A Principal-Agent Model of Bidding Firms in Multi-Unit Auctions

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Abstract. Principal-agent relationships between the supervisory board and the management of bidding firms are widespread in spectrum auctions, but they can also be observed in other multi-object markets. The management aims at winning the highest valued licenses whereas the board wants to maximize profit and limits exposure in the auction. In environments in which it is efficient for firms to coordinate on allocations with multiple winners, we show that the principals would coordinate on smaller sets of objects while the agents would not and inflate their demand to larger sets. We first analyze multi-unit markets in which principal and agent have complete information about the valuations, and show that it can be impossible for the principal to implement her equilibrium strategy with only budget constraints in a first-price sealed-bid package auction. With hidden information about the valuations a principal would need to determine contingent budgets and transfers to compensate the agent, which can be impossible with value-maximizing motives of the agent. Contrary, in an ascending package auction this is straightforward even without knowledge of valuations as long as the principals know the efficiency environment. The second-price payment rule of the ascending auction enables the principal to overcome the agency problem more easily and the dynamic mechanism further facilitates coordination for the principals.

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1 Introduction

Agency problems in bidding teams are pervasive in many auction markets. For example, it is well-known that the relationship between the supervisory board (principal) and the management (agent) of a telecom play an important role in determining the firm's bidding strategy in spectrum auctions (Chakravorti et al. 1995; Shapiro et al. 2013).Such principal-agent relationships have been put forward as one possible cause of allocative inefficiencies in spectrum auctions (Schmidt 2005). In this paper, we introduce a principal-agent model of a bidding firm in which the principal provides the agent with an upper limit on the amount to be spent in an auction (budget constraint). This model helps explain inefficient outcomes in multi-object auctions that are characterized by bidders inflating their demand to larger sets of objects (demand inflation) instead of efficiently coordinating on smaller sets. One of our central results is that principals might be more likely to overcome the agency problem if second-price auction formats are adopted, in particular ascending mechanisms.

Our motivation is a wide-spread hidden information problem in auctions in which the agent knows the valuation of different objects or packages, but the principal does not. The agent has limited liability and the principal determines upper limits on exposure in the auction. Any residual money is re-invested in the firm and the agent is unlikely to be induced to maximize profit. In this environment the agent tries to maximize the value of objects won given the budget constraint. Let us motivate our model by looking at spectrum auctions that are an important economic activity (generating billions of dollars worldwide) and that have been a catalyst for theoretical research in auctions. For example, principal-agent relationships arise between the management of a multinational telecom and the management of a national subsidiary bidding in the auction. In spectrum auctions, firms have preferences over different packages of spectrum licenses. Each of these packages can be assigned a business case with a net present value. The management knows the market best, it knows the technology, the competition, and the end consumer market, and so they can compute business cases that allow for a good estimate of the net present value of each package. The board of directors does not have this information, and the management has no incentive to reveal it truthfully. Principals often need to rely on analyst estimates that typically have an enormous variance.[2] The principal will also not learn the true valuations of the licenses after the auction, as the future profits of the firm depend on many other decisions.

The payments made by telecommunication firms in spectrum auctions are often billions of dollars, and thus the management cannot cover the cost of the auction. This means, the agent in these markets has limited liability and the principal has to pay in the auction. The budgets that need to be reserved for such an auction by the board are also such that they cannot just be transferred in total to the agent in order to induce a profit-maximizing motive. The residual budget after the auction might be in the billions, and there can be much more efficient investments elsewhere. The different incentives of principal and agent are nicely summarized in a report by a consulting firm in this field (Friend 2015):

> *"The amount of money spent by mobile operators at auction is often staggering. The money needed to pay for spectrum cannot usually be funded from the agreed capital expenditure budget of the business. As a result, spectrum payments are usually treated as a separate amount that does not impact the Key Performance Indicators of the business upon which the management team's bonuses are often based. However, the management team of a mobile business usually prefer to have more spectrum rather than less. The Chief Marketing Officer prefers more spectrum than his competitors as it allows him to advertise a bigger, faster and better network which helps him achieve his sales target. The Chief Technical Officer prefers more spectrum as it means he needs to build fewer sites to provide the same capacity and helps him achieve his capex to sales targets. The CEO is happy because the business is hitting its targets. So the management team will typically prefer to win more rather than less spectrum at auction."*

Empire-building motives are a widespread reason for such value-maximizing behavior of agents in the principal-agent literature (Jensen 1986). Note that spectrum auctions are only one example of value maximizing agents. Engelbrecht-Wiggans (1987) writes "… in bidding for mineral leases, a firm may wish to maximize expected profits while its bidder feels it should maximize the firm's proven reserves." In addition, he discusses auctions for defense systems and construction contracts. Payoff maximization is hard to defend for an agent in such relationships, and agents typically try to "win within budget." In contrast, *value maximization* is a good approximation of such agent motives. In this setting it is important to understand the impact of the agent's bias on the firm's bidding behavior and means for the principal to implement on optimal strategy.

1.1 Contributions and Outline

We introduce a principal-agent model of firms participating in a multi-unit auction. In each firm an agent bids on behalf of the principal. The agent has hidden information about the valuations of the goods and wants to win his value-maximizing allocation whereas the uninformed principal aims at maximizing profit and determines budget constraints to restrict the agent's bids.

This principal-agent problem can be seen as a form of optimal delegation in which an uninformed principal delegates decision rights to an informed but biased agent. Holmström (1977) and Alonso and Matouschek (2008) showed that the optimal mechanism for the principal when utility is not transferable consists of choosing a subset of actions, from among which the agent is allowed to pick the most desired one. A budget constraint that cannot be overbid corresponds to a restriction on the actions of the bidding agent. Budget constraints are widely used as a means to discipline the bidding agent in spectrum auctions and other high-stakes auctions (Engelbrecht-Wiggans 1987; Shapiro et al. 2013).

Whether the principal can implement an optimal bidding strategy with such a constraint crucially depends on the level of information she has about the valuations. We show that even if the principal had full information, she could not always optimally align the agent's incentives via budget constraints only. Therefore, we also analyze a situation in which the principal can pay contingent transfers to the agent to optimally incentive align the agent in an asymmetric information environment. Principal-agent relationships of bidding teams in auctions have only recently become a topic of interest in auction theory, but prior models focus on single object auctions (Burkett 2015; Burkett 2016).[3] We demonstrate that multi-object auctions are characterized by additional strategic considerations on which the agency problem has an impact that do not appear in single-object auctions.

First, we describe the environment formally as the *principal-agent* 2×2 *package auction model* in which 2 firms compete for 2 units of a homogeneous good (perfect substitutes) in Section 2.[4] The 2×2 *model* captures the central strategic challenge that can also arise in larger markets and provide examples in the conclusions. We primarily focus on the first-price sealed-bid (FPSB) and the ascending package auctions in our analysis due to their relevance in spectrum auctions.[5] However, in Section 4.2 we also discuss traditional (non-package) Simultaneous Multiple-Round Auctions (SMRA) that have frequently been employed to sell spectrum (Milgrom 2000). It is still a topic of debate among regulators which auction format to use. Ideally, a mechanism would be strategy-proof and welfare-maximizing for the agent and the principal. Unfortunately, there is no strategy-proof mechanism for value-maximizing agents (Fadaei and Bichler 2016), and therefore there cannot be a strategy-proof mechanism for both. We briefly illustrate this result in our 2×2 *model* for the Vickrey-Clarke-Groves (VCG) mechanism (Clarke 1971; Groves 1973; Vickrey 1961) in Section 2.2.

We are limiting our attention to markets in which it is efficient to have two winners (*dual-winner outcome*). This environment is called *dual-winner efficiency* and has been introduced and motivated for a procurement setting by Anton and Yao (1992). This context enables us to demonstrate strategic difficulties that can arise with principal-agent relationships in bidding firms as principals need to coordinate in the efficient equilibrium and there is a conflict of interest with the agent. Moreover, this limited setting actually allows us to derive the principal's Bayesian Nash equilibrium strategies as one can only analyze specific environments game-theoretically. The role of this model is similar to that of the stylized local-local-global (LLG) model that is often used to analyze threshold problems in combinatorial auctions (Ausubel and Baranov 2018; Goeree and Lien 2016).

Finally, the described setting models an interesting market environment that is also relevant to business practice. For example, in a spectrum auction two bidders might be interested in multiple homogeneous licenses in a band and it is the efficient solution for bidders to split the spectrum. Interestingly, one can often observe demand inflation in such situations in the field even though payoff-maximizing bidders would reduce demand from the start. Note here, that regulators tend to be legally bound to aim for efficiency as noted, for example, for the spectrum auction to sell personal communications services (PCS) licenses in 1994 by the US Federal Communications Commission (FCC) (McMillan 1994).

In Section 3, we focus on the FPSB package auction and analyze the agency problem that arises. First, in Section 3.1 we analyze the equilibrium bidding strategies of profit-maximizing principals in case they had full information about the valuations of the firm and were participating in the auction without their agents. Bayesian Nash equilibrium analysis of multi-object auctions has turned out challenging as, for example, in case of the SMRA for (asymmetric) bidders with synergies in their valuations (Goeree and Lien 2014; Krishna and Rosenthal 1996). Similar to Anton and Yao (1992) we assume prior information about the efficiency environment, i.e. *dual-winner efficiency*, and show that coordination on the efficient outcome constitutes an efficient Bayesian Nash equilibrium (*dual-winner equilibrium*) for the principals in a FPSB package auction in this environment. Under reasonable distributional assumptions this equilibrium payoff-dominates a *single-winner equilibrium*. Contrary to Anton and Yao (1992) the proofs are independent of a publicly known efficiency parameter and involve two-dimensional package valuations. Moreover, in Section 4.1 we extend these results to the ascending package auction format.

In Section 3.2 we analyze strategies of the agents if they were to participate in the auction without their principals, but with a budget constraint for one and two units. In this section, the constraints are considered exogenous and drawn from a random distribution. We show that it is the unique Bayesian Nash equilibrium for the agent not to bid on a single unit but only on the package of two units. The analysis highlights the bias of the agent and how his equilibrium bidding strategy differs from that of the principal leading to inefficiency. The same behavior continues to be optimal for the agents in the ascending package auction as shown in Section 4.1.2. We focus on the implementation of the principal's original efficient equilibrium using budget constraints in the symmetric information environment in Section 3.3. We show that even if the principal has complete information, there cannot always be budget constraints that set the right incentives for the agents and at the same time constitute the equilibrium for the principal in the FPSB package auction.

In Section 4, we extend our analysis to the ascending package auction and the SMRA. The principals' *dual-winner equilibrium* of not bidding on the package of two units at all is reminiscent of demand reduction equilibria in (ascending) uniform-price multi-unit auctions in (Ausubel et al. 2014) and in particular to (Engelbrecht-Wiggans and Kahn 1998) in which equilibrium-conditions for a zero-bid on the second unit are established. This bidding behavior is in stark contrast to the agents' equilibrium strategy which involves demand-inflation of bidding on two units only. However, the ascending auctions have advantages over the FPSB package auction in our model if the principal knows the efficiency environment in the market. In this case, she could set the budget for the package to null and implement her equilibrium strategy.

Interestingly, for the ascending package auction this advantage is a feature of the second-price payment rule which we demonstrate by establishing outcome equivalence between the ascending package auction and the VCG mechanism with respect to solving the agency dilemma in the *principal-agent* 2×2 *package auction model*. Note that a principal would still need to know that the outcome with two winners is efficient, which is often not given in the field. However, as an additional advantage over the FPSB package auction the ascending mechanism allows bidders to observe their opponents strategies and adjust accordingly. For profit-maximizing bidders this reduces the burden of starting to coordinate on the *dual-winner outcome* as the strategy can be altered anytime to pursue the *single-winner outcome* if the opponent does not cooperate. To the best of our knowledge this is the first theoretical paper to demonstrate the advantage of an ascending package auction to coordinate on efficient allocations. We also discuss how our model helps explain more complex markets with more bidders and items.

Finally, in Section 5 we study the implementation of an optimal ("second-best") contract with menus of contingent wages and budget constraints in the asymmetric information setting. We focus on a strategically interesting situation in which there is uncertainty about the package valuation as well as their corresponding ranges, but not about the efficiency environment. With knowledge about the efficiency in the market the principal can align the incentives of the agent by paying wages that compensate the agent for not aiming to win the package of two units in the FPSB package auction. However, these wages can be very high making it impossible to implement the coordination equilibrium. Similar to the symmetric information environment, in the ascending package auction, the principal would only need to set the budget for two units to zero to implement her strategy.

2 The Model

In our model we consider 2 ex-ante symmetric firms $i, j \in I$ with $|I| = 2$, competing in a multi-object package auction for 2 units of a homogeneous good in which the number of units is denoted by $l \in L = \{1, 2\}$. In each firm i, a value-maximizing and budget-constrained agent bids in the auction on behalf of a profit-maximizing principal. The auction itself corresponds to a game of incomplete information between firms and the risk-neutral auctioneer selects the revenue maximizing set of packages. We denote this setting as *principal-agent* 2×2 *package auction model*.

2.1 Bidding Firms

Each firm $i \in I$ has a value for l units of $v_i(l) \in V(l) = \left[\underline{v}(l), \overline{v}(l)\right]$ and we define the vector of package values as $v_i = \left(v_i(1), v_i(2)\right) \in V = V(1) \times V(2)$. Let us normalize the reservation utility $v_i(0) = 0$ and assume $v_i(1) < v_i(2)$ for all $i \in I$. Note that the assumption implies $\overline{v}(2) > \overline{v}(1)$ and $\underline{v}(2) > \underline{v}(1)$. Moreover, we are considering a model with decreasing marginal values and an extra assumption that the highest marginal value of the second unit is less than the lowest possible value for the first unit, i.e. $\overline{v}(2) - \underline{v}(1) < \underline{v}(1)$. We refer to this environment as *dual-winner efficiency*, as it is efficient for both firms to obtain one unit each, independent of their package valuation draws. The condition $2 \cdot \underline{v}(1) > \overline{v}(2)$ ensures *dual-winner efficiency* for all $v_i, v_j \in V$ and we also assume strictly separated package-valuation ranges, $\overline{v}(1) < \underline{v}(2)$.

Given the standard symmetry assumption, each firm i's vector of valuation draws v_i is a priori distributed according to a monotonically increasing joint cumulative distribution function $\mathrm{F}(v_i)|_{v_i(1)<v_i(2)}$ with $\mathrm{F}|_{V(1)<V(2)}: V \to [0,1]$. The marginal distribution function for package value $v_i(l) \in V(l) \; \forall \; l \in L$ is of the form $F_l\big(v_i(l)\big)$ with $F_l: V(l) \to [0,1]$ and strictly positive density function $f_l\big(v_i(l)\big) > 0$. We assume the distribution functions $\mathrm{F}(\cdot)|_{V(1)<V(2)}$, $F_l(\cdot)$ and $f_l(\cdot)$, to be common knowledge within a firm i and between both firms i and j.

The risk-neutral principal maximizes expected profit in which her profit of winning a package of l units is given by $\pi_i(l) = v_i(l) - p_i(l)$. In this expression $p_i(l)$ denotes the final price paid for a package of l units in the auction and depends on the auction format. Let the bid submitted in a sealed-bid auction (or the highest price accepted in an ascending auction) for the package of l units be $\beta_i(l) = \beta(v_i(l))$ and $\beta: V(l) \to \mathbb{R} \; \forall \; l \in L$ are $F_l(\cdot)$-measurable functions. Let us define the vector of package bids as $\beta_i = \big(\beta_i(1), \beta_i(2)\big)$. The agent's gross utility includes his value-maximizing motives and is denoted as $u_i(l) = w\big(v_i(l)\big)$ when winning a package of l units. The function $w: V(l) \to \mathbb{R}$ is strictly increasing in package value $v_i(l)$ and thus, an agent always prefers winning two units to one unit. Moreover, the function $w(\cdot)$ is assumed to be commonly known among both firms and within each firm i, which is a standard assumption in contract theory and important for the derivation of optimal transfers in Section 5.

In the symmetric information setting in Section 3 and 4 the principal determines a budget constraint for the package of l units of $\alpha_i(l) \in A(l) = \big[\underline{\alpha}(l), \overline{\alpha}(l)\big]$ with which to provide her agent to bid in the auction. We refer to the vector of all package budget constraints as $\alpha_i = \big(\alpha_i(1), \alpha_i(2)\big)$ with $\alpha_i \in A = A(1) \times A(2)$. As long as the price for a bundle of l units is weakly lower than his respective budget constraint of $\alpha_i(l)$, the agent obtains a utility of $u_i(l)$. Any price $\beta_i(l) > \alpha_i(l)$ is an unacceptable action for him, as he will be fired, for example, if payments exceed the budget constraint.

In our initial analysis of the agents' strategies without the principals in Section 3.2 (and 4.1.2), we assume that $\alpha_i(l)$ is an exogenous random variable to demonstrate the bias of the agent with respect to the equilibrium bidding strategy of the principal. Each agent $i \in I$ draws a budget for one unit of $\alpha_i(1) \in A(1)$ and a package budget for two units of $\alpha_i(2) \in A(2)$. We assume $\alpha_i(1) \leq \alpha_i(2)$ for all $i \in I$. Moreover, each agent i's vector of budget draws α_i is a priori distributed according to a monotonically increasing joint cumulative distribution function $Q(\alpha_i)|_{\alpha_i(1) \leq \alpha_i(2)}: A \to [0, 1]$ with respective marginal distribution function for budget $\alpha_i(l) \in A(l)\, \forall\, l \in L$ of $Q_l\big(\alpha_i(l)\big): A(l) \to [0, 1]$. The distribution functions $Q(\cdot)|_{\alpha_i(1) \leq \alpha_i(2)}$ and $Q_l(\cdot)$, are assumed to be common knowledge between agent i and j.

In Section 3.3 (and 4.1.3), we will remove the assumption of exogenously determined budgets and ask whether an informed principal can endogenously implement her equilibrium bidding strategy by providing the agent with budget constraints, α_i. In this setting we will denote the commonly known joint and marginal probability distribution functions of the budget constraints as $D(\alpha_i): A \to [0, 1]$ and $D_l\big(\alpha_i(l)\big): A(l) \to [0, 1]$, respectively. The *principal-agent package auction model* is illustrated in **Figure 1** below.

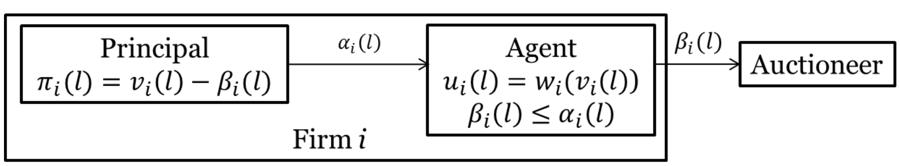

Figure 1 Illustration of the *principal-agent package auction model*

First, each principal decides on a vector of budget constraints, α_i, with which to provide her agent. Then, agents compete against each other and decide on a vector of bids, β_i, in the auction. The principal has to choose the budget constraints to implement the optimal *dual-winner equilibrium*.

In the asymmetric information setting in Section 5 the principal knows the *dual-winner efficiency* environment but she is not informed about the package value draws and their exact ranges. However, we assume that she knows bounds for these supports. In particular, we model these bounds as a random draw $d \in D = [\underline{d}, \overline{d}]$ that specifies $d \equiv \overline{v}(2) - \underline{v}(1)$ that is unknown to the principal. Note that d corresponds to the true entire range out of which valuations for one unit and the package can be drawn. The principal does know the support D and that the market satisfies *dual-winner efficiency* such that $2 \cdot \underline{v}(1) > \overline{v}(2)$. In addition to the vector of package constraints, α_i, the principal employs a vector of payments for the agent in the asymmetric setting, $m_i = (m_i(1), m_i(2))$, with payment for the package of l units of $m_i(l)$. A principal i has to design a menu of budget constraints and payments, (α_i, m_i), to implement a *dual-winner equilibrium*.

2.2 Auction Formats

It is straightforward to see that the VCG mechanism is not incentive-compatible for agents, who do not internalize payments in their utility function and, moreover, that there cannot be a strategy-proof and deterministic mechanism for value-maximizing agents (Fadaei and Bichler 2016). Therefore, we focus on the practically relevant 2×2 *FPSB package auction*, the 2×2 *ascending package auction* and the 2×2 *SMRA* with 2 units of a homogeneous good and 2 bidders. Budgets in the VCG mechanism for the *principal-agent* 2×2 *package auction model* can optimally be set as in the ascending package auction with sufficient information about the efficient allocation as we show in Section 4.2. Therefore, the VCG mechanism and the ascending package auction are outcome equivalent with respect to the agency dilemma. Nevertheless, we focus on the ascending format as it is practically relevant.

In multi-unit package auctions, each bidder $i \in I$ submits an all-or-nothing bid for every package. Here, each package is identified by the number of units, $l \in L$, it contains, i.e. a package of two units and a package of one unit. In these package auctions, we assume an XOR bid language because it is the most general bid language allowing the expression of complements and substitutes (Nisan 2006), and it is also typically used in spectrum auctions. An XOR bid language allows a bidder to specify a bid for all possible packages, but only one of the bids submitted can become winning. This avoids problems in which a bidder might win multiple non-overlapping bids for which her value is less than the sum of the winning package values.

Before the auction starts, bidders decide whether they participate or not and make this decision public. In the 2×2 *FPSB package auction*, both bidders simultaneously submit their bids $\beta_i = \big(\beta_i(1), \beta_i(2)\big)$ to the auctioneer without knowing the opponent's bids. Then, a risk-neutral auctioneer selects the revenue-maximizing combination of package bids. This can either be an allocation in which each bidder i and j gets one unit, $\beta_i(1) + \beta_j(1) \geq \max\{\beta_i(2), \beta_j(2)\}$, or an allocation in which one bidder wins the package of both units, $\max\big\{\beta_i(2), \beta_j(2)\big\} > \beta_i(1) + \beta_j(1)$. In case of a tie between an allocation with two winners or a single winner, the auctioneer allocates one unit to each of the bidders. In case of a tie between two package bids, the auctioneer randomizes. Any firm i that wins a package of l units pays a corresponding price of $p_i(l) = \beta_i(l)$ to the auctioneer.

We also study a multi-unit clock auction with package bidding. In this 2×2 *ascending package auction*, two clocks are visible to each bidder and they are used to indicate a personalized single-unit price $p_i(1)$ of the items and a personalized package price $p_i(2)$ for all $i \in I$. Bidders only see their personalized clock prices that start at zero and then increase gradually. A participating bidder needs to bid at least on one unit at the start and once she stops bidding on the single unit and on the package she cannot enter the auction again. Each active bidder can either press the button for a package bid or for a single unit or both to start the respective clock(s). A bidder i releasing the button at price $p_i(l) = \beta_i(l)$ indicates her decision to stop demanding the package of l units at any price higher than $\beta_i(l)$. The auction stops as soon as there is no over-demand anymore, and the revenue-maximizing allocation is determined on the basis of the reported prices.

In addition, we compare our results to the 2×2 *SMRA*. We use an ascending uniform-price multi-unit auction as an abstraction for a multi-unit SMRA, similar to Ausubel et al. (2014). For perfect substitutes both auction formats are identical. In this auction, there is a single uniform-price *p* which starts at zero and the auction stops when demand no longer exceeds supply. There are also two buttons similar to the ascending package auction, but no package bidding is allowed. Each button is for one unit and a bidder can first press both buttons and then release one or even both of the buttons simultaneously if she wants to stop bidding on one of the units or both units, respectively. She can also just press one button. Appendix II provides a detailed discussion of the rules of the ascending package auction and the ascending uniform-price auction and their outcomes.

3 The First-Price Sealed-Bid Package Auction

In this section, we analyze the equilibrium bidding strategy of the principal if she had full information about the valuations in the FPSB package auction. Then, we analyze equilibrium bidding strategies of agents (without principals) and show the bias of the agent with respect to the principal's equilibrium bidding strategy and the resulting inefficiencies. Finally, we examine the principal's options to incentive align the agent with budget constraints in the symmetric information environment.

3.1 Principals' Strategies

Let us first analyze how a profit-maximizing principal would bid in equilibrium without hidden information and without an agent. We will first derive necessary and sufficient conditions for the *dual-winner equilibrium* in **Theorems 1** and **2**, respectively. This is a Bayesian Nash equilibrium in which the efficient *dual-winner outcome* results for all possible valuations $v_i, v_j \in V$ in the 2×2 *FPSB package auction* in case of *dual-winner efficiency*. **Theorem 3** states conditions under which the *single-winner* outcome constitutes a *single-winner equilibrium* for all possible valuations $v_i, v_j \in V$ in *dual-winner efficiency*. This equilibrium is just an adaptation of the standard Bayesian Nash equilibrium in a FPSB single-object auction in which bidders bid only on the package. Finally, **Theorem 4** establishes conditions for the efficient *dual-winner equilibrium* to be payoff-dominant. We will first start with the necessary conditions for the *dual-winner equilibrium*.

Theorem 1: *Suppose any bidder i's bidding strategy $\beta_i = \big(\beta_i(1), \beta_i(2)\big)$ constitutes a dual-winner equilibrium in the 2×2 FPSB package auction. Then it must be true that:*

(1.) $\beta_i(1)$ is constant over $v_i(1) \in V(1)$ and denoted by $\beta_i(1) = \beta(1)$ for all $i \in I$

(2.) $\beta(1) \in [\overline{v}(2) - \underline{v}(1), \underline{v}(1)]$

(3.) $2 \cdot \beta(1) = sup_{v_i(2)}\{\beta\big(v_i(2)\big)\}$.

In the *dual-winner equilibrium* both bidders must pool at a constant single-unit bid of $\beta(1)$ from condition (1.) out of its range from condition (2.). Moreover, the single-unit bid bounds in condition (2.) show that *dual-winner efficiency* is necessary for the existence of a *dual-winner equilibrium.* Condition (3.) ensures that the auctioneer always selects the *dual-winner outcome* in equilibrium. Let us now derive the sufficient conditions for the *dual-winner equilibrium* in the next theorem.

Theorem 2: *Assume dual-winner efficiency is given, $\overline{v}(2) < 2 \cdot \underline{v}(1)$, then any principal i's vector of bids $\beta_i = \big(\beta(1), \beta_i(2)\big)$ is a dual-winner equilibrium in the 2×2 FPSB package auction if the following conditions hold:*

(1.) $\beta(1) \in [\overline{v}(2) - \underline{v}(1), \underline{v}(1)]$

(2.) $\beta_i(2) = \beta(v_i(2))$ with $\beta(.)$ continuous and strictly increasing on its support $V(2)$

(3.) $\beta\big(\overline{v}(2)\big) = 2 \cdot \beta(1)$

(4.) $G\big(v_i(2), \beta(1)\big) \leq \beta\big(v_i(2)\big)$ for all $v_i(2) \in V(2)$ and all $i \in I$

The lower bound $G(.)$ is defined as:

$$G\big(v_i(2), \beta(1)\big) \equiv \beta(1) + \frac{\beta(1) - \underline{v}(1) \cdot \big(1 - F_2(v_i(2)\big)}{F_2\big(v_i(2)\big)}$$

Conditions (2.) and (4.) restrain any bidder i's equilibrium bidding function for two units. It is not allowed to fall below the lower bound of $G\big(v_i(2), \beta(1)\big)$ in order to support the pooling bid for one unit. This condition ensures that winning the package is less profitable in expectation than obtaining a single unit in equilibrium. For our analysis we focus on the lowest pooling bid for one unit of $\beta(1) = \overline{v}(2) - \underline{v}(1)$. This maximizes the utility of both bidders and therefore serves as a natural focal point for implicit coordination in the *dual-winner equilibrium*.

Let us introduce a brief example to illustrate the *dual-winner equilibrium*. Suppose $v_i(1) \in [110{,}150]$ and $v_i(2) \in [160{,}190]$, both uniformly distributed. The payoff-dominant pooling bid in the *dual-winner equilibrium* for both bidders is $\beta(1) = \overline{v}(2) - \underline{v}(1) = 190 - 110 = 80$. The upper bound for $\beta_i(2)$ is 160. A higher bid would make the auctioneer select the package bid. Remember, in case of a tie, the auctioneer selects the *dual-winner outcome*. The bidder with the lowest type on a single unit, $v_i(1) = 110$, and the highest type on the package, $v_i(2) = 190$, has the strongest incentive to deviate. With the equilibrium bids $\beta_i = (80, 160)$, her payoff for one unit and for the package is 30. The lower bound for $\beta_i(2)$ is defined by a function $G(v_i(2), 80)$. For a low value draw of $v_i(2) = 165$ the lower bound is $G(165{,}80) = 10$, and an equilibrium bid for the package is $\beta_i(2) \in [10{,}160]$. Each bidder type has to follow this lower bound function in equilibrium to ensure the opponent has no incentive to profitable deviate on the *single-winner award*, i.e. the payoff in the *dual-winner equilibrium* of 30 for the lowest type on one unit always needs to be higher or equal to the expected payoff of the package bid. In addition, suppose the opponent *j* bids low, say zero, on the package. Then bidder *i* could also bid low on the package in an attempt to be able to bid $\beta_i(1) < 80$ on a single object and make a higher profit on one unit in a *dual-winner* outcome. The lower bound $G(v_i(2), 80)$ avoids such deviations from equilibrium, too.

The *dual-winner equilibrium* is not the only equilibrium for payoff-maximizing principals in our model, and there is also a *single-winner equilibrium*.

Theorem 3: *Any principal i's vector of bids β_i is a single-winner equilibrium in the 2×2 FPSB package auction under dual-winner efficiency if the following conditions hold:*

(1.) $\beta_i(2) = v_i(2) - F_2(v_i(2))^{-1} \cdot \int_{\underline{v}(2)}^{v_i(2)} F_2\left(v_j(2)\right) \cdot dv_j(2)$

(2.) $\beta_i(1) \in [0, \underline{v}(2) - \overline{v}(1))$

The equilibrium bid on the package in the *single-winner equilibrium* of the 2×2 *FPSB package auction* from condition (1.) corresponds to the equilibrium strategy of the well-known standard FPSB auction in which two units are sold as the sole package to two bidders. Condition (2.) ensures any bidder can enforce the *single-winner equilibrium* by making the *dual-winner outcome* unprofitable for the opponent. In our earlier example with the uniform distribution, this equilibrium bid on the package would simply be $\beta_i(2) = \frac{v_i(2)}{2} + 80$. The equilibrium bid on one unit must be low enough to veto the *dual-winner outcome* for all possible bidder types, i.e. $\beta_i(1) \in [0,10)$. If principal i with type $v_i = (130,165)$ bids $\beta_i(1) = 9$ and $\beta_i(2) = 162.5$, even opponent j with the highest value draw for one unit, $v_j(1) = 150$, cannot profitably implement the *dual-winner outcome* as the sum of single-unit bids is strictly smaller than the lowest equilibrium bid on two units, i.e. $150 + 9 < 160$.

Using payoff-dominance, we can show that for specific distributional properties, payoff-maximizing bidders prefer to coordinate in the *dual-winner equilibrium* rather than select the *single-winner equilibrium*.

Theorem 4: *Any principal i prefers the dual-winner equilibrium to the single-winner equilibrium in the dual-winner efficiency environment of the 2×2 FPSB package auction iff the expected value of two units exceeds two times the bidder-optimal dual-winner equilibrium pooling bid $\beta_i(1) = \overline{v}(2) - \underline{v}(1)$,* i.e. $2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) < E\left(v_j(2)\right)$.

Intuitively, the expected package valuation exceeds twice the pooling bid if the probability for large package value draws is high. If high value draws for the package of two units are likely, however, any bidder prefers the *dual-winner equilibrium* to the *single-winner equilibrium* because she is likely to lose in the latter equilibrium. In our earlier example, the condition is satisfied as $2 \cdot 80 < 175$. In this case the principal prefers the *dual-winner equilibrium* as it yields higher expected payoff. Let us now turn to the analysis of the agents' equilibrium strategies in the 2×2 *FPSB package auction*.

3.2 Agents' Strategies

Now, we analyze the strategies of agents assuming the budget constraints to be random variables. This assumption is sufficient to highlight the bias of the agent and the resulting inefficiency of the auction. In Section 3.3 we will then analyze if a principal can set budget constraints such that the agents implement her equilibrium bidding strategy. Let us first provide a few useful lemmas for the 2×2 *FPSB package auction* that eliminate weakly dominated strategies. These lemma allows for a succinct analysis of equilibrium bidding strategies.

Lemma 1: *Any strategy in which an agent bids strictly less than his budget for two units,* $\hat{\beta}_i(2) < \alpha_i(2)$, *is weakly dominated, independent of his bid on one unit in the* 2×2 *FPSB package auction.*

As agents are value-maximizing and strictly prefer the package to one unit, there is no reason for them to not fully bid their budget on two units.

Lemma 2: *The set of strategies* $\hat{\beta}_i = \left(\hat{\beta}_i(1), \alpha_i(2)\right)$ *with* $\hat{\beta}_i(1) \in (0, \alpha_i(1))$ *is weakly dominated by the strategy-set* $\beta_i = (\beta_i(1), \alpha_i(2))$ *with* $\beta_i(1) \in \{0, \alpha_i(1)\}$ *in the* 2×2 *FPSB package auction.*

A value-maximizing agent either expects his budget for the package to be high enough to win the package, and he vetoes the *dual-winner outcome* with a bid of zero on one unit, or he beliefs coordination on the *dual-winner outcome* allows him to win at least one unit, then he maximizes his chances by bidding his full budget for one unit. Bids in the interval $\beta_i(1) \in \left(0, \alpha_i(1)\right)$ are always weakly dominated.

With **Lemma 1** and **2**, we can now derive the agents' equilibrium strategies in the 2×2 *FPSB package auction*. Our first observation is an ex-post equilibrium in which agents do not coordinate.

Theorem 5: *It is an ex-post equilibrium for any agent i to submit a vector of bids $\beta_i = (0, \alpha_i(2))$ in the 2×2 FPSB package auction for any vector of package budgets $\alpha_i \in A$.*

There could be other Bayesian Nash equilibria. Interestingly, $\beta_i = (0, \alpha_i(2))$ is also the unique Bayesian Nash equilibrium as we will show next, i.e. there is no other Bayesian Nash equilibrium by the bidding agents. Intuitively, any agent i's opponent j would only be willing to coordinate on one unit if his valuation for two packages was low. In this case, however, it would be a best response for bidder i not to coordinate, but try to win the package of both units independent of both agents' actual values. In the ex post equilibrium arbitrary risk-averse bidders cannot coordinate on winning one unit with certainty. [6]

Theorem 6: *The strategy $\beta_i = (0, \alpha_i(2))$ is the unique Bayesian Nash equilibrium for any agent i in the 2×2 FPSB package auction for any vector of package budgets $\alpha_i \in A$.*

Note that the agent does not respond to the valuations of the firm v_i, but only to the budgets α_i that he is given as long as $v_i(1) < v_i(2)$ and $\alpha_i(1) \leq \alpha_i(2)$. Therefore, the agent's unique equilibrium bidding strategy is independent of the efficiency environment considered. More importantly, the unique equilibrium of the agent in the 2×2 *FPSB package auction* is in conflict with the efficient *dual-winner equilibrium* of the principal. In the next Section 3.3 we analyze the possibilities of the principal to implement her *dual-winner equilibrium* bidding strategy by constraining the agent with budgets in the symmetric information environment.

3.3 Principal-Agent Model

In this section, we assume an environment in which the supports of the prior distributions and the value draws are known to the principals, and we show that even in this symmetric information setting budget constraints can be insufficient to implement the principal's equilibrium bidding strategy in the *principal-agent* 2×2 *FPSB package auction model*. In Section 3.2, we assumed agent i's budget constraints, $\alpha_i(l)$ with $\alpha_i(1) \leq \alpha_i(2)$, to be exogenous random draws. As we have shown, agents would not bid on a single unit in the unique equilibrium with these types of budget constraints. There is always an agent type with a high enough package budget constraint such that only bidding on two units yields higher expected utility, independent of the opponent's strategy. The only option to counteract these incentives is to assign a relatively low budget constraint for two units for all package valuations. In particular, budget constraints of the form $\overline{\alpha}(2) < \overline{\alpha}(1)$ are required. Given opponent j wants to coordinate on the *dual-winner outcome*, even an agent i with the highest package budget of $\overline{\alpha}(2)$ cannot win two units with certainty by not bidding on the small package.

Lemma 3: *Any principal i can direct her agent on the dual-winner outcome in the symmetric information environment of the principal-agent 2×2 FPSB package auction model by assigning her package-dependent budget constraints of the form $\alpha_i = [\alpha_i(1), \alpha_i(2)]$ with $\overline{\alpha}(2) < \overline{\alpha}(1)$ that satisfy inequalities $\alpha_i(1) + \underline{\alpha}(1) \geq \alpha_i(2)$ and $u_i(1) \geq u_i(2) \cdot P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big)$.*

In **Lemma 3** the expression $P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big)$ denotes the probability of agent i's package budget constraint exceeding opponent j's budget constraints for one and two units. In order of keeping the lemma traceable we do not express the corresponding probability in terms of the marginal and joint distribution functions $D_l(\cdot)$ and $D(\cdot)$. Following backward induction, we need to analyze whether the budget constraint scheme from **Lemma 3** actually permits the implementation of the principals' *dual-winner equilibrium*. In other words, we need to understand when these budget constraints violate the principals' equilibrium bid functions from **Theorem 2.** In **Theorem 7**, we derive a distributional condition under which the principals cannot direct their agents to truthfully reveal their profit-maximizing equilibrium strategies.

Theorem 7: *There is no vector of budget constraints, $\alpha_i = [\alpha_i(1), \alpha_i(2)]$, with which any principal i can implement the dual-winner equilibrium under dual-winner efficiency in the symmetric information environment of the principal-agent 2×2 FPSB package auction model if the following inequality is true:* $2 \cdot \big(\overline{v}(2) - \underline{v}(1)\big) > \overline{v}(1)$.

It follows that a solution to the two-stage game in which principal and agent are incentive aligned, cannot exist under reasonable ranges of valuations. In our leading example with $\mathrm{v_i}(1) \in [110{,}150]$ and $\mathrm{v_i}(2) \in [160{,}190]$, both uniformly distributed, it is easy to verify that the condition in **Theorem 7** is satisfied as $2 \cdot 80 > 150$. This means that budget constraints are not always sufficient to align agent strategies in a FPSB package auction, even if the principal knows the valuations. Intuitively, any firm faces the following trade-off: On the one hand, the principal has to bid high enough on two units in equilibrium to prohibit the opponent from making a profit by deviating from the *dual-winner equilibrium*. On the other hand, the agent can only be directed on bidding for one unit if his budget constraint on the package is low enough. Both requirements cannot always be met simultaneously.

Moreover, note that **Theorem 7** highlights consequences of the principal-agent problem that are specific to the multi-unit package auction environment. In a standard single-unit auction, for example, the principal could simply provide the agent with a budget in height of her optimal bid for one unit and solve the agency problem under symmetric information within our principal-agent model.

4 Ascending Auctions

In what follows we will extend our analysis to the 2×2 *ascending package auction* and the 2×2 *SMRA*, as they are also widely used. The structure of this section follows the same logic as Section 3.

4.1 Ascending Package Auction

4.1.1 Principals' Strategies

We start with the characterization of the principals' *dual-winner equilibrium* in the ascending package auction if they had full information and were to bid alone in the auction. **Theorem 8** shows that an efficient *dual-winner outcome* can be supported as equilibrium for all possible valuations $v_i, v_j \in V$ in case of *dual-winner efficiency* in the 2×2 *ascending package auction*.

Theorem 8: *In an ex-post dual-winner equilibrium of the* 2×2 *ascending package auction, any principal* i *only bids on one unit until the respective price reaches her valuation of* $\beta_i(1) = v_i(1)$.

Similar to the 2×2 *FPSB package auction*, there is also a *single-winner equilibrium*.

Theorem 9: *In an ex-post single-winner equilibrium of the* 2×2 *ascending package auction under dual-winner efficiency, any principal* i *remains active on the single unit as long as its price is strictly below* $\beta_i(1) = \underline{v}(2) - \overline{v}(1)$ *and continues to bid on the package of two units until the respective price reaches her valuation of* $\beta_i(2) = v_i(2)$.

The *single-winner equilibrium* of the 2×2 *ascending package auction* corresponds to the equilibrium strategy of the well-known single-object ascending (English) auction in which two units are sold as the sole package to two bidders. Again, any bidder can enforce the *single-winner equilibrium* by making the *dual-winner outcome* unprofitable for the opponent. However, in the ascending package auction the *dual-winner equilibrium* always strictly dominates the *single-winner equilibrium* in payoff.

Corollary 1: *Any principal* i *always strictly prefers the dual-winner equilibrium to the single-winner equilibrium in the* 2×2 *ascending package auction in dual-winner efficiency.*

Although the 2×2 *ascending package auction* is characterized by a similar equilibrium selection problem than the 2×2 *FPSB package auction*, the former auction format possesses two advantages. First, the *dual-winner equilibrium* strictly dominates the *single-winner equilibrium* in profit and therefore serves as a natural focal point for the bidders to coordinate on. Second, bidders can observe their opponents' equilibrium choices and adjust accordingly. This means they see if the opponent wants to coordinate on a *dual-winner equilibrium*. If this is not the case, they can switch and still aim for a *single-winner equilibrium*. Note that this robustness of the 2×2 *ascending package auction* against the equilibrium selection problem might serve as an additional reason for each bidder to start coordinating on the efficient allocation in the first place.

4.1.2 Agents' Strategies

In the analysis of the 2×2 *ascending package auction*, let us first introduce an adapted definition of *straightforward bidding* (Milgrom 2000) for agents in the 2×2 *ascending package auction*.

Definition 1 (Straightforward bidding of value-maximizing agents): *For any vector of package budgets $\alpha_i \in A$, any agent i begins to bid on the most valuable package of two units and remains active until he is overbid at his corresponding budget of $\beta_i(2) = \alpha_i(2)$. As long as he is winning, he does not bid for the smaller single unit. If the agent is overbid, he starts to bid for the less valuable package and again remains active until he is overbid at his respective budget of $\beta_i(1) = \alpha_i(1)$.*

Remember that **Lemma 2** also holds for the 2×2 *ascending package auction*: An agent *i's* strategies that are not weakly dominated are to remain active on the package until the price reaches his corresponding valuation and either to quit directly on one unit or to remain active until the respective single-unit valuation is reached.

Theorem 10: *In the* 2×2 *ascending package auction straightforward bidding of any agent i constitutes an ex-post equilibrium. In this equilibrium the agent with the highest budget for two units does not get active on one unit.*

Similar to **Theorem 6** of the 2×2 *FPSB package auction,* **Theorem 10** describes an ex-post equilibrium that is robust against risk aversion. In both auction formats, agents never coordinate on winning one unit each. This result is independent of the efficiency environment. The analysis shows that in the 2×2 *package auction*, agents will only bid on the large package in equilibrium. Note, once a bidder does not care about the profit, its bidding problem in both the first-price auction and second-price auction is the same. In general, the agents' equilibrium behavior of not bidding on one unit leads to a conflict of interest with the principals' *dual-winner equilibrium*. In the next section, we discuss how to overcome this conflict via budget constraints in the 2×2 *ascending package auction*.

4.1.3 Principal-Agent Model

Unlike the 2×2 *FPSB package auction,* the principals' *dual-winner equilibrium* can easily be implemented as a solution to the *principal-agent* 2×2 *ascending package auction model,* as long as *dual-winner efficiency* is known.

Theorem 11: *Any principal i can implement the dual-winner equilibrium with the vector of budget constraints $\alpha_i = [\alpha_i(1), \alpha_i(2)]$, with $\alpha_i(1) = v_i(1)$ and $\alpha_i(2) = 0$, in the symmetric information environment of the principal-agent 2×2 ascending package auction model under dual-winner efficiency.*

Note that the budget constraint in this theorem requires complete information about the valuation for a single unit or at least a good estimate of $v_i(1)$. Alternatively, the principal could just set a budget at $\alpha_i(1) = \overline{v}(1)$. With only two agents in our model, there would not be excess demand in the initial round and the auction would close at reservation prices. Remember from Section 4.1.2 that if one principal does not start to coordinate on the *dual-winner outcome*, the auction continues and the other principal might be able to adjust budgets and also pursue the *single-winner award*. Again, this observation might constitute an additional reason why coordination on the *dual-winner outcome* in the *principal-agent* 2×2 *ascending package auction model* is easier than in the *principal-agent* 2×2 *FPSB package auction model.*

With precise information about the valuation for a single unit the principal can apply the same budgets as in **Theorem 11** to implement the efficient allocation in the 2×2 *VCG mechanism*. Then the VCG mechanism implements the *dual-winner outcome* for the principals under *dual-winner efficiency* in the 2×2 *package auction model*.[7] The outcome equivalence between the VCG mechanism (generalized second-price sealed-bid auction) and the ascending package auction in the *principal-agent* 2×2 *package auction model* is reminiscent of the outcome equivalence between the standard single-unit English and Vickrey auctions. This suggests that the payment rule causes the different results in **Theorem 7** and **Theorem 11**.

In the first-price mechanism the principal has to submit a high package bid to make deviations from the *dual-winner outcome* by her opponent unprofitable in equilibrium. This prevents a solution of the agency dilemma as the budget for two units is too high for agents to coordinate. Contrary, in the strategy-proof VCG mechanism the principal's report on the package does not affect the competing principal's bidding behavior who is indifferent between truthfully reporting her valuation for the package and hiding this valuation. Vice versa, this reasoning is true for both principals and therefore, the agency dilemma is easy to solve if the principals know that the solution with two winners is efficient. This is because the principal can also set the budget for the package bid to zero as in the ascending auction. The fact that the VCG mechanism is strategy-proof independent of the complexity of the market environment suggests that it makes it easier to solving the agency dilemma more generally.

The insights of the *principal-agent* 2×2 *package-auction model* are also helpful for the analysis of more complex markets. Suppose, for example, the number of packages strictly exceeds the number of bidders, which is common in combinatorial auctions in the field. Here, individual bidders' veto-power against coordination on a multiple-winner outcome is still strong such that the results from the 2×2 market extend. Often there are many bidders participating in an auction, but only a few are strong and they decide the outcome. For example, in the German auction in 1999 that we will discuss in the last section there were four bidders and ten units, but in the strategic analysis one can focus on the competition between Mannesmann and T-Mobile. It is reasonable to assume that the principals both understood that an outcome with two winners was efficient and payoff-dominant to a single-winner equilibrium.

Moreover, as we will show in Section 5, the ascending auction format allows for easier coordination even under asymmetric information in our *principal-agent* 2×2 *package auction model* as long as the efficiency setting is known to the principals. This is a consequence of the fact that the principals' equilibrium bids are independent of each other in a strategy-proof second-price mechanism. In more complex settings the efficiency environment might not be known by the principals. If the principals neither know the values nor *dual-winner efficiency*, they are not able to set the budget constraints such that the payoff-dominant equilibrium is selected in our model. The bidding firms compete although they would not if the principals had the same information as the agents. In the next section we show that the agency dilemma discussed for package auctions also carries over to non-combinatorial mechanisms.

4.2 Simultaneous Multiple-Round Auction

Unlike the ascending package format there is only a *dual-winner equilibrium* for principals in which both immediately reduce demand to one unit in the 2×2 *SMRA*, but no *single-winner equilibrium*. To analyze the agents' bidding strategies in the SMRA we need an adapted definition of straightforward bidding.

For any vector of package budgets $\alpha_i \in A$, suppose an agent has budget constraints of the form $\alpha_i(1) \geq \alpha_i(2)/2$, then he remains active on two units until the unit price reaches half his budget for the package, $\alpha_i(2)/2$. He then reduces his demand to one unit and remains active until the unit price reaches his single-unit budget constraint of $\alpha_i(1)$. If agent i is provided with budgets of $\alpha_i(1) < \alpha_i(2)/2$, he will bid up to $\alpha_i(1)$ for two units, then indicate demand for one unit and immediately after that drop out completely because he must not bid beyond his constraint for one unit. If he did bid beyond $\alpha_i(1)$ for two units and first dropped out at $\alpha_i(2)/2$ from both units, then he could be assigned a single unit at a price of $\alpha_i(2)/2$ beyond budget, which is inacceptable.

It is straightforward to show an equilibrium in which both agents engage in straightforward bidding that may result in the *dual-winner outcome* and in the *single-winner* award depending on the budget constraints. Similar to the ascending package auction, principals can set a zero budget constraint for two units to implement the *dual-winner equilibrium* with their agents, and the auction would stop immediately in our model in which the principals know that there is *dual-winner efficiency*. However, if the principals have uncertainty about *dual-winner efficiency* or the agents are not appropriately restricted with zero budgets for the package, they may not reduce demand which can help explain demand inflation in the SMRA in the field that we discuss in the conclusions.

5 Contracts with Contingent Transfers

This section determines the optimal contract of budget constraints and contingent transfers or wages between principal and agent in the asymmetric information setting. The principal only knows that there is *dual-winner efficiency*, but she neither knows the package value draw for l units, $v_i(l)$, nor its exact range, $V(l)$, for all l. However, we assume that she knows supports for the range bounds: the support for the lower bound $\underline{v}(l) \in \underline{V}(l) = [\underline{\underline{v}(l)}, \overline{\underline{v}(l)}]$ and the support for the upper bound $\overline{v}(l) \in \overline{V}(l) = [\underline{\overline{v}(l)}, \overline{\overline{v}(l)}]$. This notation allows us to specify the lower bound for the random variable d as $\underline{d} \equiv \underline{\overline{v}(2)} - \overline{\underline{v}(1)}$, the smallest possible range out of which all values can be drawn, and the upper bound as $\overline{d} \equiv \overline{\overline{v}(2)} - \underline{\underline{v}(1)}$, the largest possible range. Remember that the principal does not know the exact value of d, but only its support D, whereas the agent knows both. The implementation of the *dual-winner equilibrium* in the 2×2 *ascending package auction* remains as simple as in **Theorem 11** with the only difference of assigning a budget constraint of $\overline{\overline{v}(1)}$ for the package of one unit. Thus, principals do not incur any agency costs.

In the 2×2 *FPSB auction* the optimal contract to implement the *dual-winner equilibrium* is more difficult to design. A principal i has to design a menu of budget constraints and payments, (α_i, m_i), to direct her agent to submit the *dual-winner equilibrium* strategies of $\beta(1) = \overline{v}(2) - \underline{v}(1)$ and $\beta(2) = 2 \cdot (\overline{v}(2) - \underline{v}(1))$ in the auction.

Theorem 12: *Any principal i can implement her dual-winner equilibrium with the vector of budget constraints $\alpha_i = [\alpha_i(1), \alpha_i(2)]$ with $\alpha_i(1) = d$ and $\alpha_i(2) = 2d$, and the vector of constant payments $m_i = [m(1), m(2)]$ with $m(1) = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$ and $m(2) = 0$, in the asymmetric information environment of the principal-agent 2×2 FPSB package auction model given dual-winner efficiency.*

The principal does not need to set incentives for bidding on the package, but she needs to pay a constant amount that compensates any agent for not winning two units. Unfortunately, these agency costs caused by wages in the FPSB auction can be very high as the theorem shows. Also, in this environment a principal typically does not know $w(.)$. However, if any principal i now sets the optimal menu of budget constraints and payments, the set of implementable *dual-winner equilibria* become a subset of the original set. This becomes clear by looking at the range of implementable pooling prices: $\beta(1) \in [\overline{v}(2) - \underline{v}(1) + m(1), \underline{v}(1) - m(1)]$.

Note, that for a given upper range, $\overline{d} = \overline{\overline{v}(2)} - \underline{\underline{v}(1)}$, of the random draw d, the optimal transfer, $m(1) = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$, increases as the magnitude of the agent's value-maximizing motive, $w(.)$, rises. With larger $m(1)$ the lower bound of supportable pooling prices, $\overline{v}(2) - \underline{v}(1) + m(1)$, increases whereas the upper bound, $\underline{v}(1) - m(1)$, decreases. Therefore, it can happen that no range of supportable pooling prices, $[\overline{v}(2) - \underline{v}(1) + m(1), \underline{v}(1) - m(1)]$, is left. In this case the principal cannot implement an efficient *dual-winner equilibrium* even with contingent transfers in the asymmetric information setting. Moreover, if the principal was unsure about *dual-winner efficiency*, she might pay transfers and implement the *dual-winner equilibrium* although it is not maximizing her expected payoff and is even inefficient.

6 Conclusions

We analyze a hidden information model in which the agent bids on behalf of the principal in a multi-unit auction. He knows the goods valuations but has limited liability and the principal has to pay. In this model, there is no reason for the agent to maximize payoff, but he tries to win the most valuable allocation within budget. We show that there is a conflict of interest between principal and agent in efficiency settings in which it is payoff dominant for the principals to coordinate. The types of manipulation discussed in this paper are specific to multi-object auctions, and differ from the problems in single-object auctions (Burkett 2015; Burkett 2016).

If the auctioneer is concerned about efficiency and is aware of agency problems among bidding firms he should favor ascending package auctions. First, if the principals understand that there is *dual-winner efficiency*, the ascending package auction and the VCG mechanism allow to implement the efficient equilibrium that is also payoff-dominant in our model. Second, ascending package auctions further alleviate coordination problems in multi-object markets and might even have advantages in environments in which the principals do not know the efficiency environment ex ante because they can learn new information about the competition during the auction.

However, note that in our principal-agent package auction model, the principal would need to know that there is *dual-winner efficiency* to set budget constraints appropriately in an ascending package auction or the VCG mechanism. In a FPSB package auction, this might not even be possible in equilibrium with budget constraints only, even if the principal had precise information about the valuations. The wide use of budget constraints in bidding firms might be due to the fact that there is often considerable uncertainty about the valuations, the efficiency environment and the prior type distributions for the principal in the field and only the agent has detailed information. With uncertainty about the efficiency environment, the equilibrium bidding strategies in our model are unknown. In such an environment, principals often try to at least limit the risk of the agent overbidding substantially via budget constraints. However, overall budget constraints are insufficient to make the agent bid payoff-maximizing in general as we have shown. In our principal-agent package auction model,

Let us leverage the insights from our model and revisit some well-known examples of spectrum auctions. In the German spectrum auction in 1999 the two strong players Mannesmann and T-Mobile reduced demand to five blocks each in the initial rounds of the ascending auction (Grimm et al. 2003). One could assume that in this simple environment with 10 homogeneous objects it was clear to the principal that the *dual-winner outcome* would be payoff-dominant. The auction was criticized for low revenue, but it can well have been the efficient allocation.[8]

While the 1999 auction result is compatible with enforcing coordination on a dual winner outcome, other auctions show that demand inflation is also observed, which is compatible with the agency problem generally described in this paper. An example is the German auction in 2015 (Bichler et al. 2017).[9] The bidders did not coordinate and several observers reported "demand inflation" rather than demand reduction.[10] The analysts' estimates before the auction differed substantially and it is likely that the principals had little information about the value of a package making it very hard to set appropriate budget constraints.[11]

There is not much public information about FPSB combinatorial auctions. France used this auction format for selling spectrum in the 800MHz auction and the 2.6 GHz auction in 2011 and the average prices in these auctions were among the highest in Europe.[12] This could be the result of strong competition on larger packages by agents who are not appropriately constrained to coordinate on demand-reduction equilibria. Of course, one must not over-interpret these observations. The comparison of prices in spectrum auctions in the field is far from trivial and a number of factors influence the final prices. Apart from the specifics of the auction format, the competitive situation, the reservation prices and spectrum caps, the types of bands in the auction, and country-level idiosyncrasies matter to name just a few. In summary, the traditional assumption in which bidders are modeled as payoff-maximizing individuals might be too simple and the presence of principal-agent relationships in the bidding firms can have significant negative impact on the efficiency of auctions as a means to allocate scarce resources.

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Appendix I: Proofs

Proof of Theorem 1

For the following argument suppose opponent j follows the *dual-winner equilibrium* strategy $\beta_j = (\beta_j(1), \beta_j(2))$. Regarding condition (1.), suppose any bidder i's equilibrium bidding function for one unit $\beta_i(1)$ varies with $v_i(1)$ in the *dual-winner equilibrium*, so that $\beta(v_i(1)) < \beta(\hat{v}_i(1))$ with $v_i(1) \neq \hat{v}_i(1)$. Then bidder i with value $\hat{v}_i(1)$ for one unit has an incentive to bid $\beta_i(1) = \beta(v_i(1))$ to raise her profit: $\hat{v}_i(1) - \beta(v_i(1)) > \hat{v}_i(1) - \beta(\hat{v}_i(1))$. Thus, any bidder i with single-unit values of $v_i(1)$ or $\hat{v}_i(1)$ bids $\beta(v_i(1))$ independent of her valuation. This reasoning is true for any bidder with any value and results in the equilibrium bidding function for one unit of $\beta(v_i(1)) = \beta(\hat{v}_i(1)) = \beta(1)$ for all $v_i(1), \hat{v}_i(1) \in V(1)$, the pooling bid.

The upper bound in condition (2.) ensures that any bidder with the lowest single-unit value $\underline{v}(1)$ will not make a negative profit in the *dual-winner equilibrium*: $\underline{v}(1) \geq \beta(1)$. Note further that any bidder i with vector of valuations $v_i = [\underline{v}(1), \overline{v}(2)]$ has the highest incentive to deviate from the *dual-winner outcome*. The lower bound in condition (2.) makes sure this bidder does not deviate at any pooling price $\beta(1) \geq \overline{v}(2) - \underline{v}(1)$. The respective bidder i could marginally overbid twice the pooling bid with her bid on the package of two units to obtain the profit of the *single-winner* outcome with certainty: $\pi_i(2) = \overline{v}(2) - 2 \cdot \beta(1) - \varepsilon$ for $\varepsilon \to 0$. Note, that depending on her type, opponent j might in fact bid twice the pooling bid on the package of two units. It is the highest bid on two units that still supports the *dual-winner outcome* as stated in condition (3.). For this deviation not to be profitable, the pooling bid has to be of the form $\beta(1) \geq \overline{v}(2) - \underline{v}(1)$. Note that bidder i's profit in the *dual-winner equilibrium* is given by $\pi_i(1) = \underline{v}(1) - \beta(1)$. If this bidder with vector of valuations $v_i = [\underline{v}(1), \overline{v}(2)]$ has no incentive to deviate from the *dual-winner equilibrium*, then no other bidder j with valuations $v_j(1) \geq \underline{v}(1)$ and $v_j(2) \leq \overline{v}(2)$ deviates either. Note that if bidder i did not bid $\beta(1)$, but zero for example, then it would not be in equilibrium for bidder j to bid $\beta(1)$.

For the *dual-winner outcome* to be chosen by the revenue-maximizing auctioneer for all possible package bids, it has to be true that $2 \cdot \beta(1) \geq sup_{v_i(2)}\{\beta(v_i(2))\}$ as stated in condition (3.). The supremum is defined as the smallest upper bound or the greatest element in the set. Suppose $2 \cdot \beta(1) > sup_{v_i(2)}\{\beta(v_i(2))\}$, then for any bidder i, it is a best response to deviate from her equilibrium strategy by underbidding the pooling bid slightly with her single-unit bid (and thus raising her profit in the *dual-winner equilibrium*). This cannot be optimal and therefore we obtain the equilibrium requirement of $2 \cdot \beta(1) = sup_{v_i(2)}\{\beta(v_i(2))\}$. **QED.**

Proof of Theorem 2

The *dual-winner equilibrium* requires mutually best responses of both bidders. Assume opponent j sticks to her *dual-winner equilibrium* strategy for all possible package values $v_j(1) \in V(1)$ and $v_j(2) \in V(2)$. Then we demonstrate that under conditions (1.) to (4.), $\beta_i = [\beta(1), \beta_i(2)]$ is a *dual-winner equilibrium* supporting strategy in relation to any other deviating bidding strategy $\hat{\beta}_i = [\hat{\beta}_i(1), \hat{\beta}_i(2)]$. Note that the profit in a *dual-winner equilibrium* for any bidder i is given by $\pi_i(1) = v_i(1) - \beta_i(1)$ for all $v_i(1) \in V(1)$. Let us refer to this as equilibrium profit. Now we consider three different possible cases that can result from bidder i playing any deviating strategy $\hat{\beta}_i = [\hat{\beta}_i(1), \hat{\beta}_i(2)]$ instead of the equilibrium strategy $\beta_i = [\beta(1), \beta_i(2)]$. Any deviating strategy might involve changing only the bid for one unit, the bid for two units or both bids. Nevertheless, any of the three mentioned deviations will always end up in one of the following three outcomes given opponent j sticks to the proposed equilibrium strategy $\beta_j = [\beta(1), \beta_j(2)]$: no *single-winner* outcome for principal i, $\beta(1) + \hat{\beta}_i(1) > \hat{\beta}_i(2)$, a *single-winner* outcome, $\beta(1) + \hat{\beta}_i(1) < \hat{\beta}_i(2)$, and an indifference condition for bidder i, $\beta(1) + \hat{\beta}_i(1) = \hat{\beta}_i(2)$. Let us consider each case in turn:

- Case 1: $\beta(1) + \hat{\beta}_i(1) > \hat{\beta}_i(2)$

Bidder i deviates to a different *dual-winner outcome*. However, bidder i would never reach a *dual-winner outcome* if $\beta(1) + \hat{\beta}_i(1) < \beta(v_j(2))$ for all $v_j(2) \in V(2)$, i.e. $\beta(1) + \hat{\beta}_i(1) < \beta(\underline{v}(2))$. Thus, it is a necessary condition for $\hat{\beta}(1)$ to exceed $\beta(\underline{v}(2)) - \beta(1)$ to satisfy Case 1. Due to condition (3.), raising $\hat{\beta}_i(1)$ above $\beta(1)$ does not increase the probability of winning, which is already equal to one in the *dual-winner equilibrium*, but strictly lowers profits. Therefore, the rationalizable range for the deviating bid is defined by $\hat{\beta}_i(1) \in [\beta(\underline{v}(2)) - \beta(1), \beta(1)]$. The expected profit, $\hat{\pi}_i(1)$, of submitting a $\hat{\beta}_i(1)$ from this range for any single-unit value $v_i(1)$ is given by equation (I):

$$\hat{\pi}_i(1) = (v_i(1) - \hat{\beta}_i(1)) \cdot P(\beta(v_j(2)) \leq \beta(1) + \hat{\beta}_i(1)) \qquad \textbf{(I)}$$

Now, (I) can be simplified as follows: As $\beta(\cdot)$ is continuous and strictly increasing, for any $\hat{\beta}_i(1) \in [\beta(\underline{v}(2)) - \beta(1), \beta(1)]$, a unique proxy valuation for the package of two units $\hat{v}_i(2) \in V(2)$ can be defined, so that $\hat{\beta}_i(1) = \beta(\hat{v}_i(2)) - \beta(1)$. Using this expression for $\hat{\beta}_i(1)$ to rewrite equation (I) we obtain equation (II):

$$\hat{\pi}_i(1) = (v_i(1) - \beta\big(\hat{v}_i(2)\big) + \beta(1)) \cdot F_2(\hat{v}_i(2)) \qquad \textbf{(II)}$$

Deviating single-unit bids of the form $\hat{\beta}_i(1) \in [\beta(\underline{v}(2)) - \beta(1), \beta(1)]$ imply a focus on a deviation weakly below the pooling equilibrium price of $\beta(1)$. In addition, any bidder i with single-unit value of $v_i(1) = \underline{v}(1)$ earns least in a *dual-winner outcome* and therefore has the highest incentive to deviate to a lower bid on one unit in equilibrium. To cancel this incentive, her *dual-winner equilibrium* profit of $\pi_i(1) = \underline{v}(1) - \beta_i(1)$ has to exceed her profit from deviating, $\hat{\pi}_i(1)$, which is ensured in inequality (III):

$$\underline{v}(1) - \beta_i(1) \geq (\underline{v}(1) - \beta\big(\hat{v}_i(2)\big) + \beta(1)) \cdot F_2(\hat{v}_i(2)) \qquad \textbf{(III)}$$

Rearranging and adding $v_i(1)$, we obtain condition (4.) for all $v_i(1) \in V(1)$ and $\hat{v}_i(2) \in V(2)$ because $v_i(1) \geq \underline{v}(1)$:

$$\beta\big(\hat{v}_i(2)\big) \geq \beta(1) + \frac{\beta(1) - \underline{v}(1) \cdot \big(1 - F_2(\hat{v}_i(2)\big)}{F_2(\hat{v}_i(2)} \equiv G(\hat{v}_i(2), \beta(1))$$

If bidder i with the lowest value for one unit has no incentive to deviate, then in fact no player with a higher value can have an incentive to deviate independent of the valuation for two units. The proposed *dual-winner equilibrium* is preferred to a Case 1 deviation by any bidder i as long as all deviating bids for two units are bounded from below by $G(v_i(2), \beta(1))$. This is true for all valuations $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$.

- Case 2: $\beta(1) + \hat{\beta}_i(1) < \hat{\beta}_i(2)$

Bidder i deviates in a way that the auctioneer never selects any *dual-winner outcome*. definitely does not win both units either if $\hat{\beta}_i(2) < \beta(\underline{v}(2))$, which then defines the lower bound of her deviating bid for the package. If $\hat{\beta}_i(2) > \beta(\overline{v}(2))$, the bidder wins both units, but lowering the respective bid until $\hat{\beta}_i(2) = \beta(\overline{v}(2))$ strictly dominates in profit without changing the probability of winning. Thus, we obtain a rationalizable range for bidder i's deviating package bid of $\hat{\beta}_i(2) \in [\beta(\underline{v}(2)), \beta(\overline{v}(2))]$ with an expected deviating profit of $\hat{\pi}_i(2) = (v_i(2) - \hat{\beta}_i(2)) \cdot P(\beta(v_j(2)) \leq \hat{\beta}_i(2))$ for all $v_i(2) \in V(2)$. Let us again use the proxy notation $\hat{\beta}_i(2) = \beta(\hat{v}_i(2))$ to rewrite above profit as $\hat{\pi}_i(2) = (v_i(2) - \hat{\beta}_i(2)) \cdot F_2(\hat{v}_i(2))$. Bidder i prefers the *dual-winner equilibrium* profit $\pi_i(1)$ to the deviating profit of $\hat{\pi}_i(2)$ if the following inequality (IV) holds:

$$0 \geq F_2\big(\hat{v}_i(2)\big) \cdot [\frac{\beta(1) - v_i(1)}{F_2\big(\hat{v}_i(2)\big)} + v_i(2) - \hat{\beta}_i(2)] \qquad \textbf{(IV)}$$

The above inequality is true for all valuations $v_i(1) \in V(1)$, $v_i(2) \in V(2)$ and $\hat{v}_i(2) \in V(2)$ if the term in squared brackets is weakly negative. We already know the sufficient condition to satisfy Case 1 is $\beta\big(\hat{v}_i(2)\big) \geq G(\hat{v}_i(2), \beta(1))$. It ensures opponent j's equilibrium-supporting bid on two units to be sufficiently high. Then, principal i's deviation to a different *dual-winner outcome* is less likely to succeed and therefore does not offer high enough expected profit for the deviation to be worthwhile. Note, that a similar reasoning applies in Case 2: Again, principal j's equilibrium-supporting bid on two units must be sufficiently high. In this case, principal i's deviation to a *single-winner* award is less likely to succeed and therefore not profitable enough in expectation. As a result the sufficient condition from Case 1 can be used in Case 2 as well. Thus, the term in squared brackets in (IV) is in fact weakly negative for all $v_i(1) \in V(1)$, $v_i(2) \in V(2)$ and $\hat{v}(2) \epsilon V(2)$ because the following inequality (V) is true:

$$\frac{\beta(1) - v_i(1)}{F_2\big(\hat{v}_i(2)\big)} + v_i(2) \leq G(\hat{v}_i(2), \beta(1)) \qquad \textbf{(V)}$$

Using the definition of $G(\hat{v}_i(2), \beta(1))$ and rearranging, we obtain (VI):

$$\underline{v}(1) - v_i(1) \leq F_2(\hat{v}_i(2) \cdot [\beta(1) + \underline{v}(1) - v_i(2)] \qquad \textbf{(VI)}$$

The LHS of the above inequality is weakly negative. Now we have to distinguish two cases regarding the RHS of inequality (VI): If $\beta(1) + \underline{v}(1) - v_i(2) \geq 0$, the inequality always holds. If $\beta(1) + \underline{v}(1) - v_i(2) < 0$, we have to show that $\beta(1) + \underline{v}(1) - v_i(2) \geq \underline{v}(1) - v_i(1)$. This is true for all $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$ given the lower bound of condition (1.). As inequality (VI) holds, inequality (V) must also be true. Remember from Case 1 that $\beta\big(\hat{v}_i(2)\big) \geq G(\hat{v}_i(2), \beta(1))$ must be given, which implies that inequality (IV) is satisfied. Therefore, any deviation considered in Case 2 is not profitable.

- Case 3: $\beta(1) + \hat{\beta}_i(1) = \hat{\beta}_i(2)$

In this case, bidder i deviates as if she were indifferent between the *dual-winner* and *single-winner outcome*. Remember from condition (1.) that any bidder i with valuations of $v_i = [\underline{v}(1), \overline{v}(2)]$ is indifferent between the *dual-winner equilibrium* and any *single-winner* outcome at the pooling price. Hence, the deviating behavior in Case 3 does in fact define her equilibrium strategy. Rewrite as the deviation in Case 3 to $\beta(1) = \hat{\beta}_i(2) - \hat{\beta}_i(1)$ and bear in mind the lower bound from condition (1.):

$\beta(1) \geq \overline{v}(2) - \underline{v}(1)$. Combining these two equations by substituting for $\beta(1)$ and rearranging, we obtain inequality (VII):

$$\underline{v}(1) - \hat{\beta}_i(1) \geq \overline{v}(2) - \hat{\beta}_i(2) \qquad \textbf{(VII)}$$

Now, consider player i with values of $v_i = [v_i(1), v_i(2)]$ in which $v_i(1) > \underline{v}(1)$ and $v_i(2) \leq \overline{v}(2)$. For any such bidder, (VII) holds with strict inequality and she strictly prefers any deviating *dual-winner outcome* (LHS) to any *single-winner* award, which contradicts Case 3. Note that for bidder i with valuations of $v_i = [v_i(1), v_i(2)]$ in which $v_i(1) \geq \underline{v}(1)$ and $v_i(2) < \overline{v}(2)$, the same reasoning holds.

Note in particular that by strictly decreasing the deviating bid on two units from $\hat{\beta}_i(2)$ to $\hat{\beta}_i(2)'$, so that the deviation from Case 3 becomes $\beta(1) + \hat{\beta}_i(1) > \hat{\beta}_i(2)'$, the bidder changes from a Case 3 deviation to a Case 1 deviation. As the latter always leads to some *dual-winner outcome*, it dominates Case 3 for all $v_i = [v_i(1), v_i(2)]$ with $v_i(1) > \underline{v}(1)$ and $v_i(2) \leq \overline{v}(2)$, and for all $v_i = [v_i(1), v_i(2)]$ in which $v_i(1) \geq \underline{v}(1)$ and $v_i(2) < \overline{v}(2)$. Finally, as a Case 1 deviation is not beneficial, a Case 3 deviation cannot possibly be either. **QED.**

Proof of Theorem 3

In a *single-winner equilibrium*, any bidder i solely aims for the package of two units for all package valuations of $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$. This scenario is strategically equivalent to the well-known FPSB auction for a single package in which two units are sold as the unique bundle. In this standard auction format, the equilibrium strategy of any bidder i takes the form of $\beta_i(2)$ from condition *(1.)*.

Note that the *single-winner equilibrium* requires any bidder i to possess ultimate “veto” power on the *dual-winner outcome* to make it unprofitable for her opponent to deviate from equilibrium. Suppose opponent j follows the proposed equilibrium strategy and submits a very low “veto” bid $\beta_j(1)$ on one unit, such as $\beta_j(1) = 0$ for example. Then bidder i would have to submit a deviating single-unit bid, $\hat{\beta}_i(1)$, to retain the chance of winning the *dual-winner outcome* in which $\hat{\beta}_i(1)$ is defined by the next inequality (I):

$$\hat{\beta}_i(1) > \beta_i(\underline{v}(2)) = \underline{v}(2) \qquad \textbf{(I)}$$

In inequality (I), $\beta_i(\underline{v}(2))$ is the optimal bid on the package of bidder i with lowest valuation for two units. Add the valuation for one unit $v_i(1)$ on both sides of inequality (I) and rearrange to obtain inequality (I'):

$$v_i(1) - \underline{v}(2) > v_i(1) - \hat{\beta}(1) \qquad \textbf{(I')}$$

The LHS of inequality (I') is strictly negative if $\underline{v}(2) > v_i(1)$ for all single-unit valuations $v_i(1) \in V(1)$, i.e. if $\underline{v}(2) > \overline{v}(1)$ is true. The last inequality holds by assumption. As the LHS of (I') is strictly negative, the RHS of inequality (I') must be strictly negative. Note that the RHS corresponds to bidder i's profit in the forced deviating *dual-winner outcome*. Thus, if opponent j submits a "veto" bid in form of condition (2.), any deviating single-unit bid $\hat{\beta}_i(1)$ of bidder i to enforce the *dual-winner outcome* results in strictly negative profit. As she receives weakly positive expected profit in the *single-winner equilibrium*, a deviating bid of $\hat{\beta}_i(1)$ is strictly dominated by any single-unit bid that supports the *single-winner equilibrium*. By symmetry, only a bid of the form $\beta_i(1) < \underline{v}(2) - \overline{v}(1)$ supports the *single-winner equilibrium* for all $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$ with certainty. **QED.**

Proof of Theorem 4

Remember from **Theorem 2** that any principal i submits the payoff-maximizing pooling bid of $\beta_i(1) = \overline{v}(2) - \underline{v}(1)$ in the *dual-winner equilibrium* and obtains respective equilibrium profit of $v_i(1) - \overline{v}(2) + \underline{v}(1)$ with certainty. The principal's expected equilibrium profit in the *single-winner equilibrium* is

$\int_{\underline{v}(2)}^{v_i(2)} F_2(v_j(2)) \cdot dv_j(2)$, as in the standard FPSB auction in which two units are sold as the sole package to two bidders. For principal i let us define the difference between expected profits in the *dual-winner* and *single-winner equilibrium* as a function $\Delta[v_i(1), v_i(2)]: V \to \mathbb{R}$ on the compact set $V \subset \mathbb{R}^2$, with

$$\Delta[v_i(1), v_i(2)] = v_i(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{v_i(2)} F_2(v_j(2)) \cdot dv_j(2)$$

The above function is continuous, due to the differentiability of its constituents. It follows that it possesses a global maximum and a global minimum on V. Moreover, $\Delta[v_i(1), v_i(2)]$ is strictly increasing in its first argument and strictly decreasing in its second argument. Consequently, the function does not have a critical point in the interior of its domain, but on the boundary. Its maximum occurs at $(\overline{v}(1), \underline{v}(2))$ and the minimum at $(\underline{v}(1), \overline{v}(2))$, with values of $\Delta[\overline{v}(1), \underline{v}(2)] = \overline{v}(1) - \overline{v}(2) + \underline{v}(1)$ and $\Delta[\underline{v}(1), \overline{v}(2)] = \underline{v}(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{\overline{v}(2)} F_2(v_j(2)) \cdot dv_j(2)$, respectively. Remember that *dual-winner efficiency* is defined by $\bar{v}(2) < 2\underline{v}(1)$. This implies the maximum $\Delta[\overline{v}(1), \underline{v}(2)]$ is always strictly positive and the minimum $\Delta[\underline{v}(1), \overline{v}(2)]$ is strictly positive for all package valuations $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$ if inequality (I) holds:

$$\underline{v}(1) - \overline{v}(2) + \underline{v}(1) - \int_{\underline{v}(2)}^{\overline{v}(2)} F_2(v_j(2)) \cdot dv_j(2) > 0 \qquad \textbf{(I)}$$

Using integration by parts, inequality (I) can be rewritten to (II):

$$2 \cdot \underline{v}(1) - 2 \cdot \overline{v}(2) + \int_{\underline{v}(2)}^{\overline{v}(2)} f_2(v_j(2)) \cdot v_j(2) \cdot dv_j(2) > 0 \qquad \textbf{(II)}$$

Finally, it follows that $\Delta[v_i(1), v_i(2)]$ is strictly positive for all package valuations $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$ if $E\left(v_j(2)\right) > 2 \cdot \beta(1)$. **QED.**

Proof of Lemma 1:

We prove the lemma by eliminating weakly dominated strategies. A strategy is weakly dominated if, regardless of what the other agent does, the strategy earns an agent a payoff at most as high as any other strategy, and, the strategy earns a strictly lower payoff for some profile of the other agent's strategies. Suppose agent i follows the set of strategies $\hat{\beta}_i = \left(\beta(1), \hat{\beta}_i(2)\right)$ with fixed $\beta(1) \in [0, \alpha_i(1)]$ and $\hat{\beta}_i(2) < \alpha_i(2)$. Let us first focus on the setting in which opponent j submits bids of $\beta(1) + \beta_j(1) < \beta_j(2)$. Then,

a) if $\beta_j(2) < \hat{\beta}_i(2)$ agent i wins both units. However, he also wins the package with strategy $\beta_i = (\beta(1), \alpha_i(2))$, and the higher bid on two units does not impact his utility.

b) If $\beta_j(2) = \hat{\beta}_i(2)$ agent i might win two units through randomization by the auctioneer. With strategy β_i agent i would win the package, which provides a strictly higher utility.

c) If $\beta_j(2) > \hat{\beta}_i(2)$ agent i wins nothing, but might have won the package with strategy β_i in case $\alpha_i(2) > \beta_j(2)$, . such that $\hat{\beta}_i$ was strictly dominated. Otherwise he is indifferent between strategies $\hat{\beta}_i$ and β_i.

Let us now consider the setting in which opponent j submits bids of $\beta(1) + \beta_j(1) \geq \beta_j(2)$. Here,

d) if $\beta(1) + \beta_j(1) < \hat{\beta}_i(2)$ agent i wins the package in any case and is indifferent between $\hat{\beta}_i$ and β_i.

e) If $\beta(1) + \beta_j(1) \geq \hat{\beta}_i(2)$ agent i wins one unit, but could win the package with strategy β_i as long as $\beta(1) + \beta_j(1) < \alpha_i(2)$, which would strictly dominate $\hat{\beta}_i$. For $\beta(1) + \beta_j(1) \geq \alpha_i(2)$ agent i is indifferent between strategies $\hat{\beta}_i$ and β_i.

It can be useful to analyze $\hat{\beta}_i(2) < \alpha_i(2)$ also under two strategic goals: winning the package or winning only one unit. Obviously, if a value-maximizing agent wants to win the package, any $\hat{\beta}_i(2) < \alpha_i(2)$ will be weakly dominated. However, $\beta_i(2) = \alpha_i(2)$ can also not decrease the utility of the agent if he aims to win a single unit only. If an agent wanted to maximize his probability of winning one unit only, then he will bid $\beta_i(1) = \alpha_i(1)$. A bid $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$ weakly dominates a bid $\beta_i = \left(\alpha_i(1), \hat{\beta}_i(2)\right)$ with $\hat{\beta}_i(2) < \alpha_i(2)$. This is because the utility for the package is higher, and bidding $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$ increases the probability of $\alpha_i(1) + \beta_j(1) < \beta_i(2)$, but it does not decrease the probability of $\alpha_i(1) + \beta_j(1) > \beta_j(2)$. Thus, any strategy set $\hat{\beta}_i = \left(\beta(1), \hat{\beta}_i(2)\right)$ is weakly dominated by the set of strategies $\beta_i = (\beta(1), \alpha_i(2))$ for all $\beta(1) \in [0, \alpha_i(1)]$. **QED.**

Proof of Lemma 2:

We prove this lemma by eliminating weakly dominated strategies. From **Lemma 1**, we know that any strategy $\beta_i(2) < \alpha_i(2)$ is weakly dominated. Now, we can concentrate on strategies $\beta_i{}' = (0, \alpha_i(2))$, $\beta_i{}'' = (\alpha_i(1), \alpha_i(2))$, and the strategy set $\hat{\beta}_i = \left(\hat{\beta}_i(1), \alpha_i(2)\right)$ with $\hat{\beta}_i(1) \in (0, \alpha_i(1))$. In what follows, we show that $\hat{\beta}_i$ is weakly dominated. Let us first analyze agent j submitting bids of $\hat{\beta}_i(1) + \beta_j(1) < \alpha_j(2)$. Then,

a) if $\alpha_j(2) < \alpha_i(2)$ agent i wins both units just as with strategy $\beta_i{}' = (0, \alpha_i(2))$. A strategy $\beta_i{}'' = (\alpha_i(1), \alpha_i(2))$ might still lead to winning the package, but it could also lead to $\alpha_i(1) + \beta_j(1) > \alpha_i(2)$, such that the agent wins only one unit. In this case, strategy $\beta_i{}''$ is strictly dominated. $\hat{\beta}_i$ is weakly dominated in each of the cases because a payment is not considered in utility u_i.

b) If $\alpha_j(2) = \alpha_i(2)$ he might win the package or nothing due to the randomization of the auctioneer as with strategy $\beta_i{}'$. Strategy $\beta_i{}''$ does either not change the outcome or it does lead to winning one unit with certainty in case of $\alpha_i(1) + \beta_j(1) > \alpha_i(2)$. In the latter case, $\beta_i{}''$ dominates the other strategies if $u_i(1) > 0.5u_i(2)$. If $u_i(1) < 0.5u_i(2)$, then the agent is indifferent between $\hat{\beta}_i$ and $\beta_i{}'$. Strategy $\hat{\beta}_i$ is again weakly dominated in each of the cases

c) If $\alpha_j(2) > \alpha_i(2)$ he wins nothing independent of strategy $\hat{\beta}_i$ or $\beta_i{}'$. With strategy $\beta_i{}''$ agent i will not win anything if $\alpha_i(1) + \beta_j(1) < \alpha_j(2)$, or he will win one unit if $\alpha_i(1) + \beta_j(1) > \alpha_j(2)$. In the earlier case the bid on a single unit is irrelevant, in the latter case, $\beta_i{}''$ strictly dominates the other strategies $\beta_i{}'$ and weakly dominates $\hat{\beta}_i$.

Let us now consider opponent j submitting bids of $\hat{\beta}_i(1) + \beta_j(1) \geq \alpha_j(2)$. Then,

d) if $\hat{\beta}_i(1) + \beta_j(1) < \alpha_i(2)$, the outcomes of this case correspond to a) for agent i.

e) If $\hat{\beta}_i(1) + \beta_j(1) = \alpha_i(2)$ he wins one unit just as with strategy $\beta_i{}''$. If $\alpha_i(2) > \alpha_j(2)$, then $\beta_i{}'$ strictly dominates $\hat{\beta}_i$. Also, if $\alpha_i(2) = \alpha_j(2)$ and $u_i(1) < 0.5u_i(2)$, then $\beta_i{}'$ strictly dominates $\hat{\beta}_i$. If $\alpha_i(2) = \alpha_j(2)$ and $u_i(1) > 0.5u_i(2)$, then $\beta_i{}''$ strictly dominates $\beta_i{}'$. Strategy $\hat{\beta}_i$ is weakly dominated in all cases.

f) If $\hat{\beta}_i(1) + \beta_j(1) > \alpha_i(2)$ he wins one unit just as with strategy $\beta_i{}''$. With strategy $\beta_i{}'$ we have to distinguish different cases. Case $\alpha_j(2) > \alpha_i(2)$ corresponds to the outcome in c) and $\beta_i{}'$ is strictly dominated by $\beta_i{}''$ in which agent *i* would win one unit. If in case $\alpha_j(2) < \alpha_i(2)$, then $\beta_i{}'$ strictly dominates the other strategies. Finally, case $\alpha_j(2) = \alpha_i(2)$ is reflected in b) and $\beta_i{}'$ is strictly dominated by $\beta_i{}''$ if $u_i(1) > 0.5u_i(2)$. With $u_i(1) < 0.5u_i(2)$ then strategy $\beta_i{}'$ would strictly dominate $\beta_i{}'$. Also here strategy $\hat{\beta}_i$ is weakly dominated.

In summary, the strategy set $\hat{\beta}_i = \left(\hat{\beta}_i(1), \alpha_i(2)\right)$ with $\hat{\beta}_i(1) \in (0, \alpha_i(1))$ is always weakly dominated by the set of strategies $\beta_i = (\beta_i(1), \alpha_i(2))$ with $\beta_i(1) \in \{0, \alpha_i(1)\}$. **QED.**

Proof of Theorem 5

Suppose opponent j follows the proposed equilibrium strategy and bids $\beta_j = \left(0, \alpha_j(2)\right)$. In this case bidder i cannot win one unit independent of which strategy he chooses from the set of weakly dominant strategies defined in **Lemma 2**. Remember, he always bids his full budget on two units that exceeds his single unit budget by assumption. With $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$ he either wins the package or nothing because $\alpha_i(1) \leq \alpha_i(2)$. Therefore, agent i is indifferent between both strategies and $\beta_i = \left(0, \alpha_i(2)\right)$ is an equilibrium. In this equilibrium no agent i would want to deviate even if he knew the opponent's type. Again, assume opponent j to follow the proposed equilibrium strategy. For package budgets of the form $\alpha_i(2) > \alpha_j(2)$, bidder i wins two units following the equilibrium strategy and has no incentive to adjust. In case of $\alpha_i(2) < \alpha_j(2)$, agent i does not win anything independent of his strategy and therefore is indifferent to deviating. **QED.**

Proof of Theorem 6

We prove the theorem by contradiction. Assume there is a set of budget combinations $S \subset \mathrm{A}$ for which bidders submit bids $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$. For all draws of package budget combinations not in S bidders bid on the large package only, i.e. $\beta_i = \left(0, \alpha_i(2)\right)$. Let us focus on a bidder i with any draws of package budgets $\alpha_i \in A$ and suppose his opponent j possesses budget draws of $\alpha_j \in S$. Based on **Lemma 2** opponent j employs the strategy $\beta_j = \left(\alpha_j(1), \alpha_j(2)\right)$. Bidder i's expected payoff of bidding on the large package only is $\pi_p = u_i(2) \cdot P(\alpha_i(2) \geq \alpha_j(2))$ whereas his expected payoff of using $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$ is

$$\pi_{sp} = u_i(2) \cdot P\left(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_i(1) + \alpha_j(1)\right) +$$

$$+ u_i(1) \cdot P\left(\alpha_i(1) + \alpha_j(1) \geq \alpha_i(2) \cap \alpha_i(1) + \alpha_j(1) \geq \alpha_j(2)\right),$$

in which $P(X)$ denotes the probability of event X to occur. To keep the proof traceable we will not express the relevant probabilities as functions of $Q_l(\cdot)$ and $Q(\cdot)$. Define the difference between bidder i's expected payoffs, $\Delta\pi$, as $\Delta\pi = \pi_{sp} - \pi_p$. We now demonstrate that $\Delta\pi \leq 0 \; \forall \; \alpha_i \in A$ which corresponds to showing that set S is empty, i.e. there cannot exist a Bayesian Nash equilibrium strategy in which any agent i bids $\beta_i = \left(\alpha_i(1), \alpha_i(2)\right)$.

First, observe that bidder i's package draw for two-units of $\alpha_i(2) \in [\hat{\alpha}_i(2), \overline{\alpha}(2)]$ cannot belong to set S independent of the budget for one unit. For the highest possible budget draw for two units, $\alpha_i(2) = \overline{\alpha}(2)$, the difference in bidder i's expected payoffs is weakly negative for all possible single-unit budgets, i.e. $\Delta\pi \leq 0 \; \forall \; \alpha_i(1) \in A(1)$, as

$$u_i(2) \cdot P\left(\overline{\alpha}(2) \geq \alpha_i(1) + \alpha_j(1)\right) + u_i(1) \cdot P\left(\alpha_i(1) + \alpha_j(1) \geq \overline{\alpha}(2)\right) \leq u_i(2). \qquad \textbf{(I)}$$

Let us distinguish two different cases. If $\overline{\alpha}(2) < 2 \cdot \overline{\alpha}(1)$ then $P\left(\overline{\alpha}(2) \geq \alpha_i(1) + \alpha_j(1)\right) < 1$ and $P\left(\alpha_i(1) + \alpha_j(1) \geq \overline{\alpha}(2)\right) > 0$. As $u_i(1)$ is strictly smaller than $u_i(2)$ the LHS is strictly smaller than the RHS and (I) holds strictly for bidder i with highest package budget and any single-unit budget draw. If $\overline{\alpha}(2) \geq 2 \cdot \overline{\alpha}(1)$ then $P\left(\overline{\alpha}(2) \geq \alpha_i(1) + \alpha_j(1)\right) = 1$ and $P\left(\alpha_i(1) + \alpha_j(1) \geq \overline{\alpha}(2)\right) = 0$, so (I) holds with equality. Bidder i is indifferent between bidding on the package only and bidding on one and two units for all possible single-unit budget draws because he wins the large package anyway. WLOG in this case we can assume bidder i with package budget draws $\alpha_i(2) \geq \breve{\alpha}_i(2)$ to bid on the large package only, independent of the single-unit budget draw, with $\breve{\alpha}_i(2)$ being defined as the lowest package budget such that $P\left(\breve{\alpha}_i(2) \geq \alpha_i(1) + \alpha_j(1)\right) = 1$. Note that for both cases (I) also holds strictly for slightly lower two-unit budgets, $\alpha_i(2) \in [\hat{\alpha}_i(2), \overline{\alpha}(2))$, independent of the budget draw for one unit.

Second, define the set of budget combinations with the highest package budget draw in S as $H \subseteq S$. Let us from now on focus on bidder i with budget draws of $\alpha_i \in H$. By definition if his package budget draw, $\alpha_i(2)$, is marginally increased he does not belong to set S anymore. Bidder i's expected payoff from bidding on the large package only, $\beta_i = \left(0, \alpha_i(2)\right)$, remains unaltered whereas his expected payoff from bidding on the single unit and the package corresponds to

$$\pi_{sp} = u_i(2) \cdot P\left(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_i(1) + \alpha_j(1)\right) +$$

$$+ u_i(1) \cdot P\left(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(1) + \alpha_j(1) \geq \alpha_i(2)\right).$$

The bidder's probability of winning the single unit must include the opponent not having a higher package budget draw than himself, $\alpha_j(2) \leq \alpha_i(2)$. Otherwise, by symmetry, opponent j would not bid on the single unit independent of his corresponding budget and bidder i could not win one unit anyway. Note at this stage, there might be budget combinations with lower budget draws for two units for which opponent j bids on the large package only. However, if we can show that bidder i prefers to bid on the large package only if we assume all bidders with lower package budget draws to bid on both packages he will not change his preferences if some bidders with lower package budgets bid on the large package only.

By definition for bidder i with a budget draw from set S the difference in his expected payoff must be positive for all his budget combinations, i.e. $\Delta\pi > 0 \; \forall \; \alpha_i \in H \subseteq S$, which corresponds to

$$u_i(2) \cdot P\left(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_i(1) + \alpha_j(1)\right) +$$

$$+ \, u_i(1) \cdot P\left(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(1) + \alpha_j(1) \geq \alpha_i(2)\right) >$$

$$> u_i(2) \cdot P(\alpha_i(2) \geq \alpha_j(2)). \qquad \textbf{(I')}$$

We use conditional probability to rewrite the LHS of (I') and cancel out $P(\alpha_i(2) \geq \alpha_j(2))$ to obtain

$$u_i(2) \cdot P\left(\alpha_i(2) \geq \alpha_i(1) + \alpha_j(1) | \alpha_i(2) \geq \alpha_j(2)\right) +$$

$$+ \, u_i(1) \cdot P\left(\alpha_i(2) \leq \alpha_i(1) + \alpha_j(1) | \alpha_i(2) \geq \alpha_j(2)\right) > \alpha_i(2).$$

As $u_i(1)$ is strictly smaller than $u_i(2)$ the LHS is strictly smaller than the RHS in (I') and in fact $\Delta\pi < 0 \; \forall \; \alpha_i \in H \subseteq S$. Hence, bidder i has an incentive to deviate and bid for the large package only. Thus, set H cannot belong to S. Finally, as it is always possible to define a subset H in S in which bidder i has the highest budget for two units in the set S, there cannot be a set $S \subset V$ as defined above, and the argument unravels for all types. Therefore, the proposed Bayesian Nash equilibrium strategy is in fact unique. **QED.**

Proof of Lemma 3

Any agent i can be coordinated on winning one unit together with his opponent with certainty. For this, the sum of both single-unit bids must exceed each agent's package bid. Remember from **Lemma 1** that any agent always spends his entire package budget constraint and if he submits a non-zero bid on one unit, he bids his entire single-unit budget constraint. Therefore, both principals have to implement a budget constraint scheme so that the sum of both single-unit budget constraints exceeds each agent's package budget constraint in the *dual-winner outcome***.**

In this allocation, principal i chooses the vector of budget constraints α_i such that condition $\alpha_i(1) + \underline{\alpha}(1) \geq \alpha_i(2)$ is satisfied. She does not know firm j's budget for one unit. Thus, she has to make sure his two-unit budget constraint is below the sum of both single-unit budget constraints. This has to be true for all possible single-unit budget constraints of his opponent, especially the smallest budget constraint of $\underline{\alpha}(1)$. Thus, we get $\alpha_i(1) + \underline{\alpha}(1) \geq \alpha_i(2)$.

For agent i to in fact submit a positive single-unit bid, his certain utility from bidding on this package must exceed his expected utility from winning two units. This is ensured in condition $u_i(1) \geq u_i(2) \cdot P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big)$. On the RHS, agent i does not bid on one unit, but can win two units instead. Here, $P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big)$ is the probability with which his package budget constraint exceeds opponent j's single- and package budget constraints. The budget constraint $\alpha_i(2)$ is chosen so that $P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big)$ is low enough for the RHS to be lower than the LHS. Thus, agent i prefers to bid on one unit and win with certainty. **QED.**

Proof of Theorem 7

For the following line of argument suppose the opposing principal j manages to implement the *dual-winner equilibrium* strategy. In **Theorem 2**, every principal chooses the same pooling price of $\beta(1)$ in the *dual-winner equilibrium*. Thus, according to **Lemma 3**, any principal i has to provide her agent with the same single-unit budget constraint. This budget constraint must be in the amount of the pooling bid, i.e. $\alpha_i(1) = \beta(1)\ \forall\ i \in I$. Moreover, any principal i selects a package budget constraint that corresponds to her equilibrium bid on two units and thus, $\alpha_i(2) = \beta(v_i(2))$. Finally, according to **Lemma 1** every agent always truthfully bids her entire package budget constraint for two units, $\alpha_i(2)$. The following two conditions (I) to (II) then have to be satisfied to implement the *dual-winner equilibrium*:

$$2 \cdot \beta(1) \geq \alpha_i(2) \qquad \textbf{(I)}$$

$$\alpha_i(2) \geq G(v_i(2), \beta(1)) \qquad \textbf{(II)}$$

$$u_i(1) \geq u_i(2) \cdot P\big(\alpha_i(2) \geq \alpha_j(2) \cap \alpha_i(2) \geq \alpha_j(1)\big) \qquad \textbf{(III)}$$

Condition (I) ensures that the auctioneer will choose the *dual-winner outcome* in equilibrium. The last two conditions (II) and (III) ensure that no deviation from obtaining the single unit will increase payoff for principal and agent in equilibrium. They correspond to conditions (5.) from **Theorem 2** and the second condition from **Lemma 3**, respectively. Let us now check whether the above three conditions can be satisfied for the focal point pooling bid of $\beta(1) = \overline{v}(2) - \underline{v}(1)$. Suppose firm i has the highest possible value for the package of two units $v_i(2) = \overline{v}(2)$. In this case, condition (II) becomes (II') by definition of $G(v_i(2), \beta(1))$:

$$\alpha_i(2) \geq 2 \cdot (\overline{v}(2) - \underline{v}(1)) \qquad \textbf{(II')}$$

Given **Theorem 2** (4.), for a package value of $v_i(2) = \overline{v}(2)$ it must be true that $\beta_i(\overline{v}(2)) = 2 \cdot \big(\overline{v}(2) - \underline{v}(1)\big)$. Agent i's bid on two units equals twice the pooling bid. Remember, principal i chooses a package budget constraint in the amount of her respective equilibrium bid, which thereby also determines the highest possible budget for two units: $\overline{\alpha}(2) = \beta_i\big(\overline{v}(2)\big)$. This implies $\overline{\alpha}(2) = 2 \cdot \big(\overline{v}(2) - \underline{v}(1)\big)$, which satisfies condition (I) and forces condition (II') to hold with equality. Moreover, remember that condition (III) reflects $\alpha_i(2) \leq \overline{\alpha}(1)$, given the assumption of $\overline{\alpha}(2) \leq \overline{\alpha}(1)$. Applying this insight to the fact that $\overline{\alpha}(1) \leq \overline{v}(1)$, by definition of the pooling price and its range, we obtain condition (III'):

$$\alpha_i(2) \leq \overline{v}(1) \qquad \textbf{(III')}$$

Finally, combining conditions (II') and (III') is not possible if the condition from **Theorem 7** is true. In this case, any firm i with package value of $v_i(2) = \overline{v}(2)$ cannot implement budget constraints that satisfy two restrictions: They correspond to its principal's equilibrium strategy and at the same time direct its agent to bid truthfully on both packages. Hence, the *dual-winner equilibrium* cannot be supported as a solution to the *principal-agent* 2×2 *FPSB package auction model*. **QED.**

Proof of Theorem 8

Let both bidders not bid on two units and solely start bidding on the single unit. Then there is no over-demand, and the auction immediately stops at a price of zero for the package of one unit. Any bidder i receives equilibrium profit of $v_i(1)$ with certainty. In the remaining proof, we assume opponent j follows this proposed equilibrium strategy.

Assume principal i tries to win two units and suppose she does not bid on the single unit. If player i wins two units at a price of $p_i(2) = v_j(1)$, she obtains a profit of $v_i(2) - v_j(1)$. Remember, *dual-winner efficiency* implies $v_i(1) + v_j(1) > v_i(2)$ for all possible valuations $v_i(1), v_j(1) \in V(1)$ and $v_i(2) \in V(2)$. Thus, for any principal i, equilibrium profit of $v_i(1)$ strictly exceeds $v_i(2) - v_j(1)$. Now, let bidder i also bid on the package of one unit. This cannot possibly raise i's profit on two units compared to not bidding on one unit. Finally, note that the proposed equilibrium strategy is independent of any bidder's package valuations and, therefore, constitutes an ex-post equilibrium. **QED.**

Proof of Theorem 9

Suppose both bidders do not begin to bid on one unit, but start bidding on two units and continue to be active until their respective values are reached, i.e. $p_i(2) = v_i(2)$**.** Any bidder i obtains an expected equilibrium profit of $F_2\big(v_i(2)\big) \cdot (v_i(2) - v_j(2))$. Bidder i has the highest package value with probability of $F_2\big(v_i(2)\big)$. In this case, she wins two units at a price of the second highest value, $v_j(2)$, and receives a profit of $v_i(2) - v_j(2)$. From now on, assume opponent j follows the proposed equilibrium strategy.

Bidder i has no chance to profitably enforce the *dual-winner outcome* because opponent j does in fact use her "veto" bid, given $\underline{v}(2) > \overline{v}(1)$ is true. The reasoning is analogue to the proof of **Theorem 3** and therefore omitted. Bidder i is indifferent between submitting any single-unit bid of $\beta_i(1) \in [0, v_i(1)]$ as long as she continues to be active on the large bundle until the price reaches her corresponding value of $\beta_i(2) = v_i(2)$. She obtains an expected profit of $F_2\big(v_i(2)\big) \cdot (v_i(2) - v_j(2))$. However, if bidder i decides to drop out on two units before the price reaches her value of $v_i(2)$, she strictly lowers her probability of winning. This strictly decreases her expected profit and cannot be optimal. In the *single-winner equilibrium*, by symmetry, both bidders quit bidding on one unit before its price reaches $\underline{v}(2) - \overline{v}(1)$ and remain active on the package of two units until its price reaches their respective valuations. With *dual-winner efficiency* being common knowledge, the proposed equilibrium strategies are independent of the bidders' actual package valuations and therefore are rationalizable ex-post. **QED.**

Proof of Corollary 1

In the *dual-winner equilibrium* from **Theorem 8**, the bidder with the lowest value for one unit obtains the lowest profit of $\underline{v}(1)$ with certainty. According to **Theorem 9**, the highest possible profit achievable in the *single-winner equilibrium* is $\overline{v}(2) - \underline{v}(2)$. Using the definition of *dual-winner efficiency* we get $\overline{v}(2) - \underline{v}(2) < 2\underline{v}(1) - \underline{v}(2) < 2\underline{v}(1) - \underline{v}(1)$. The last inequality stems from the fact that $\underline{v}(2) > \underline{v}(1)$.

Therefore, the profit in the *dual-winner equilibrium* is strictly greater than in the *single-winner equilibrium* for all possible bidder's valuations $v_i(1) \in V(1)$ and $v_i(2) \in V(2)$. **QED.**

Proof of Theorem 10

Lemma 1 implies that both agents start bidding on the two-unit package. Each of them remains active until his budget for two units is reached. The bidder *i*, who is outbid on the package with $\alpha_i(2) < \alpha_j(2)$, can then start bidding on one unit. However, since $\alpha_i(1) < \alpha_i(2)$, the bidder *i* cannot become winning unilaterally with his bid on one unit. Both bidders have a higher utility for the package, and therefore agent *j*, who is the standing winner on the package, would not bid on a single unit. Note that the low bidder *i* on the package can also not win by starting to bid on the single unit only because he would be overbid by opponent *j* as well, who submits his equilibrium bid on the package only. In summary, even knowing the opponent's type, no agent can benefit by deviating from his equilibrium strategy. Thus, straightforward bidding constitutes an ex-post equilibrium in the second stage of the *principal-agent* 2×2 *ascending package auction model*. **QED.**

Proof of Theorem 11

In **Theorem 8**, any principal i does not bid on the package of two units. She remains active on the package of one unit at most until the price reaches her corresponding valuation of $\beta_i(1) = v_i(1)$. To implement the principal's *dual-winner equilibrium* strategy for her agent, the principal provides a zero budget constraint on the large package. This eliminates the agent's possibility to win the package and the principal can simply provide her agent with a budget constraint in the amount of her valuation for one unit. From the beginning there is no over-demand and the auction terminates immediately. **QED.**

Proof of Theorem 12

Note first, that any firm i's optimal contract menu must involve budget constraints of the form $\alpha_i(1) = d$ and $\alpha_i(2) = 2 \cdot d$, i.e. $\alpha_i(2) = 2 \cdot \alpha_i(1)$ due to **Theorem 2**. Next, assume opponent j truthfully submits bids $\alpha_j(1) = d$ and $\alpha_j(2) = 2 \cdot d$. Now, suppose agent i chooses a deviating budget constraint of $\hat{\alpha}_i(1) > \alpha_i(1)$ for one unit from the menu, which by construction implies a deviating package budget constraint of $\hat{\alpha}_i(2) = 2 \cdot \hat{\alpha}_i(1)$. This deviation results in the *single-winner* award as $2 \cdot \hat{\alpha}_i(1) > \hat{\alpha}_i(1) + \alpha_j(1)$ and $2 \cdot \hat{\alpha}_i(1) > 2 \cdot \alpha_j(1)$ and agent i wins the package. For this deviation to be unprofitable, the following incentive-compatibility constraint (IC1) needs to be satisfied:

$$w\big(v(1)\big) + m_i(1) \geq w\big(v(2)\big) + m_i(2) \qquad \textbf{(IC1)}$$

Any profit-maximizing principal chooses $m_i(2) = 0$, and thus IC1 can be rewritten as: $m_i(1) \geq w\big(v_i(2)\big) - w\big(v_i(1)\big)$. Principal i could now offer a menu of payments $m_i(1) = w\big(\overline{v}(2)\big) - w\left(\underline{v}(1)\right)$ with corresponding budget constraints of $\alpha_i(1) = d$ and $\alpha_i(2) = 2 \cdot d$. However, as d could be the result of various $\overline{v}(2)$ and $\underline{v}(1)$ pairings agent i has an incentive to choose the pairing that offers the highest payment and still satisfies $d = \overline{v}(2) - \underline{v}(1)$. Agent i with package valuations of $v_i = (\overline{\overline{v}(2)}, \underline{\underline{v}(1)})$ has the highest incentive to deviate the optimal payment scheme must involve constant transfers in height of $m(1) = w\left(\overline{\overline{v}(2)}\right) - w\left(\underline{\underline{v}(1)}\right)$.

Finally, suppose agent i chooses a deviating single-unit budget constraint of $\hat{\alpha}_i(1) < \alpha_i(1)$ from the menu, which by construction implies a deviating budget of $\hat{\alpha}_i(2) = 2 \cdot \hat{\alpha}_i(1)$ for two units. Note that this deviation results in the *single-winner* award as $2 \cdot \hat{\alpha}_i(1) < \hat{\alpha}_i(1) + \alpha_j(1)$ and $2 \cdot \hat{\alpha}_i(1) < 2 \cdot \alpha_j(1)$. Here, agent j wins the package and agent i wins nothing. According to IC1, this deviation cannot be optimal for agent i. **QED.**

Appendix II: Formal Description of the Ascending Auctions

In what follows, we discuss the possible outcomes of the 2×2 *ascending package auction*:

- Case 1: Both bidders only bid on one unit and the auction stops immediately with both bidders i and j winning one unit each at a price of zero.
- Case 2: Bidder i bids on the package, while bidder j only bids on a single unit. The respective prices $p_i(2)$ and $p_j(1)$ rise on equal level. If bidder i releases the button and does not switch to one unit, then bidder j wins the package at price $p_j(2) = \beta_i(2)$ because of $v_j(1) < v_j(2)$. If bidder i reduces demand to one unit, we are in case 1 and each bidder i wins one unit at a price of $p_i(1) = \beta_i(2)/2$. If bidder j releases the button on one unit and does not switch to the package, then the package is assigned to bidder i at a price of $p_i(2) = \beta_j(1)$. If bidder j switches to the package, we are in case 3.
- Case 3: Both bidders only bid on the package and the prices for $p_i(2)$ and $p_j(2)$ continue to rise at the same level until one bidder stops bidding on the package. The bidder, who dropped out on two units can still bid on one unit, such that we are in case 2. If one bidder i stops pressing both buttons, the auction stops and the opponent j wins the package at a price of $p_j(2) = \beta_i(2)$. If both bidders reduce demand to one unit simultaneously, we are in case 1 and each bidder i wins one unit at a price of $p_i(1) = \beta_i(2)/2$. If both bidders drop out from the auction at the same time, the auctioneer randomly assigns the package to one of the bidders at the current price.
- Case 4: One bidder i bids on the package and the single unit, the other bidder j only on a single unit. Prices rise on $p_i(1)$, $p_i(2)$, and $p_j(1)$ according to the indifference condition of the auctioneer, $p_i(1) + p_j(1) = p_i(2)$. If bidder i stops bidding on the package, we are in case 1 and each bidder i wins one unit at a price of $p_i(1) = \beta_i(2)/2$. If bidder i stops bidding on the one unit, we are in case 2. If bidder i releases both buttons, bidder j wins the package at price $p_j(2) = \beta_i(2)$. If bidder j stops bidding on one unit, bidder i wins the package at price $p_i(2) = 2\beta_j(1)$.

- Case 5: One bidder i bids on the package and the single unit, the other bidder j only on the package. Prices rise on $p_i(2)$, $p_j(2)$ and $p_i(1)$ on the same level. If bidder *i* stops bidding on the single unit, we are in case 3 but have to take into account the standing price on one unit of $p_i(1) = \beta_i(1)$. If bidder *i* stops bidding on the package, we are in case 2. If bidder *j* stops bidding on the package, the opponent wins the package at price $p_i(2) = \beta_j(2)$.
- Case 6: Both bidders bid on the package and the single unit. Prices rise on all clocks and the single-unit prices are in sum at the level of the package prices, which are equal. If one of them stops bidding on one unit, we are in case 5. If one of them stops bidding on the package, we are in case 4. If both release the button for the package, this is case 1 and each bidder i wins one unit at price of $p_i(1) = \beta_i(2)/2$. If both simultaneously release the button for the single unit, we are in case 3. In case one bidder stops bidding on the package and her opponent stops bidding on one unit, we are in case 2. If both bidders simultaneously release both buttons, the auctioneer assigns each bidder $i$one object at price of $p_i(1) = \beta_i(2)/2$.

In contrast, the following outcomes are possible for the 2×2 *ascending uniform-price auction*:

- Case 1: Both bidders only bid on one unit and the auction stops immediately with both bidders winning one unit at unit price of zero.
- Case 2: Bidder *i* bids on two units, while bidder *j* only bids on a single unit. The price increases. If bidder *i* stops bidding on the second unit, the auction stops and both bidders win one unit at current unit price of *p*. If bidder *j* stops bidding on her one unit, the auction stops and bidder *i* wins both units at twice the current unit price of *p*. If bidder *i* stops bidding on both units simultaneously, then the auction stops and the auctioneer awards one unit to both bidders at the current unit price of *p*.
- Case 3: Both bidders bid on two units. If one bidder stops bidding on both units simultaneously, the other bidder gets the package of both units at twice the current unit price of *p*. If one bidder

drops out on one of the items, we are in case 2. If both bidders simultaneously stop bidding on the second unit, we are in case 1.

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[2] Bulow et al. (2009) writes that "Prior to the AWS auction, analyst estimates of auction revenue ranged from $7 to $15 billion. For the recent 700 MHz auction, they varied over an enormous range – from $10 to $30 billion." This is by no means an exception and estimates of investment banks and other external observers can be quite different from the actual revenue of the auction. Prior to the German spectrum auction in 2010, most analysts expected low revenue (Berenberg Bank estimated €1.67 bn. and the LBBW bank estimated €2.10 bn.). The actual revenue from the auction was €5.00 bn.

[3] Burkett (2015) has to be given credit for introducing principal-agent relationships in a single-object auction context. He showed how the fact that budget constraints are endogenously set by the principal to mitigate the agency problem affects the standard revenue comparisons between FPSB and SPSB auctions. Later, Burkett (2016) studies a principal's optimal choice of the budget constraint for an agent participating in an auction-like direct-revelations mechanism. Principal and agent are assumed to be equity holders in the firm, interested in maximizing the firm's expected return at the auction, but the bidder receives an additional private payoff when the firm wins the good. In contrast, we model a complementary environment with multiple objects in which agents are no equity holders. The types of manipulation possible for agents in such multi-object markets are quite different from single-object auctions. We show that the information asymmetry and the different preferences result in an agency dilemma that is difficult to resolve.

[4] Many spectrum auctions are for homogeneous goods, i.e. multiple licenses of 5 MHz spectrum in a particular band. While the strategic problems discussed can also be found with heterogeneous goods, markets with homogeneous goods require less notational burden.

[5] For example, France (2011) and Norway (2013) used a FPSB package auction, whereas Romania (2012) used an ascending combinatorial clock auction: https://www.ofcom.org.uk/__data/assets/pdf_file/0021/74109/telefonica_response.pdf

[6] We refer to risk aversion as is implied by a concave utility function $w(\cdot)$ over possible outcomes of the auction (lottery) $\{0, v_i(1), v_i(2)\}$ for some agent i. Regarding **Theorem 6** and **Theorem 10** one could expect a risk averse agent to prefer the certain *dual-winner outcome* with utility of $w\big(v_i(1)\big)$ over the lottery of winning the *single-winner* outcome with utility of $w\big(v_i(2)\big) \cdot F_2\big(v_i(2)\big)$ and not winning at all.

[7] Given firm j reports zero on two units, principal i is indifferent between reporting truthfully and fully shading on the package. Fully shading on one unit is weakly dominated. Thus, the budget regime from **Theorem 11** carries over to the *2×2 package auction model* under the VCG mechanism. The agents are not affected by the generalized second-price payment rule and their reporting behavior corresponds to the one in **Theorem 5**.

[8] We leave revenue maximization as a topic for future research and focus on efficient auction design, which is the primary goal of regulators.

[9] http://www.bundesnetzagentur.de/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OeffentlicheNetze/Mobilfunknetze/Projekt2016/projekt2016-node.html

[10] http://telecoms.com/opinion/the-german-spectrum-auction-failure-to-negotiate/

[11] The strategic analysis of the wide-spread two-stage combinatorial clock auction is more involved, which is why we do not discuss it in this context (Bichler and Goeree 2017).

[12] https://www.ofcom.org.uk/__data/assets/pdf_file/0021/74109/telefonica_response.pdf