Trading Airport Time Slots: Market Design with Complex Constraints

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Abstract

Due to the lasting growth in air traffic, many international airports have reached their capacity limits. Access to major airports is granted through the assignment of airport time slots. Current practices of allocating these time slots via grandfathering are widely regarded as inefficient by experts. New market mechanisms need to take into account synergistic valuations of airlines for departure and arrival time slots, as well as financial constraints of the participating airlines for the many time slots available. Unfortunately, computing core-stable outcomes in such environments is Σ_2^p-hard. Such problems are typically considered intractable. We introduce bilevel integer optimization models for airport time slot trading and compute core-stable outcomes, i.e. allocations and prices such that no coalition can beneficially deviate. Interestingly, despite the computational hardness of the underlying problem numerical experiments show that instances of practically relevant size can be solved in due time. The proposed market design provides a solution that addresses the specific constraints of airport time slot markets, a precondition for adoption in the field.

1 Introduction

Air traffic has grown dramatically in the past decades and is still expected to rise in the future. In fact, the widely used commercial market outlook by Boeing for the next twenty years predicts passenger flow by air travel in Europe and North America to double and flow in China to more than triple (Boeing, 2019). This continuing increase in traffic has made airport capacity a very scarce resource at major airports. The lack of airport capacity is a constraint for the development of air traffic because building of new runways is strongly limited due to cost, land availability, or political reasons. Therefore, the efficient use of scarce airport resources is important. We will refer to the "efficient" or "welfare-maximizing" allocation as the one that maximizes the gains from trade, i.e. the difference between what the buyers are willing to bid and the sellers are asking for subject to allocation and budget constraints. This is a reasonable objective as it treats all market participants equally.

The design of efficient trading mechanisms is challenging, however. It requires the consideration of preferences for packages of time slots, but also allocation and budget constraints of airlines to achieve welfare-maximizing, stable outcomes for all participants, as we will discuss below. Importantly, the bidding language needs to be simple and allow bidders to express their preferences and constraints in a parsimonious way. Unfortunately, the combination of these desiderata leads to a computationally very hard market design problem. Let us first give an overview of the state-of-the-practice before we then introduce market design challenges and our contribution.

1.1 Current Practice

Almost all European airports and three major US airports use slot allocation systems based on the IATA (International Air Transport Association) guidelines (Ball et al., 2018). A slot at an airport is defined as a time interval available for the arrival or departure of a flight. Airports declare their capacity in number of available slots per hour, where the number of slots reflects the amount of air traffic an airport can handle in the specified time frame. Then, slots are allocated in two steps.

In the *primary allocation*, which takes place twice a year, slots for the upcoming summer or winter season are allocated from airports to airlines in Europe. Slots are first allocated according to grandfather rights. Airlines only lose these rights in case slots are underutilized. Half of the underutilized slots are distributed to new entrants. The remaining slots are allocated to interested airlines regardless of market position. Due to the limited number of slots, incumbent airlines want to keep these slots. An example of inefficiency in the allocation is the occasional practice of "babysitting" slots, in which airlines only use them for the minimum required amount in order to retain their grandfather rights Ball et al. (2018). Actually, some airlines organize ghost flights with empty seats just not to lose valuable airport time slots (Mirror, 2018). Grandfather rights make it hard for newcomers to enter the market and compete with major carriers. Currently only three airports in the U.S. (JFK, LGA, DCA) are slot constrained, but this number can grow as air traffic increases. Under the current system, the slots at slot-controlled airports are allocated by a central coordinator (or facilitator). In Europe, there is one coordinator per country, responsible for this country's slot allocation. Research on efficiently allocating slots without implementing market-based mechanisms has put varied emphasis on simultaneous allocation at several airports (Pellegrini et al., 2017), fast heuristics (Benlic, 2018), fairness (Fairbrother et al., 2020), or adherence to IATA regulations (Fairbrother et al., 2020), we refer the interested reader to the respective publication.

Secondary slot trading has been taking place at US airports since 1986 for logistical reasons. Airlines often swap slots or lease them out, because they are hard to re-acquire. The main difference between Europe and the United States is that monetary exchange is not permitted in Europe. In spite of European regulations, in 1999 the High Court of the United Kingdom authorized some transactions of slots involving monetary compensations and opened the way to a grey market operating at UK airports (Condorelli, 2007). However, participation in these secondary markets is low (De Wit et al., 2007). Currently, secondary trading only occurs through bilateral bargaining. Bilateral bargaining might enhance efficiency, but for a welfare-maximizing allocation all bids and asks and all constraints need to be considered.

Ball et al. (2018) discuss primary markets and argue that "it [is] clear that current slot allocation policies do not encourage allocating slots to their highest and best use, and in some respects actually discourage it." There are similar arguments for the use of market mechanisms for secondary trading (Dot Econ, 2001; Pellegrini et al., 2012). In fact, the secondary market is the only part of the allocation process where market-based instruments are permissible under current EU regulations (Odoni, 2020).

Despite widespread support and arguments in favor for market mechanisms, IATA has stated that it "would oppose any consideration of market-based primary slot allocation mechanisms" since there is no "no clear indications that such mechanisms improve the utilization of already-congested airport capacity or provide benefits to improving customer experience and choice in connectivity and fares" (IATA, 2016). Although one might or might not agree to this statement, it does not include secondary trading which in the most recent IATA fact sheet is considered "a better solution than primary auctions" as it "provides for some flexibility in the system and allows slots to be traded which may allow new entrants or new routes to more in-demand destinations" (IATA, 2020). While we focus on markets for secondary slot trading in this paper, we note that the mechanisms can also be adapted for primary slot allocation if desired.

1.2 Market Design Challenges

Although it is widely accepted that "market mechanisms would improve allocative efficiency" (Madas and Zografos, 2010), the design of such markets is considered very challenging. This constitutes a major barrier to the adoption of market-based solutions. Jones et al. (2004) write:

> ··· auctions of 10% of slots, combined with secondary trading could, in theory, achieve the most efficient allocation of slots possible. But in practice, many of the auctions are likely to be so complex, both for auction organizers and for airlines bidding for slots, that it is probably unlikely that an efficient allocation of slots will emerge from this process.

An obvious source of complexity is the need for *package bids*: a takeoff slot at a flight originating airport is only valuable with a landing slot at the flight destination airport. We will refer to such packages as slot pairs for a "connection" between two airports. As a consequence, combinatorial exchange mechanisms have long been proposed for the primary allocation of airport time slots (Rassenti et al., 1982; Cramton et al., 2002; Ball et al., 2006, 2007; Castelli et al., 2011; Pellegrini et al., 2012; Ball et al., 2018). Combinatorial exchanges not only allow for the expression of package bids, but regulators can also enforce various allocation and market share constraints. The latter are important if the market is dominated by a few strong airlines and competition in the downstream market is at risk (De Wit and Burghouwt, 2008). Ball et al. (2018) provide an excellent summary of the advantages of combinatorial markets for airport time slots over congestion pricing and administrative slot controls (e.g., grandfathering).

Most proposals for airport time slot auctions assume a fully enumerative XOR bid language as it is also standard in spectrum auction design (Bichler and Goeree, 2017). In such a bid language, airlines can submit bids on all possible, exponentially many, packages of slots, and at most one of these packages can win. This bid language can express general valuations with substitutes and complements. For example, in 2008 the U.S. Federal Aviation Administration (FAA) proposed a single-round combinatorial auction for 10% of the slots at all three New York Airports (i.e., 32-34 slots per airport). The proposed market design also used an XOR bidding language where bidders could only win one out of 2000 bids that they could submit. Such limit on the number of bids are used to guarantee computational tractability, because the resulting winner determination problem is NP-hard (Lehmann et al., 2006).

1.2.1 Bidder-Optimal Core-Selecting Payments

In addition prices in the FAA proposal should be computed as bidder-optimal core-selecting prices (Day and Raghavan, 2007). The *core* is the most important stability concept in game theory, and core-selecting prices are such that no coalition of buyers and sellers (i.e., also an individual) has an incentive to deviate from the outcome of the auction. The well-known Vickrey-Clarke-Groves mechanism is incentive-compatible for purely payoff-maximizing bidders, but not always in the core, which can lead to very low revenue and outcomes that are considered unfair by participants (Ausubel and Milgrom, 2006). In other words, it can easily happen that a winning buyer for a set of slots has to pay less than what one or more losing buyers were willing to pay for these slots. This is the reason, why bidder-optimal core-selecting payment rules are nowadays the de facto standard for combinatorial auctions used by governments world-wide to sell spectrum licenses (Bichler and Goeree, 2017). Bidder-optimal core-selecting payments are arguably the most promising rule for airport time slots and other large combinatorial auction applications (Goetzendorff et al., 2015), and they extends the traditional concept of a competitive equilibrium to combinatorial markets (Bikhchandani and Mamer, 1997). Importantly, the Vickrey-Clarke-Groves mechanism loses its properties in multi-object auctions with budget constraints (Dobzinski et al., 2008) that we analyze, while we can still compute core-selecting outcomes as we will show. Just, the computation is computationally very hard.

The planned 2008 auction by the FAA was not implemented[1] due to resistance of airlines against slot trading in general,[2] but the design highlights the importance of *package bids* and *core stability* in markets for airport time slots.

1.2.2 Bid Languages and Budget Constraints

While the XOR bid language might be an option for small markets with three airports only as in the FAA design, it quickly becomes impractical for larger markets. The first reason is related to communication complexity (Nisan and Segal, 2006). With 6 airports and 30 slots per airport an airline could already bid on 887 million packages of slots. Those packages for which a bidder does not submit a bid are treated as if the bidder had no value for them. This is also referred to as the 'missing bids' problem, which can lead to enormous welfare losses (Bichler et al., 2014). Second, if bidders bid on many thousand packages, then the allocation problem easily becomes intractable.

A simple either-or (OR) bid language allows bidders to specify their preferences for many connections one-by-one and it reduces the number of bids substantially compared to an XOR language where only one package can win. Airlines can now win more than one package bid. Some of these bids might be substitutes, such as multiple slot pairs for the same connection. Simple extensions of the bid language can make sure that only one of these substitutes slot pairs can win (Nisan, 2006). However, given the many possible connections, the consideration of *budget constraints* becomes essential for airlines. In particular smaller airlines might not be able to pay up to their net present value for all packages they can win in an OR language, but they can only pay up to a budget constraint as is highlighted by several authors.[3] Actually, given the many package bids that airlines need to submit for individual connections between airports it is very hard to submit them in a way that their budget is not exceeded as the market prices are not known a priori. Note that if bidders only submitted budget-capped valuations, then the auctioneer does not know the true valuations and will not be able to maximize total welfare and achieve stability. Let us illustrate the problems that can arise in an example.

Consider three airlines A, B, and C. Let (NY_1, CH) be a possible connection using a starting time slot NY_1 and landing time slot CH and let (NY_2, LA) be another possible connection using starting time slot NY_2 and a landing time slot LA. The valuation of the airlines over these slots and the budget of airlines are given as in Table 1. In reality, airlines bid for a much larger number of possible connections in order to maximize their expected payoff considering their budget

[1] `https://www.gao.gov/products/B-316796`, accessed: 2020-03-30

[2] `https://www.nytimes.com/2007/02/18/nyregion/18laguardia.html`, accessed: 2020-03-30

[3] Harsha (2009) writes that "an aspect of airport slot market environments, which we argue must be considered in auction design, is the fact that the participating airlines are budget-constrained." Also, Cramton et al. (2002) motivate the importance of budget constraints in their proposal for a combinatorial airport time slot auction.

and capacity constraints (e.g., the number of planes available). Airline A has high value for both connections but is constrained by its budget.

Airline	(NY_1, CH)	(NY_2, LA)	Budget
A	5	5	7
B	4	0	4
C	0	4	4

Table 1: Example for three airlines and two possible connections where one airline is constrained by its budget. Numbers describe net present values in millions of dollars.

Suppose the airlines bid truthfully on the individual connections, but they cannot express their budget constraints. A market clearing price considering only the bids would require airline A to make a payment of at least 8 for both connections. Since this is above A's budget, the airline incurs a loss. If, on the other hand, airline A only submits bids up to 7 in total, it needs to decide how to shade its bids for each of the connections. If A shades its bids equally to 3.5, then it will not win any of the connections. This leads to an inefficient allocation of slots with a welfare of 8 whereas any allocation where A gets one of the connections and a competitor gets the other one has a welfare of 9. Airlines do not have complete information about the bids of others as in our complete information example. Shading bids appropriately would require an unreasonable amount of information about the valuations of others, and it makes bidding strategically challenging. Note that an airline with multiple bids might not want to bid beyond its net present value of a connection. For example, if airline A bids 10 for the connection (NY_1, CH) rather than its true net present value of 5, it could win this connection at 7 and make an effective loss of 2.

The simple example illustrates that even in such a small stylized market, bidding is strategically hard without the possibility to communicate budget constraints, and it can easily lead to outcomes that are neither individually rational (such that airlines make a loss) nor allocatively efficient (i.e., they do not maximize welfare). These effects illustrated in this example are amplified in larger markets, as we show in our paper.

Ideally, the auctioneer takes budget constraints into account and computes an allocation and prices such that no coalition of airlines could deviate, but that the gains from trade (i.e., welfare) are maximized subject to these constraints. This makes bidding simple for airlines and maximizes welfare subject to all relevant constraints. Unfortunately, the computation of welfare-maximizing outcomes in a combinatorial market with budget-constrained bidders is a Σ_2^p-hard optimization problem in general (Bichler and Waldherr, 2019). Σ_2^p problems are significantly harder to solve than NP-hard problems. Roughly, even if we had access to an efficient procedure that solves NP-hard subproblems, it would still not be possible to design a polynomial-time algorithm that solves a Σ_2^p-hard unless P=NP. While NP-hard problems are widely studied and there are effective computational techniques to solve them, Σ_2^p-hard problems are rarely analyzed and there are no general-purpose solvers as there are for integer programs.

1.3 Contributions

The fact that computing market equilibria can be computationally so hard is an important insight and relevant to the design of markets for airport time slots. One would think that even with today's computing power we cannot expect to solve real-world problems in this complexity class. This paper demonstrates that we can hope for market designs that are core-stable, respect budget constraints and maximize welfare. The OR bid language in combination with allocation and budget constraints limits the number of bids that airlines need to submit and effectively addresses the communication complexity and the missing bids problem for bidders.

We contribute appropriate optimization models for primary and secondary airport time slot markets and effective column- and constraint-generation algorithms that leverage the specifics of the airport time-slot problem. Extensive experimental evaluation based on the widely used and publicly available CATS instance generator (Leyton-Brown et al., 2000) provide evidence that in spite of the fundamental computational hardness of these problems, we can solve problem sizes that are relevant for the field. This provides a new market design for airport time slots with design desiderata that were so far considered impossible to achieve. The fact that it is possible to consider these complex constraints in a market-based allocation of airport time slots can be a central argument for the adoption of airport time slot auctions in practice.

The paper is structured as follows. In Section 1.4, we discuss related literature on airport congestion management. In Section 2.2, the mathematical model is presented and the mechanism to allocate airport time slots and derive prices is introduced in Section 3. We evaluate our mechanism in Section 4 before drawing conclusions in Section 5.

1.4 Airport Congestion Management

The past few decades have have led to significant research on airport congestion management (Churchill et al., 2012; Vaze and Barnhart, 2012; Pyrgiotis and Odoni, 2015; Gillen et al., 2016). In what follows, we provide a brief discussion of alternative streams in the literature. Methods to distribute slots effectively can roughly be divided into groups of *non-monetary mechanisms* and *mechanisms with monetary transfer*.

Adhering to current practices, researcher from the first group argue for centralised mechanisms to further optimise the allocation of slots while keeping changes to service levels and passenger demand minimal (Le et al., 2008; Vaze and Barnhart, 2012). In these approaches airlines state their preferred time for take-off/landing and specify a time interval within which displacement is acceptable. Subsequently an optimization method is applied to find a feasible allocation that minimizes some displacement metric (e.g., total sum of displacements, number of refused requests, average displacement per airline). Differences within this group can be found in regard to scope (single or multiple airports), modelling of airport capacity (deterministic or stochastic), and methods applied (linear programming or heuristics) (Koesters, 2007; Castelli et al., 2011; Corolli et al., 2014; Zografos et al., 2012). Recent contributions include fair distribution of shifted flights (Jacquillat and Vaze, 2018) and the use of meta-heuristics (Castelli et al., 2011; Ribeiro et al., 2019). Zografos et al. (2017) provide an extensive study on the latest results.

The second group argues in favour of market mechanisms where *monetary transfers* are used to elicit preferences or promote desirable behaviour. We need to distinguish between *congestion pricing* and *auctions or exchanges*, both of which are market mechanisms, but different in how prices are determined. Congestion pricing aims to price the marginal social cost of delays, i.e. making an airline pay the cost of the delays it caused. Carlin and Park (1970) provided one of the earliest articles on congestion pricing in the airport time slot context albeit on a very simple setting with only one runway and constant demand. The main difference to road congestion pricing lies in the nature of the user. Drivers can be considered truly atomistic users, but the same is not true for airlines which typically operate several flights on the same airport resulting in a greater internalization of social costs. However, recent empirical evidence is mixed (Ater, 2012; Morrison and Winston, 2007; Daniel and Harback, 2008). The main question is how to elicit cost internalization (Daniel, 1995).

Market mechanisms such as auctions and exchanges are a way to elicit preferences of airlines systematically. Various proposals for combinatorial trading of airport time slots have already been discussed in the introduction. An up-to-date discussion of auctions and alternative means of allocating airport time slots can be found in Ball et al. (2018). They introduce basics of the market design for airport time slot markets including the product definition, the need for package bids, allocation and market share constraints, auction frequency, and the consideration of grandfather rights. Our paper draws on these considerations and earlier research on market design for airport time slot auctions (Rassenti et al., 1982; Cramton et al., 2002; Ball et al., 2006, 2007; Castelli et al., 2011; Pellegrini et al., 2012; Ball et al., 2018). For example, Pellegrini et al. (2012) provide an interesting design for a combinatorial secondary market with an XOR bid language, while Ball et al. (2007) propose an ascending clock-proxy auction, similar to what is nowadays being used in spectrum auctions (Bichler and Goeree, 2017). Such designs with an XOR bid language suffer from the missing bids problem and are limited to small applications with a few airports only. Besides, even with an XOR language an auctioneer cannot guarantee welfare maximization if he does not know the values and budget constraints.

We propose a simple OR bidding language that allows airlines to express additional (exclusive-or) allocation constraints, their budget limits and values for individual connections and win several of these slot packages. Substitutes connections can easily be excluded with the exclusive-or constraints, and budget constraints make sure that bidders do not have to pay more than they can afford. This makes bidding simple for airlines also in larger applications, but it leads to a Σ_2^p-hard optimization problem for the auctioneer. We provide a market design and algorithms and show that realistic problem sizes can be solved.

A note on strategic manipulation is in order. There is a large literature on incentive-compatibility in auction markets. It is well-known that the Vickrey-Clarke-Groves (VCG) mechanism is the unique design that is incentive-compatible in dominant strategies (Green and Laffont, 1979). However, the VCG mechanism assumes pure quasi-linear preferences where participants maximize payoff and they do not have budget constraints. Unfortunately, it was shown that the addition of budget constraints does not allow for incentive-compatible mechanisms (Dobzinski et al., 2008). Even if we assume that there are no budget constraints, the VCG mechanism is no option in exchanges, because it is not budget-balanced. Actually, Myerson and Satterthwaite (1983) showed that there is no efficient way for two parties and in two-sided markets to trade when they each have private and probabilistically varying valuations, without the risk of forcing one party to trade at a loss. These results are important for smaller markets with only a few participants. In large markets (with many items and bidders) the power of participants to manipulate prices is small (Roberts and Postlewaite, 1976). Therefore, complete information models where participants are price takers are standard in competitive equilibrium theory (Bikhchandani and Mamer, 1997; Bikhchandani and Ostroy, 2002; Baldwin and Klemperer, 2019). This is also a stantard assumption in the study of large markets such as electricity or financial markets where individuals can be assumed to be price takers.

2 Model

Let us now introduce a bilevel integer optimization problem for the computation of welfare-maximizing and core-stable allocations and prices that respect allocation and budget constraints. We draw on basic results by Bichler and Waldherr (2019) but leverage the specific conditions in secondary markets for airport slot trading.

To reach an outcome which is core-stable, we need to ensure that no coalition of bidders (buyers and sellers) could reach a more efficient outcome for themselves by only trading among each other. If that was the case, bidders would prefer the outcome of a decentralized mechanism over participating in the centralized market. To ensure core-stability, the allocation and prices must be chosen in a way such that no such coalition of bidders exists. This can be modeled through bilevel programming which we will introduce in Section 2.1. In the bilevel program, the allocation problem is solved in the upper level, while potential coalitions of bidders are modeled within the lower level. In Sections 2.2 and 2.3 we will show how bilevel programming can be used to model secondary markets for airport slot trading, while in Section 2.4, we describe how the model needs to be adjusted for primary markets. To reach an outcome which is core-stable, we need to ensure that no coalition of bidders (buyers and sellers) could reach a more efficient outcome for themselves by only trading among each other.

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2.1 Bilevel Programming

Bilevel linear programs are frequently used to model sequential distributed decision making. In these situations, typically a *leader* makes the first decision and a *follower* reacts after observing the leader's decision. The follower's action is important to the leader as it might interfere with the leader's objective. The challenge of the leader is to predict the follower's reaction and take action in such a way that after the follower's reaction the leader's objective is reached to the highest possible degree. More technically, a bilevel linear program is a linear program that is constrained by another linear optimization problem. Usually the first optimization problem is called the upper level problem (leader) while the constraining problem is referred to as the lower level problem (follower). Given an upper level solution, the lower level computes an optimal solution under consideration of its respective constraints. This in turn affects the upper level by altering the value of the objective function or violating constraints, possibly making the overall solution infeasible. Let X be the set of variables in the upper level problem and Y be the set of variables in the lower level problem. The general form of this problem is

$$\max_{x \in X} \; F(x,y) \tag{1a}$$

$$\text{s.t. } G(x,y) \leq 0 \tag{1b}$$

$$\min_{y \in Y} \; f(x,y) \tag{1c}$$

$$\text{s.t. } g(x,y) \leq 0 \tag{1d}$$

where $F, f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^1, G : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p, g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$ are continuous, twice differentiable functions. Note that in MIBLP, F and f are represented by linear objective functions of the upper and lower level, while G and g are the respective linear constraints. X, and Y include continuous as well as integer variables.

A bilevel program can be applied to obtain stable outcomes in combinatorial exchanges of goods. In this case, x is the vector of allocation and payments that are derived by the mechanism in the upper level (the outcome of the upper level). $F(x, y)$ is the social welfare function that needs to be maximized (e.g., gains from trade) and $G(x, y)$ contains all allocation and pricing constraints such that the resulting outcome determined in the upper level is feasible.

In order to model core-stability as a constraint, possible blocking coalitions are modeled in the lower level. The lower level constraints $g(x, y)$ define a feasible allocation for a possible blocking coalition which leads to a minimal improvement d that a member of the coalition can achieve in comparison to the upper level outcome. The lower level objective function maximizes the minimal improvement d.

If a coalition exists such that they could improve upon the upper level outcome, the upper level outcome is instable. Hence, by adding the constraint '$d \leq 0$' to the upper level, the MIBLP becomes infeasible whenever a blocking coalition exits. If d cannot be maximized beyond 0 in the lower level, the upper level outcome is stable since there exists no coalition that can deviate from its outcome such that all members in the coalition can make a profit by doing so.

In the following, we describe a bilevel program to model airport time slot trading such that the outcome respects budget constraints and maximizes gains from trade among all stable outcomes. We first describe the secondary slot market in which bidders consist only of airlines that trade endowed slots. The primary slot market in which airlines only attend as buyers will be introduced thereafter as a special case of this market.

2.2 Bilevel Programming for Slot Trading

Let I be the set of bidders (airlines) and K be the set of slots available in the exchange. A subset $S \subseteq K$ defines a package of slots that can contain a slot pair for single or multiple connections. For instance, a package can consist of a sequence of slot pairs at a given time slot during the whole season.

For each $S \subseteq K$, $v_i(S)$ defines the true value of bidder i for package S. For each bidder i, B_i denotes i's budget and the set $\mathcal{G}_i$ defines a set of different *flight groups* i is interested in. Flight groups are comprised of similar packages of slots for one or more connections (e.g., having the same origin and destination) which only differ slightly in the arrival and departure slots in order to allow for flexibility in assigning these slots. For each flight group $g \in \mathcal{G}_i$ several bids may be placed by i, but he/she is only interested in obtaining one of the connections within the group. Flight groups can be used to model whether packages of slots are substitutes for each other and allow for some flexibility in bidding. Note that our model does not allow for any complementarities and synergies between packages outside of flight groups. While it is possible to incorporate these notions by additional constraints and larger packages, this would also lead to a significantly more complicated (and computationally more demanding) model. However, synergies between packages are not a big issue in primary markets: The overall operational strategy of the airline is defined by the primary allocation and the secondary market served to make small adjustments in order to increase the efficiency of the primary allocation. Therefore, synergies in relation to slots available to the airline (which are not traded in the secondary market) can easily be reflected in the valuations for the packages that are up for trade in the secondary market.

For all $i \in I, S \subseteq K$, let $x_i(S) \in \{0, 1\}$ define a binary variable whether i

buys S in the upper level and let $y_i(S) \in \{0,1\}$ define whether i sells package S in the upper level. For each bidder i, $y_i(S)$ are only defined for packages S that are owned by i. Let $p_i \in \mathbb{R}$ be the corresponding payment for i in the upper level. In case p_i is positive, i receives this payment in the exchange, while he/she pays $|p_i|$ in case p_i is negative. The model differs from two-sided markets (Bichler and Waldherr, 2019) where the set of buyers and sellers are disjoint (i.e., bidders could not buy and sell at the same time) and implements the (OR) bid language and constraints of the airport time slot market.

Let $d \in \mathbb{R}$ be the minimal profit of a member of a blocking coalition that is determined in the lower level. Further, let $\chi_i(S) \in \{0,1\}$ describe whether i is assigned S in the lower level, $\gamma_i(S) \in \{0,1\}$ describe whether i sells S in the lower level and $\rho_i \in \mathbb{R}$ describe the payment of i in the lower level. For each bidder we also introduce an auxiliary variable a_i to keep track of whether i is a member of the coalition blocking the upper level outcome. Then, the MIBLP can be written as follows.

$$
\begin{aligned}
\max_{x,y} \; & \sum_{S \subseteq K} \sum_{i \in I} v_i(S) \cdot (x_i(S) - y_i(S)) && && (\mathbf{CEx}) \\
\text{s.t.} \; & \sum_{i \in I} p_i = 0 && && \text{(2a)} \\
& \sum_{S:k \subseteq K} \sum_{i \in I} x_i(S) \leq \sum_{S \subseteq K} y_i(S) && \forall k \in K && \text{(2b)} \\
& p_i \geq \sum_{S \subseteq K} \sum_{i \in I} v_i(S) \cdot (y_i(S) - x_i(S)) && \forall i \in I && \text{(2c)} \\
& -p_i \leq B_i && \forall i \in I && \text{(2d)} \\
& \sum_{S \in g} x_i(S) \leq 1 && \forall g \in \mathcal{G}_i, i \in I && \text{(2e)} \\
& d \leq 0 && && \text{(2f)} \\
& d = \max d && && (\textbf{Lower Level}) \\
& \text{s.t.} \quad \sum_{i \in I} \sum_{S \subseteq K} \rho_i(S) = 0 && && \text{(2g)} \\
& \qquad \sum_{S:k \subseteq K} \sum_{i \in I} \chi_i(S) \leq \sum_{S \subseteq K} \gamma_i(S) && \forall k \in K && \text{(2h)} \\
& \qquad \rho_i \geq \sum_{S \subseteq K} \sum_{i \in I} v_i(S) \cdot (\gamma_i(S) - \chi_i(S)) && \forall i \in I && \text{(2i)} \\
& \qquad -\rho_i(S) \leq B_i && \forall i \in I && \text{(2j)} \\
& \qquad a_i \geq \chi_i(S) && \forall S \subseteq K, i \in I && \text{(2k)} \\
& \qquad a_i \geq \gamma_i(S) && \forall S \subseteq K, i \in I && \text{(2l)} \\
& \qquad d \leq M \cdot (1 - a_i) + \sum_{S \subseteq K} v_i(S) \left(\chi_i(S) - \gamma_i(S)\right) +
\end{aligned}
$$

$$\rho_i - \left[\sum_{S \subseteq K} v_i(S) \left(x_i(S) - y_i(S) \right) + p_i \right] \qquad \forall i \in I \tag{2m}$$

$$\sum_{i \in I} a_i \geq 1 \tag{2n}$$

$$\chi_i(S), \gamma_i(S) \in \{0, 1\} \qquad \forall S \subseteq K, i \in I \tag{2o}$$

$$\sum_{S \in g} \chi_i(S) \leq 1 \qquad \forall g \in \mathcal{G}_i, i \in I \tag{2p}$$

$$a_i \in \{0, 1\} \qquad \forall i \in I \tag{2q}$$

$$\rho_i(S) \in \mathbb{R} \qquad \forall S \subseteq K, i \in I \tag{2r}$$

$$d \in \mathbb{R} \tag{2s}$$

$$x_i(S), y_i(S) \in \{0, 1\} \qquad \forall S \subseteq K, i \in I \tag{2t}$$

$$p_i(S) \in \mathbb{R} \qquad \forall S \subseteq K, i \in I \tag{2u}$$

(**CEx**) determines an outcome that maximizes the gains from trade under all feasible and stable outcomes. The upper level constraints include economic pricing and allocation constraints such as budget balance (2a), balance of supply and demand (2b), individual rationality (2c), and respects the budget constraints of all bidders (2d). Buyers are allowed to state that they are not interested in simultaneously winning more than one bid out of a group of bids $\mathcal{G}_i$ and specify multiple such groups (2e). Constraint (2f) states that the solution is infeasible (unstable) if the lower level finds a coalition with positive deviation value d.

Given the upper level solution, the lower level problem tries to find a coalition that can deviate in a beneficial way from the upper level. This is modeled as a maximization problem similar to the upper level. Trades in the lower level also have to fulfill all economic pricing and allocation constraints (2g)-(2j).

Constraints (2k) and (2l) set $a_i = 1$ if bidder i performs a trade in the lower level. In this case, i is member of a blocking coalition. Constraints (2m) models the gains from deviation for each individual bidder, comparing her outcomes (and hence her payoff) in the upper and the lower level. Only if $i \in I$ actually participates in a blocking coalition (i.e., when $a_i = 1$), her difference in payoffs should be considered. We define a very large number M such that d is not affected by bidders that are not members of the blocking coalition. To avoid empty coalitions with only trivial bounds on the d value a constraint (2n) ensures that at least one participant makes a trade in the lower level. The corresponding domains of the upper and lower level variables are defined in (2o) - (2u). Large coalitions are difficult for participants to form, and the computation of the coalitional value of these small coalitions is NP-hard in general. A way to reduce the computational burden is to limit attention to coalitions of a particular size. Such a restriction on the coalitions reduces computational costs for the lower-level programs, and consequently the computation times, as we will show.

2.3 Discussion of Bid Language and Extensions

The MIBLP program introduced above draws on an either-or (OR) bid language in which bidders can specify their preferences over many connections while limiting the number of bids that would be necessary in XOR bid languages. In order to allow expressing substitutes of slots, bidders are also able to indicate flight groups. By incorporating the additional XOR constraint (2e) in the bilevel program, it is ensured that bidders cannot be assigned multiple conflicting slots within the same flight group.

The bid language can easily be extended by additional features. Instead of modeling unique slots and requiring bidders to compose large packages if they are interested in obtaining the same slot for a sequence of weeks, the elements $k \in K$ could directly model these sequences. This may lead to a significant reduction in the size of the model. However, it also requires some additional constraints in the allocation problem to assure that overlapping sequences are not traded (e.g. to ensure that a slot cannot be sold for the first 60% and the last 50% of the season at the same time). From an operational point of view, airlines are particularly interested in obtaining slots that minimize their turnaround time at airports in order to allow for time-efficient round trips between airports. Instead of submitting bids on connections that only consist of a single departure and landing slot, airlines can also bid on larger packages comprised of multiple slots that constitute such round trips. One can also allow for additional either-or constraints among packages similar to that for flight groups or other allocation constraints such as to enforce banking. The latter can either be introduced as an extension to the bidding language or as a (soft) allocation constraint at hub airports. Introducing additional allocation constraints for the airports does not impact the fundamental hardness of the problem, that we discuss in this paper.

2.4 Bilevel Programming for Primary Slot Trading

The MIBLP can also easily be adapted for primary slot allocation. In the primary market, bidders no longer can buy and sell slots at the same time, but each bidder is either a buyer (airline) or a seller (airport). In this case, we only define variables x_i and χ_i for bidders $i \in I$ that are buyers and only define variables y_i and γ_i for those bidders in I that are sellers. Otherwise, the MILP that models the primary market is identical to (**CEx**). Additional constraints that may be relevant in primary slot allocation markets (grandfather rights, limitation of market shares, etc.) can easily be incorporated in the model's upper and lower levels.

3 Algorithmic Solution

Bard and Moore (1990) initiated research on algorithmic solutions to MIBLPs. Their algorithm converges if either all leader variables are integer, or when the follower subproblem is an LP. Until recently, general MIBLPs were considered "still unsolved by the operations research community" (Delgadillo et al., 2010).

Only two years ago, two general purpose branch-and-cut MIBLP algorithms have been proposed by Fischetti et al. (2017) and Tahernejad et al. (2020). Fischetti et al. (2017) extend their earlier algorithm for MIBLPs with binary first-level variables to general MIBLPs where the linking variables (i.e., those variables that appear in both, the upper and lower level) are discrete. Tahernejad et al. (2020) also propose another general-purpose MIBLP solver based on branch-and-cut which is available open source in the MibS solver. However, MibS also requires the linking variables to be integer. In our case, the linking variables consist of the upper level allocation and prices, the latter not being integer in general.

If the lower level problem does not contain integer variables and is an LP, bilevel programs can be reformulated as single-level problem by replacing the lower level with its optimality conditions (e.g., Karush-Kuhn-Tucker (KKT)) and then solving the resulting problem using standard integer programming techniques (Bard and Moore, 1990). For general mixed-integer bilevel problems, in which linking variables can be both, integer and continuous, Zeng and An (2014) proposed a column-and-constraint generation framework that fixes integer variables in the lower level (thereby transforming it into an LP) and then adds optimality conditions for these solutions to the upper level problem. Bichler and Waldherr (2019) extend this framework for problems in which the lower level is always feasible regardless of the upper level and the objective of the lower level is to make the upper level (and hence the whole MIBLP) infeasible. This is the case in (**CEx**) since simply making the same trades as in the upper level is always feasible for the lower level and the upper level is feasible if and only if an objective value of $d < 0$ is obtained in the lower level.

In the following, we give a short outline of the algorithm. First, the upper level U of the bilevel program is solved, ignoring all lower level variables and constraints, resulting in optimal allocations x^*, y^* and prices p^*. Afterwards, the lower level is solved for the optimal upper level outcome. If for the optimal solution d^* of the lower level, it holds that $d^* < 0$, the upper level outcome is stable. Otherwise, a blocking coalition can be read from the lower level variables χ^*, γ^*, ρ^*. In this case, the KKT conditions for the fixed variables χ^* and γ^* are added to the upper level U and the upper level is solved again. The KKT conditions assure that in this iteration the upper level U will return an outcome that is stable against a coalition with assignments χ^*, γ^*. The process is repeated iteratively until either no more blocking coalition can be determined in the lower level (in which case the last upper level outcome is the one that maximizes welfare among all stable outcomes) or U is no longer feasible itself (in which case there is no stable outcome). In the worst case, this requires adding KKT conditions for all possible blocking coalitions and their allocations, but in practice the algorithm converges much faster.

Iteratively adding KKT conditions to the upper level adds substantially to an already large integer program as in each iteration we have to add a large number of lower level primal and dual constraints and variables, as well as the complementary slackness conditions. The latter, especially, complicate the program since they either require quadratic constraints or linearization by using even more big M constraints. In the following, we describe an alternative to adding KKT conditions that vastly reduces the number of necessary constraints and variables while ensuring stability against the same blocking coalitions in each iteration. In comparison to KKT conditions, we refer to this set of constrains as the blocking coalition elimination (BCE) constraints. This approach enables us to solve realistic problem sizes as we will show.

The BCE constraints can be applied to general MIBLP for finding stable outcomes in combinatorial exchanges. Since the key idea behind these constraints is that we can compute the amount a buyer is willing to pay in a blocking coalition C and what payment sellers demand in order to deviate, we will first describe the constraints for markets in which each bidder is either a buyer or a seller, as is the case in primary markets for airport time slots.

For all bidders we know their utility π_i in the upper level solution. Let v_i^C denote a bidder i's true valuation for the allocation determined within a possible blocking coalition C. Let $C(I)$ be the buyers that participate in C and $C(J)$ be the the sellers that participate in C. Within this coalition, a buyer $i \in C(I)$ is willing to pay the difference of v_i^C (her utility given the lower level allocation) and π (her utility given the current upper level allocation and prices) in combination with a small ϵ he/she wants to gain by deviating. We define this amount as her willingness to pay w_i^C. Then, the BCE constraints are as follows:

$$\pi_i = \sum_{S \subseteq K} v_i(S)\,(x_i(S)) - p_i \qquad \forall i \in C(I) \tag{UB}$$

$$\pi_j = \sum_{S \subseteq K} -\left(x_j(Z) \cdot v_i(Z)\right) + p_j \qquad \forall j \in C(J) \tag{US}$$

$$w_i^C = \min\{B_i, v_i^C - \pi_i - \epsilon\} \qquad \forall i \in C(I) \tag{WtP}$$

$$\sum_{i \in C(I)} w_i^C \leq \sum_{j \in C(J)} (\pi_j + \epsilon) + \epsilon \tag{Ex}$$

$$w_i^C \in \mathbb{R} \qquad \forall i \in C \tag{Real}$$

Constraints (UB) and (US) define the utility of all bidders in the upper level. For a buyer this is her valuation for the allocated package minus the price he/she has to pay. Similarly, a seller's utility is the payment he/she receives minus her valuation for the package he/she sold. Constraint (WtP) defines the willingness of a buyer within the blocking coalition. Note that this willingness to pay w_i^C is still constrained by her budget. Then, if the sum of w_i^C is negative sellers in the coalition demand more money for selling their items than the participating buyers are willing to pay for these items and the coalition will not deviate (Ex). Note that while the minimization function in constraint (WtP) also results in either quadratic constraints or the necessity of linearization, the overhead is way lower than for KKT conditions.

The constraints differ only slightly when considering secondary markets. If a participant is both buying and selling, her upper level utility is the difference of valuations for obtained and sold items plus her payment (which might take on negative values if her expenses exceed her revenues) (UB). The constraint for w_i^C remains unchanged (WtP), but has a slightly different interpretation if a participant by the allocation alone has negative utility $v_i^C < 0$ in the coalition. In this case, he/she will demand a payment of amount w_i^C instead of offering to contribute to the coalition by paying.

$$\pi_i = \sum_{S \subseteq K} v_i(S)\,(x_i(S) - v_i(S)) + p_i \qquad \forall i \in C \qquad \text{(UB)}$$

$$w_i^C = \min\{B_i, v_i^C - \pi_i - \epsilon\} \qquad \forall i \in C \qquad \text{(WtP)}$$

$$\sum_{i \in C} w_i^C \leq \epsilon \qquad \text{(Ex)}$$

$$w_i^C \in \mathbb{R} \qquad \forall i \in C \qquad \text{(Real)}$$

The careful reader might wonder whether a coalition would deviate when a participant that only has the role of a buyer demands a payment in the coalition (because the upper level allocation and prices are preferable to her). One could argue that this coalition would not form and that since we are not accounting for this case we restrict the upper level in inadmissible ways. However, if (Ex) is still violated, i.e. the coalition collectively is willing to pay enough money to cover for the demanding buyer, a smaller coalition could form by excluding the buyer in question. The constraints in this sense not only ensure stability against coalitions with known allocations but also some similar coalitions whose formation is predictable. After these constraints have been added to the upper level based on a lower level allocation, all solutions that satisfy these constraints are resistant to deviations with the given or very similar allocations. Iteratively adding BCE constraints either leads to an outcome for which there exists no more blocking coalition (a stable outcome) or to infeasibility in the upper level (proving non-existence of a stable outcome).

4 Experimental Results

In the following, we provide evidence that our algorithm can compute realistic problem instances and improves the welfare in slot trading markets as opposed to mechanisms that ignore budget constraints. We limit our reports to secondary slot markets in which participants act as both, buyers and sellers. The primary slot allocation is a special case of our MIBLP for the secondary market and the results with regards to scaleability and the negative effects of ignoring budget constraints presented in this section are in line with those that we found for primary markets.

First, we describe the data set that we used to evaluate our mechanism in Section 4.1. Next, we analyse the benefits of using the new BCE constraints in comparison to the classical KKT conditions in Section 4.2 before showing that with the help of these new constraints our mechanism is capable of solving instances of practically relevant size in Section 4.3. Finally, we demonstrate the necessity of a mechanism that elicits budget constraints by demonstrating that other standard markets without considering budget constraints lead to very undesirable results. In Section 4.4 we discuss the risks of bidding true valuation without considering the budget, in Section 4.5 we show that shading strategies lead to significant welfare losses and instabilities. All algorithms were implemented in Java. The experiments were executed on a laptop with Intel core i5-6600k (4 cores, 3.5 GHz) and 8GB RAM. Gurobi 7.5.2 was used for solving linear programs.

4.1 Data

To model the valuations of slots, we use the widely used combinatorial auction test suite (CATS) (Leyton-Brown et al., 2000), which allows for replication of our results. In the original version, participants are interested in buying slot pairs on two of four coordinated U.S. airports. Participants place bids on packages of slots in two airports where takeoff/landing times deviate by a small margin from their preferred time slots. This requires an evaluation of the network value of a slot pair(s) to each potential acquirer. In practice, time of day and logistics play a big role. For example, prime slots for the Trans Atlantic and Asian or intra-Europe markets are very different and use different aircraft types that produce different revenue and cost profiles. In our experiments, the valuation of the most preferred connection is based on common valuations of the slots and a private random deviation. For substitute slot pairs within a *flight group* $g \in \mathcal{G}_i$, the valuations are based on the valuation of the preferred slot pair and a deviation. The duration of flights and the selection of corresponding slots are based on the coordinates of the respective airports. We incorporate a larger number of airports, using up to 20 coordinated European airports and their real-world location. For each bidder, we generate several flight groups, each containing 4 to 6 connections. An overview of the treatment variables is given in Table 2.

Treatment variable	min	max
#Airlines	10	20
#Airports	10	20
#Slots (total)	1400	2800
#Flight groups $\lvert\mathcal{G}_i\rvert$ (per airline)	12	25
Budget β	0.2	0.8

Table 2: Treatment variables

CATS only generates valuations for airport time slots, but does not consider buyers' budgets. For each airline $i \in I$ let $M_i = \sum_{g \in \mathcal{G}_i} \max_{S \in g} v_i(S)$ be the sum of valuations of the most valuable packages for each of i's flight groups. In order to take into account financial constraints of airlines, we simulate budgets by specifying the level $\beta \in [0, 1]$ of how severely airlines are constrained from bidding up to their true valuations. Each airline i is only allowed to spend a maximum of βM_i Dollars.

On the sellers' side, CATS assumes a single seller without ask prices. In order to simulate secondary markets, we endow participants with slots and compute ask prices based on the procedure proposed by CATS. We randomly distribute these endowments among the participants. Ask prices for these slots are calculated as the common valuation for this slot scaled down by a factor of 0.3 to reflect the seller's intention to sell this slot. All problem instances are available upon request.

4.2 Comparison of KKT to BCE conditions

First, we evaluate the benefits of the BCE introduced in Section 3 as opposed to using the KKT conditions. For this, we consider two scenarios with 5 participants trading 700 slots and 10 participants trading 1400 slots, respectively. We apply three settings with $\beta_1 = 0.2$, $\beta_2 = 0.5$ and $\beta_3 = 0.8$ to both scenarios. We solved 30 instances for each parameter combination, one time using standard KKT conditions, the other time using our BCE constraints. The results are summarized in Tables 3 and 4.

Result 1 *The BCE approach to the MIBLP for the airport time slot trading problem is computationally more effective than the use of KKT conditions.*

Both approaches can solve all instances in the first scenario where only 5 participants trade (Table 3). It is evident however that the KKT conditions lead to longer solving times due to the more complex constraints added to the upper level. Even for these smaller instances where typically less than 10 iterations are needed to find a stable solution this led to longer solving times for all parameter combinations.

#Bidders	#Flight groups	Budget	Method	solved <1000s	avg. time (s)
5	25	0.2	KKT	30	9.15
5	25	0.2	BCE	30	6.05
5	25	0.5	KKT	30	14.29
5	25	0.5	BCE	30	10.16
5	25	0.8	KKT	30	40.91
5	25	0.8	BCE	30	11.68

Table 3: Comparison of KKT and BCE constraints (1/2)

The performance of the KKT approach further declines substantially after a small number of iterations. We observed that typically the computation of the upper level solution with KKT conditions takes several minutes after 12 iterations while the BCE approach still solves in less than 1s even after 40 iterations of adding constraints. Neither for low nor high budgets can the KKT approach solve a single instance within a time limit of 1000s in the 10 bidder scenario (Table 4). Using BCE all instances can be computed in less than 375s on average.

#Bidders	#Flight groups	Budget	Method	solved <1000s
10	25	0.2	KKT	0
10	25	0.2	BCE	30
10	25	0.5	KKT	0
10	25	0.5	BCE	30
10	25	0.8	KKT	0
10	25	0.8	BCE	30

Table 4: Comparison of KKT and BCE constraints (2/2)

Using a paired Wilcoxon rank-sum test, pairwise differences in solution time for all parameter combinations are significant at the level of $p < 0.001$. Based on these results, we report all remaining experiments in this section using BCE constraints only.

4.3 Scaleability

In this section, we show that despite the computational hardness of the underlying optimization problem, we are able to solve instances of practical relevance with our approach. We first discuss problem sizes that one can expect in slot trading markets. For most airports, no public records exist how many takeoff and landing slots are available for each hour. In an analysis of airport slot control at EU airports requested by the European Parliament's Committee on Transportation and Tourism, 70-80 slots per hour where mentioned. If we assume 20 hours of operations for major airports in Europe, this adds up to approximately 1,400 distinct time slots per day and airport. In the same report, the authors advised that annually 10% of these slots should be made available in an auction.

In order to simulate the full trade volume of time slots for one day, we generated 140 slots for each airport to be traded among participants. We consider 10 to 20 different airports with 1400 to 2800 total slots. This scenario matches the number of coordinated airports in three of the major regions (The Americas, Middle East and Africa or North Asia). For our experiments we generated instances for 10, 15 and 20 bidders (airlines), which can be considered a realistic number of bidders even for larger airports. We generated 10 instances for each parameter configuration.

Result 2 *Problem instances with 10 participants on the secondary market with up to 1400 time slots can be computed to optimality in less than one hour. The number of time slots has little impact on the runtime, the number of participants has a strong impact. With more participants the problem instances were* *intractable and needed to be terminated after 3 hours. If we restrict ourselves to smaller 5-core or 7-core-stable outcomes, also problem sizes with 20 buyers and sellers can be solved to optimality.*

We report the aggregated results for 10 instances each in Table 5. For 10 to 20 bidders, we report how many of the 10 instances we are able to solve (i.e. obtain a welfare-maximal stable outcome or proof that no such outcome exists) within a time limit of 3 hours. We also report the average computation time (in seconds) of the solved instances. It can be seen that all of the smaller instances can be solved in very short time.

In practice, blocking coalitions of arbitrary large size may not form since coordinating towards blocking an outcome may be hard for large groups of (self-interested) airlines. Hence, we also determined welfare-optimal outcomes that are only stable against blocking coalitions of size at most 5 (5-stable) or 7 (7-stable). We could solve all problem instances in a matter of minutes. While solving instances of increasing size becomes computationally more challenging, these results show that 5- and 7-stable outcomes can be computed even for large numbers of bidders.

#Bidders	Stable Outcome		5-Stable Outcome		7-Stable Outcome	
	#Solved	Time (s)	#Solved	Time (s)	#Solved	Time(s)
10	10	234	10	86	10	147
15	10	9916	10	84	10	1613
20	0	n.a.	10	323	10	1121

Table 5: Computation times

4.4 Outcomes without the Possibility to State Budget Constraints

In most markets, bidders cannot communicate their budgets and can either choose to submit truthful bids (i.e., bidding the true valuations for each of desired connections) or shade their bids heuristically. In the following, we demonstrate that the truthful bidding puts airlines at a significant risk of making a loss by having to pay more than they can afford according to their budgets. We analyse secondary slot trading markets with 10 buyers, 10 sellers and 1400 slots traded. We use the number of flight groups per buyer (12 or 25) and budget factor (0.2, 0.5 or 0.8) as treatments. For each combination of number of flight groups and budgets, we report the average results over 10 instances.

Result 3 *If bidders cannot communicate their budgets and bid their true valuations, up to 30% of participants face payments that exceed their budgets. Even with high budgets, 4% of bidders make a loss. The required payments exceed participants' budgets by more than 200% in the worst case.*

For our tests, we determine core-stable solutions in the standard combinatorial auction setting where all bidders submit their true valuations as bids. Finding core prices for allocation problems without budget constraints does not require solving a Σ_2^p-complete optimization problem, but 'only' a NP-complete problem (Lehmann et al., 2006). We say that a participant overpays in this auction if he is required to pay more than their budgets would allow them if they were constrained by the stated budget factor. In Table 6 we report which fraction of participants that win at least one package overpay, how large this overpayment is on average and how large the largest overpayment is on average. Bidding truthfully without taking budgets into consideration exposes airlines to overpaying in all scenarios. For small budgets around 30% of the participants overpay. These buyers exceed their budget by around 200% on average and around 300% in the worst cases. Higher budgets ($\beta = 0.5$) lead to smaller yet substantial overpayment of 40% on average for 1 in 5 participants. Even for the highest budget class there are still overpaying buyers and albeit their overpayment is lower than in the other scenarios participants are still at a risk of exceeding their budgets. Bidding truthfully is thus potentially harmful to the solvency of an airline that is constraint by budgets.

#Flight groups	Budget	% Overpaying Participants	Avg. Overpayment (% of B_i)	Avg. Maximum Overp. (% of B_i)
12	0.2	29	221	314
12	0.5	21	46	64
12	0.8	4	8	8
25	0.2	31	202	293
25	0.5	22	41	56
25	0.8	4	5	5

Table 6: Overpayment with truthful bids

4.5 Inefficiencies due to Bid Shading

The preceding results show that bidders risk making significant losses when they cannot communicate their budget constraints and bid their true valuations. In order to avoid these risks, bidders have to bid strategically and shade their bids in order to avoid overpaying. Obviously, optimal bid shading is challenging. We analyse the effect of strategies where participants report bids in such a way that the sum of their submitted bids does not exceed their budgets. In the following, we refer to the classical auction scenario in which bidders only submit their capped bids as the capped market, and to the scenario where bidders are allowed to communicate their true valuations and budgets as the uncapped market.

We consider two strategies for the airlines in capped markets: In the first strategy, **BS** (bid shading), airlines submit bids in such a way that they shade all bids by their budget factor β, but continue to bid on all of their desired packages. In the second strategy, **BR** (bid reduction), airlines no longer bid on all of their desired flight groups. Instead, we greedily determine a subset $\tilde{g}_i \subseteq g_i$ of flight groups such that $\sum_{\tilde{g}_i} \max_{S \in g_i} v_i(S) \leq \beta M_i$.

We first discuss potential welfare losses that occur because of these bid shading strategies. The BR strategy leads to less welfare because as participants report bids for fewer connections, fewer connections are traded in the resulting allocation. First, we only consider the BS strategy in estimating the welfare loss. Then, we show that the resulting outcomes, while stable for the reported shaded bids, lead to instabilities when the real valuations and budgets of participants are taken into account.

Result 4 *The average welfare loss in secondary markets where bidders shade bids (BS) are between 6 and 34 percent. The worst-case efficiency loss is 40 percent.*

We use the set of instances introduced in Section 4.4, determining a welfare-maximal stable outcome for both the capped and uncapped market for each instance. We calculate the loss in welfare by comparing the sum of true (uncapped) payoffs achieved by the winners in the capped market to the sum of payoffs achieved by winners in the uncapped market. Table 7 summarizes the results, showing for each combination of treatment variables the percentage of the uncapped market's welfare that is achieved in the capped market. We report the average welfare over the ten instances as well as the worst and best welfare achieved in the capped market for these ten instances. The average welfare loss ranged from 6% to 34%. Lower budgets lead to higher welfare loss as buyers are more restricted in what valuations they can communicate. The number of flight groups did not influence the average welfare loss significantly. Note that the capped market never led to a fully efficient outcome. Even in the best instances the welfare loss was at least 4% and even around 30% in some cases.

#Flight groups	Budget	Avg Welfare	Worst Instance	Best Instance	std. dev.
12	0.2	0.66	0.60	0.71	0.04
12	0.5	0.82	0.78	0.84	0.02
12	0.8	0.94	0.93	0.95	0.01
25	0.2	0.64	0.61	0.70	0.03
25	0.5	0.81	0.78	0.84	0.02
25	0.8	0.94	0.93	0.95	0.01

Table 7: Welfare losses due to capped bidding (BS)

In addition to these substantial losses in welfare, the inability to communicate budgets can also lead to instabilities. An outcome that is stable in the capped market may not be stable when the true valuations of bidders are considered. To analyse this effect, we determined a welfare-maximal stable outcome in the capped market and then tested whether a blocking coalition forms when true valuations are considered. We consider the BS as well as the BR strategy for this analysis. We report for how many of the instances, a blocking coalition would form against the outcome of the capped market given the true valuations as well as how often reasonable small coalitions (of size up to 7 or 5) block the capped market's outcome. While it may be unrealistic for larger coalitions to form, these small coalitions constitute a significant risk to stability.

Result 5 *Outcomes assumed to be stable based on non-truthful bids (BS or BR) were always unstable considering the true valuations in our experiments. While they are mostly stable against small coalitions of size 5, outcomes are unstable most of the time in the presence of tight financial constraints.*

Table 8 summarizes the results for the same instances as in the experiments outlined above. The amount of budget is crucial when it comes to the stability of capped markets with a BS strategy. When budgets are low even small coalitions of size 5 can block almost all outcomes in the uncapped market. Regardless of the budget amount all computed solutions are unstable when considering real valuations, even for smaller instances with only 12 flight groups. Determining allocations and prices based on capped valuations thus seem impractical in general as bidders almost always have an incentive to deviate from the outcome.

#Flight groups	Budget	Instances with blocking coalitions		
		in general	of size up to 7	of size up to 5
12	0.2	10	10	10
12	0.5	10	10	0
12	0.8	10	8	0
25	0.2	10	10	8
25	0.5	10	10	0
25	0.8	10	4	0

Table 8: Instability due to capped bidding with (BS) strategy

In Table 9 we report results for secondary markets where buyers implement a BR strategy. Evidently, low budgets lead to solutions that are unstable even when considering only small coalitions. This is especially true for instances with low budgets and 12 flight groups which are unstable against coalitions of size 5 in all cases, but also instances with a higher budget ($|G_i| = 12, \beta = 0.5$) or more flight groups ($|G_i| = 25, \beta = 0.2$) can suffer from deviation by small coalitions. Again, no overall stable solution was found.

5 Conclusion

Airport time slots are a widely adapted legislative answer to growing air traffic and congested airports. Primary markets are not taking place as of now and while secondary trading of time slots is already in place its current implementation is limited to bilateral trades. The large number of connections requires an adequate bid language. Capacity constraints need to be taken into account,

#Flight groups	Budget	Instances with blocking coalitions		
		in general	of size up to 7	of size up to 5
12	0.2	10	10	10
12	0.5	10	10	7
12	0.8	10	7	0
25	0.2	10	10	9
25	0.5	10	10	0
25	0.8	10	10	0

Table 9: Instability due to capped bidding with (BR) strategy

but also package bids, and budget constraints of airlines. The latter incurs significant computational complexity and the resulting allocation and pricing problem is Σ_2^p-hard. If airlines are not allowed to express budget constraints adequately, this leads to significant strategic complexities, because airlines need to submit bids such that their budgets are not exceeded without knowing the market prices. Unfortunately, Σ_2^p-hard computational problems were considered intractable until recently, even for small problem instances. Algorithms to solve bilevel mixed-integer programs are in their infancy.

In this paper, we introduced a model and algorithms for the trading of airport time slots in primary and secondary markets. The market design maximizes welfare subject to core-constraints and budget constraints. We introduce new computational techniques to solve the resulting bilevel integer programs, and report results of experiments.

Our experiments show that ignoring budgets leads to substantial welfare losses. Besides, outcomes determined with capped bids lead to substantial instabilities and airlines could form blocking coalitions considering their true valuations. Overall, a mechanism that considers budget and core-constraints makes bidding for airlines simple and achieves significantly higher revenue. Interestingly, we can solve small but realistic problem sizes with 10 buyers and sellers and up to 2000 time slots. If we limit our attention to deviating coalitions of restricted size, we can even solve problem sizes with many more participants. For real-world airport time slot auctions, it might be unrealistic to assume very large deviating coalitions, and n-core stability could provide a sufficient level of stability. With further algorithmic advances, this technology can provide a solution to a notoriously challenging market design problem.

CRediT author statement

Martin Bichler: Resources, Writing - Review & Editing, Project Administration, Funding acquisition, **Richard Littmann:** Software, Validation, Formal Analysis, Investigation, **Stefan Waldherr:** Methodology, Writing - Original Draft, Visualization, Supervision

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