focus of this paper. Others, such as the Equilibrium-Constrained (EC) pricing framework by Azizan et al. (2020) are restricted to price-inelastic demand. Moreover, many rules have only been investigated under very specific assumption (e.g., Generalized Uplift pricing, Semi-Lagrangean pricing).

Toczyłowski and Zoltowska (2009) introduce the DMU approach, which postulates a bid-ask-spread between the market prices for buyers and generators. DMU pricing aims at finding a spread that allows for minimal side-payments and that compensates lost opportunity costs. The side-payments are designed as uniform per-unit payments for buyers and sellers. However, the ISO has to give up a single linear price vector. DMU pricing has been proposed for multi-period power flow problems with price-sensitive demand.

More recently, Azizan et al. (2020) proposed the EC pricing scheme that is applicable to general non-convex settings with price-inelastic demand. Dispatch and payments are determined simultaneously to achieve EF and IR, as well as to ensure no incentives to deviate, rendering penalties unnecessary. Consequently, the price and payment functions must be general enough and hence allow for non-linear and personalized components. One upside is the broad applicability of their pricing framework to established price and payment functions. Moreover, the authors provide a polynomial-time approximation algorithm for general non-convex cost functions. The authors do not account for price-sensitive demand. Therefore, their settings are restricted to those where a PBE is feasible. In contrast to PBE-A, equilibrium-constrained pricing gives up budget balance and linear and anonymous payment functions to ensure stability without further penalties. Similar to O’Neill et al. (2019), we instead argue for maintaining budget balance with linear and anonymous payments and treat lost opportunity costs as secondary objective. In regulated electricity markets penalties are an accepted means to achieve stability.

Generalized Uplift pricing, introduced by Motto and Galiana (2002) and Galiana et al. (2003), has been proposed for a single-period problem with price-inelastic demand and seeks to find minimum zero-sum uplift payments that ensure stability. Minimum Zero-Sum Uplift Pricing by Liberopoulos and Andrianesis (2016) seeks the minimum prices that ensure a PBE. In contrast to PBE-A, Minimum Zero-Sum Uplift pricing allows for uplift charges for profitable generators. Starting at marginal cost, it increases prices and redistributes the additional gains of profitable generators to the loss-making generators. It terminates as soon as individual rationality is ensured for every generator. The Semi-Lagrangean pricing scheme by Araoz and Jörnsten (2011) also achieves a PBE, but their formulation is restricted to price-inelastic demand. The Primal-Dual pricing rule by Ruiz et al. (2012) aims at uniform IR prices with price-inelastic demand by relaxing efficiency. Finally, O'Neill et al. (2016) introduce the Dual Pricing algorithm which starts with the dual of the IP pricing problem and adds restrictions to ensure individual rationality and budget balance. By employing Ramsey-Boiteux pricing, it results in personalized prices for buyers. In Section 5 we will focus only on IP pricing, ELMP and AIC pricing for the reasons mentioned above.

5. Numerical Experiments

In what follows, we compare the different pricing rules experimentally. We start with small illustrative examples, before we report aggregate results for the IEEE RTS System, which is frequently used as a benchmark.

5.1. Illustrative Examples

In our illustrative examples we go from simple to more complex environments. We start with a simple convex setting, consisting of two generators G1 and G2 and two buyers B1 and B2 at a single node and over three time periods (i.e., hours). We will gradually extend this example to reflect non-convexities, as well as price-sensitive and flexible demand-side bids. We will benchmark IP and ELMP pricing as established rules used by ISOs and further include AIC pricing as a promising rule that has not yet been employed in practice. These existing rules are compared to PBE-A and PE-A, respectively.

5.1.1. Base case: convex supply, price-inelastic demand G1 offers up to 15 MW for \$5/MWh, and G2 offers up to 20 MW for \$3/MWh. B1 and B2 schedule price-inelastic demand according to the following table:

[MWh]	B1	B2
t=1	4	3
t=2	6	6
t=3	10	12

Table 1 Base Case: Price-inelastic Demand

	G1	G2
Max Load [MW]	15	20
Offer Price [$/MWh]	5	3

Table 2 Base Case: Convex Supply

The optimal solution is obviously to let G2 satisfy the entire demand in the first two periods, while G1 satisfies only the residual demand of 2 MWh in excess of the maximum load of G2 in the third period. IP, ELMP, AIC, and PBE-A prices are identically set and constitute a WE, PBE, and PE.

[$/MWh]	IP	ELMP	AIC	PBE-A
t=1	3.00	3.00	3.00	3.00
t=2	3.00	3.00	3.00	3.00
t=3	5.00	5.00	5.00	5.00
MWP	0.00	0.00	0.00	0.00

Table 3 Base Case: Prices

[MWh]	G1	G2	B1	B2
t=1	0	7	4	3
t=2	0	12	6	6
t=3	2	20	10	12

Table 4 Base Case: Dispatch

5.1.2. Non-convexities Next, we introduce non-convexities for the generators, i.e., G1 has a minimum load of 2 MW per period as well as no-load costs of $8 that occur as fixed costs when G1 is committed. G2 has a minimum load of 10 MW and no-load costs of $10. Therefore, G2 can no longer satisfy the demand in the first period and is replaced by G1. It is also assumed that G1 requires a minimum runtime of three periods. That is, if G1 is committed, it must sell at least its minimum load in every period. Consequently, the optimal dispatch now involves G1 satisfying the entire demand in $t = 1$ and running at a minimum load in the remaining periods, while G2 satisfies the residual demand.

[$/MWh]	IP	ELMP	AIC	PBE-A
t=1	5.00	3.50	6.14	6.14
t=2	3.00	3.50	3.00	9.00
t=3	5.00	6.10	9.00	9.00
MWP	38.00	40.29	22.00	0.00

Table 5 Non-convexities: Prices

[MWh]	G1	G2	B1	B2
t=1	7	0	4	3
t=2	2	10	6	6
t=3	2	20	10	12

Table 6 Non-convexities: Dispatch

Neither pricing rule yields a WE. IP, ELMP, and AIC result in individual losses, at least for some of the hours, and thus fail to produce a PBE. PBE-A avoids any make-whole payments and yields a PBE. Even if the aggregate profits were considered, AIC pricing cannot ensure individual rationality, at least after a single pricing run.

5.1.3. Price-sensitive demand We now introduce price-sensitive demand. We assume that half of the price-inelastic demand is retained as price-inelastic. For the remaining half, B1 bids $10/MWh and B2 bids $2/MWh in each period, respectively. The dispatch thus changes as it is not welfare-optimal to satisfy the entire demand. Due to the price-sensitive demand, we now use PE-A instead of PBE-A.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	6.45	6.45
t=2	2.00	3.50	4.00	4.17
t=3	3.00	3.50	3.71	8.58
MWP	64.00	50.75	26.57	17.00

Table 7 Price-Sensitive Demand: Prices

[MWh]	G1	G2	B1	B2
t=1	5.5	0	4	1.5
t=2	2	10	6	6
t=3	2	14	10	6

Table 8 Price-Sensitive Demand: Dispatch

Under the welfare-optimal dispatch, no PBE is possible. In order to satisfy the price-inelastic fraction of demand, both generators need to produce at least at their minimum loads, and make-whole payments thus become inevitable. PE-A achieves the lowest aggregate make-whole payments ($10.50 to G1, $6.50 to B2), which are as close to budget balance as possible.

5.1.4. Flexible demand We now additionally convert some of the inflexible demand into flexible demand. B1 has converted 2 MWh from $t = 1$ and 1 MWh from $t = 2$ into a shiftable volume of 3 MWh that can be satisfied in an arbitrary pattern over the considered time frame.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	7.29	7.29
t=2	2.00	3.50	4.00	9.00
t=3	3.00	3.50	3.71	9.00
MWP	64.00	44.75	22.57	7.00

Table 9 Flexible Demand I: Prices

[MWh]	G1	G2	B1	B2
t=1	3.5	0	2	1.5
t=2	2	10	8	4
t=3	2	14	10	6

Table 10 Flexible Demand I: Dispatch

The shiftable volume of B1 is completely served in $t = 2$ and replaces some of the price-sensitive demand of B2. This allows to reduce make-whole payments significantly for PE-A. Assume now that B1 adds an additional 1 MWh from $t = 2$ to the shiftable volume. Making use of this flexibility allows for PBE prices.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	7.29	7.29
t=2	5.00	3.50	6.14	6.14
t=3	3.00	3.50	3.56	9.00
MWP	38.00	43.75	10.89	0.00

Table 11 Flexible Demand II: Prices

[MWh]	G1	G2	B1	B2
t=1	3.5	0	2	1.5
t=2	7	0	4	3
t=3	2	18	14	6

Table 12 Flexible Demand II: Dispatch

This example also illustrates the advantages of demand-side bidding and bid languages that permit the expression of flexibility dimensions.

5.2. Experiments based on the IEEE RTS System

Finally, we report results of numerical experiments based on the IEEE RTS System introduced by Grigg et al. (1999), in order to better understand prices in a larger and realistic test system. This system has been used in a variety of contributions on electricity markets (Garcia-Bertrand et al. 2006, Morales et al. 2009, Zoltowska 2016, Hytowitz et al. 2020, Zocca and Zwart 2021) and includes non-convexities (no-load costs, minimum loads, minimum runtimes), price-sensitive demand, as well as several nodes and time periods. Therefore, it is well suited to study prices and make-whole payments under different pricing schemes.

Grigg et al. (1999) provide a stylized system topology, transmission network parameters, hourly (nodal) demand data as well as characteristics of generating units. In accordance with Zoltowska (2016), we select the single area, 24-node topology by Grigg et al. (1999) for a representative 24-hour winter day with 32 generators (total capacity: 6.81 GW) and 17 consumers (average hourly demand: 2.60 GWh). For data on (non-convex) generation costs or demand valuations we rely on the bid and offer curves provided by the cases studies of Garcia-Bertrand et al. (2006) and Zoltowska (2016) on this system. The experiments were conducted on an Intel(R) Core(TM) i7-8565U CPU with 16 GB RAM.

Our base setting includes 32 generators with minimum and maximum loads, minimum runtimes, as well as no-load costs and an offer curve representing variable costs. The demand of the 17 consumers is assumed to be price-inelastic at first, and later extended to price-sensitive and flexible demand. Generators and consumers are embedded in a DC power flow model with 24 nodes. Appendix C provides heatmaps of the hourly nodal prices and Table 13 reports statistics on prices, make-whole payments (MWP) as well as the magnitude of penalties necessary to avoid generators to deviate from the efficient dispatch. Note that instead of penalties an ISO could also just prohibit deviations from the efficient dispatch. In all scenarios we will see that the make-whole payments for PBE-A (in case of price-inelastic demand) or PE-A (in case of price-sensitive demand) are zero or very low compared to other pricing rules. Also, the make-whole payments for AIC prices are reduced compared to IP pricing, but remain significant after a single pricing run and with hourly loss compensation. Make-whole payments per generator can be found in Appendix B.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	22.32	8.81	35,749.28	0.00	0.00	0.00	4.63%	1.39s
ELMP	22.62	6.40	6,193.37	0.00	2,460.20	0.00	0.80%	1.39s
AIC	29.62	16.92	26,114.72	0.00	46,095.51	0.00	3.38%	2.68s
PBE-A	23.35	7.53	0.00	0.00	17,966.49	0.00	0.00%	1.46s

Table 13 IEEE RTS Statistics with Price-Inelastic Demand

Under price-inelastic demand, a PBE (Definition 2) is only achieved by PBE-A. All other pricing rules require make-whole payments to ensure individual rationality. Classical IP pricing requires high make-whole payments to the generators, resulting in a violation of budget balance for the market operator. AIC prices are high on average, especially in the peak periods $t = 18$ and $t = 19$ (see Figure 5 in Appendix C), contributing to a large standard deviation of the prices at the same time. The price peaks allow for overall profitability for the generators, but as discussed before, individual periodic losses are still compensated, resulting in make-whole payments during low-price periods. In contrast, ELMP produces a smooth price profile with little volatility and low lost opportunity costs (as reflected by the sum of make-whole payments and penalties). PBE-A adjusts this price profile only slightly in order to ensure a PBE, mainly by increasing prices at the nodes 101 and 115, where most of the otherwise unprofitable generators are situated. As a consequence, the price average and standard deviation are slightly increased, but no make-whole payments are required outside the market price. Penalties are necessary but are still lower than the lost opportunity costs required under IP or AIC pricing.

Next, we consider price-sensitive demand, taking into account the bid curves as described by Garcia-Bertrand et al. (2006). In particular, each buyer submits some minimum price-inelastic demand and a

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.63	5.03	14,272.57	0.33	0.00	0.42	2.52%	1.57s
ELMP	21.02	5.35	490.39	781.43	257.43	803.04	0.22%	1.55s
AIC	21.27	6.36	11,048.85	945.38	0.00	0.00	2.12%	2.96s
PE-A	20.85	5.23	0.00	112.43	929.52	804.66	0.02%	1.66s

Table 14 IEEE RTS Statistics with Price-Sensitive Demand

piecewise-constant demand curve on top of that. Accounting for buyer valuations naturally decreases prices compared to the price-inelastic case, which is also evident from Table 14 and Figure 6 in Appendix C.

It is not possible to achieve a PBE in this environment. IP prices produce the lowest average price and standard deviation. Similar to the price-inelastic case, it diverges most from budget balance, with make-whole payments amounting to 2.5% of the total incurred generation costs. ELMP prices are higher on average, resulting in less make-whole payments for the generators. However, these prices do not minimize total make-whole payments. PE-A requires make-whole payments of only $112.41. Only IP prices are on average lower than PE-A prices, and the total lost opportunity costs of PE-A (as reflected by the sum of make-whole payments and penalties) are minimal among the pricing rules under consideration. PE-A prices are minimal in make-whole payments, closest to stability, and imply low and smooth price profiles.

Finally, we introduce demand flexibility. The following Tables 15 and 16 reflect prices where 10% of the previously price-inelastic demand is converted to either shiftable profiles or shiftable volumes. Here, each shiftable demand is a randomly sampled 5-hour interval of inelastic demand that can either be shifted as a profile by 4 hours (shiftable profile) or the aggregate volume can be satisfied within the original 5 hours in an arbitrary fashion (shiftable volume).

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.63	5.03	14,275.82	9,116.75	0.00	0.00	4.14%	1.76s
ELMP	20.74	5.14	1,034.49	21,420.72	257.43	0.00	3.97%	1.71s
AIC	20.51	5.76	11,882.69	20,773.64	0.00	0.00	5.78%	3.27s
PE-A	20.42	4.92	188.54	322.96	929.52	538.11	0.09%	1.84s

Table 15 IEEE RTS Statistics with 20% Shiftable Profiles

In both cases, welfare gains can be realized by using the demand-side flexibility in a welfare-maximizing fashion. The increase in make-whole payments for the buyers is a result of the modeling decision to assign the highest valuation in the bid curve to the – formerly price-inelastic and now price-sensitive and flexible – demand. As the flexible demand needs to be satisfied within the boundaries set by the flexibility parameters, this can create a loss on part of the buyer if her highest valuation is still below the generation cost. This results in the significantly higher make-whole payments for buyers. Again PE-A has by far the lowest make-whole payments and little price volatility, etc.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.98	5.29	12,725.92	10,491.12	0.00	0.00	4.18%	1.83s
ELMP	21.04	5.36	436.60	22,686.77	257.43	0.00	4.16%	1.74s
AIC	21.66	6.65	10,888.60	35,141.68	0.00	0.00	8.29%	3.34s
PE-A	20.74	5.17	564.35	198.47	741.80	687.45	0.14%	1.84s

Table 16 IEEE RTS Statistics with 20% Shiftable Volumes

The numerical tests indicate that PBE-A and PE-A can substantially reduce or even eliminate make-whole payments compared to conventional pricing schemes. As a result, there are no or only very low side-payments that are not reflected in the public market price anymore. Approaching budget balance comes at the expense of higher penalties to ensure a stable market outcome. As discussed in the previous sections, we argue that penalties are less of a concern, since they are already established and enforced in highly regulated electricity markets (O'Neill et al. 2020).

6. Conclusions

Electricity markets have seen significant change among U.S. ISOs recently. While all ISOs moved to mixed-integer programming in order to determine the efficient dispatch, there is still a significant discussion about out-of-market make-whole payments paid by the ISOs to some of the generators. These payments can be significant and they distort the market price signals as has been pointed out by the U.S. FERC and domain experts. We show that with the standard assumption of price-inelastic demand and demand-supply equivalence no make-whole payments are necessary.

With the advent of variable energy sources, demand response becomes increasingly important. To adequately reflect flexibility on the demand side, ISOs need new bid formats that likely lead to additional non-convexities and price-sensitive demand. We prove that in such markets zero make-whole payments are impossible in general. Based on this insight, we introduce the PE-A pricing rule that minimizes make-whole payments, and compare it to existing payment rules used by ISOs and the AIC pricing rule. Rather than trying to mimic competitive equilibrium prices based on linear relaxations of the underlying non-convex allocation problem, we treat envy-freeness as second-order design goal and optimize these objectives directly. The results show that high side-payments on electricity markets as they are challenged by regulators can either be avoided or reduced substantially.

The experiments provide evidence that prices under PE-A do not increase on average compared to established pricing rules, and the changes in the overall payments of market participants are very small. Moreover, make-whole payments are avoided or they are negligible in all experiments that we ran. The new pricing rules are based on optimization problems that can be solved in polynomial time and whose principles are easy to understand and communicate. The new pricing rule is also general without dependencies on the specifics of the underlying allocation problem and can be applied to other non-convex markets as well.

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Appendix A: Notation for DCOPF

Sets

- $\mathcal{I} = \{1, ..., I\}$: Buyers (index i)
- $\mathcal{J} = \{1, ..., J\}$: Generators (index j)
- $\mathcal{T} = \{1, ..., T\}$: Time periods (index t)
- $\mathcal{N} = \{1, ..., N\}$: Nodes (index n)
- $\mathcal{L} = \{1, ..., L\}$: Lines (index l)

Parameters

- $v \in \mathbb{R}^{IT}$: Buyer valuations
- $c \in \mathbb{R}^{JT}$: Generator variable cost
- $h \in \mathbb{R}^{sJT}$: Generator fixed costs
- $A \in \mathbb{R}^{m \times JT}$: Generator constraint matrix I
- $G \in \mathbb{R}^{m \times sJT}$: Generator constraint matrix II
- $b \in \mathbb{R}^{m}$: Generator constraint right-hand side
- $Q \in \mathbb{R}^{k \times IT}$: Buyer constraint matrix I
- $R \in \mathbb{R}^{k \times rIT}$: Buyer constraint matrix II
- $e \in \mathbb{R}^{k}$: Buyer right-hand side
- $P \in \mathbb{R}^{NT \times LT}$: Inverse PTDF matrix (calculated from susceptance and network incidence matrix; also includes reference node)
- $W \in \mathbb{R}^{NT \times JT}$: Generator to node and period mapping matrix
- $Z \in \mathbb{R}^{NT \times IT}$: Buyer to node and period mapping matrix
- $\overline{W} \in \mathbb{R}^{T \times JT}$: Generator to period mapping matrix
- $\overline{Z} \in \mathbb{R}^{T \times IT}$: Buyer to period mapping matrix
- $\overline{F} \in \mathbb{R}^{LT}$: Upper flow limits
- $\underline{F} \in \mathbb{R}^{LT}$: Lower flow limits

Decision Variables

- $x \in \mathbb{R}^{IT}$: Buying quantities
- $d \in \{0, 1\}^{rIT}$: Buy-side binary variables (r as integer multiplier to account for several binaries [different dimensions of flexibility, etc.])
- $y \in \mathbb{R}^{JT}$: Selling quantities
- $u \in \{0, 1\}^{sJT}$: Generator commitment and other binaries (s as integer multiplier to account for several binaries [commitment, start-up, etc.])
- $f \in \mathbb{R}^{LT}$: Line flows

Appendix B: Make-Whole Payments

The following figures provide histograms of make-whole payments per generator (in $) in the different environments with price-inelastic, price-sensitive, and flexible demand (shiftable profiles and shiftable volumes) for the IEEE RTS system. With IP pricing, ELMP, and AIC some generators receive very high make-whole payments that are not reflected in the public prices. With PBE-A / PE-A such make-whole payments are negligible.

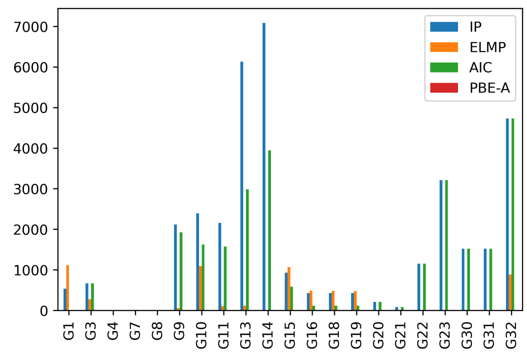

Figure 1 **Make-Whole Payments with Price-Inelastic Demand**

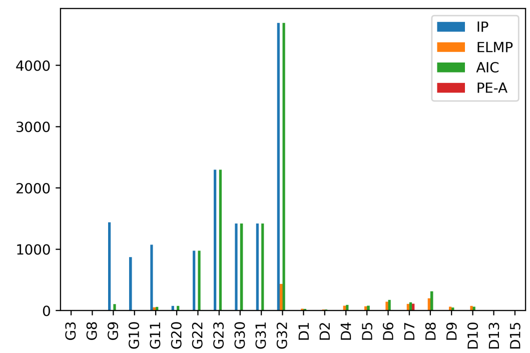

Figure 2 **Make-Whole Payments with Price-Sensitive Demand**

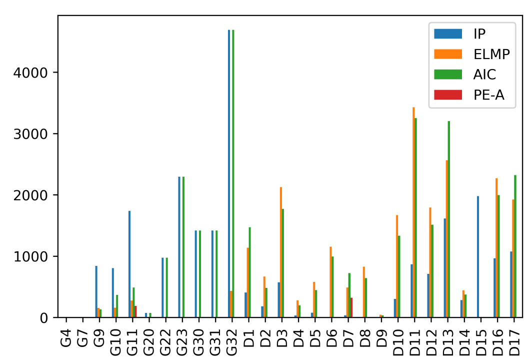

Figure 3 **Make-Whole Payments with 20% Shiftable Profiles**

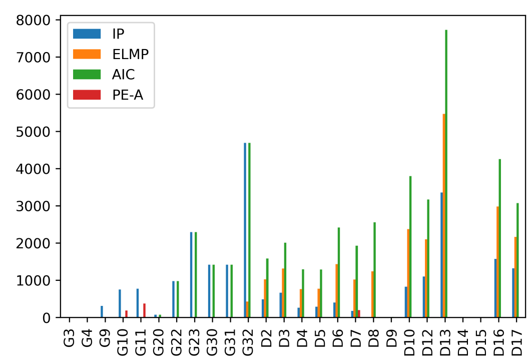

Figure 4 **Make-Whole Payments with 20% Shiftable Volumes**

Appendix C: Heatmaps of Prices

The following heatmaps describe hourly nodal prices (in $/MWh) for different nodes across the day for the IEEE RTS system. Darker colors describe higher prices. Each panel describes the outcome of one pricing rule (IP, ELMP, AIC, and PBE-A or PE-A) for the environments with price-inelastic demand, price-sensitive demand, 20% shiftable profiles or 20% shiftable volumes. In general, the prices with price-sensitive and flexible demand tend to be higher. Interestingly, the prices of PBE-A / PE-A and ELMP tend to be similar in spite of significantly lower make-whole payments with PBE-A / PE-A.