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Coalition-based Pricing in Ascending Combinatorial Auctions

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Bidders in larger ascending combinatorial auctions face a substantial coordination problem, which has received little attention in the literature. The coordination problem manifests itself by the fact that losing bidders need to submit non-overlapping package bids which are high enough to outbid the standing winners. We propose an auction format, which leverages the information that the auctioneer collects throughout the auction about the preferences of individual bidders and suggests prices for the members of losing bidder coalitions, which in total would make a given coalition winning. We model the bidder's bundle selection problem as a coordination game, which provides a theoretical rationale for bidders to agree to these prices, and highlights the role of the auctioneer in providing relevant information feedback. Results of extensive numerical simulations and experiments with human participants demonstrate that this type of pricing substantially reduces the number of auction rounds and bids necessary to find a competitive equilibrium, and at the same time significantly increases auction efficiency in the lab. This rapid convergence is crucial for the practical viability of combinatorial auctions in larger markets.

Key words: multi-object auctions, coordination problem, coalition-based pricing, bidder behavior

1. Introduction

The need to buy or sell multiple objects arises in areas such as industrial procurement, logistics, and government allocation of spectrum licenses or other assets. It is a truly fundamental problem and, due to the advances in modern computing and communication capabilities which allow to adopt advanced auction mechanisms in increasingly broader and larger-scale online settings, the theory on how multiple indivisible objects should be allocated via an auction has enjoyed renewed interest in recent years (Krishna 2002, Cramton et al. 2006, Bichler et al. 2010). One of the key goals in this research literature is to develop mechanisms that achieve high (allocative) efficiency. *Allocative efficiency* measures whether the auctioned objects end up with the bidders who have the highest valuations for them, representing a measure of social welfare.

In this paper we aim to design highly efficient ascending combinatorial auctions. Among the problems that make this goal hard to achieve, the *coordination* of bidders is a key problem that has been largely underexplored in prior work. Specifically, the currently losing bidders have a task of identifying individually profitable and collectively complementary item bundles to bid on from a exponentially-sized set of all possible bundles (and determine appropriate bid prices for them), which together stand a chance of becoming a winning bid set in the next round. Identifying such bundle bids is possible either via a large number of auction rounds or requires a highly non-trivial coordination among coalitions of losing bidders, which is difficult without appropriate price feedback. We address this challenge by proposing a *coalitional pricing rule*, which is able to draw on the data that the auctioneer collects about bidders preferences throughout the auction and, as a result, helps currently losing bidders to coordinate. The proposed combinatorial auction mechanism exhibits substantially improved convergence and increased efficiency in lab experiments.

1.1. The Need for Ascending Combinatorial Auctions

Combinatorial auctions are among the most general types of multi-object market mechanisms, as they allow selling (or buying) a set of heterogeneous items to (or from) multiple bidders. Bidders can specify package (or bundle) bids, i.e., prices are defined individually for subsets of items that are auctioned (Cramton et al. 2006). The price is only valid for the entire bundle, and the bid is indivisible. For example, in a combinatorial auction a bidder might be willing to buy a bundle, consisting of items A and B, for a bundle price of €100, which might be more than the sum of the item prices for A (€30) and B (€50) that the bidder is willing to pay, if items are bought individually. The ability to submit bundle bids allows the bidders to express their economic preferences precisely, which is valuable in settings where bidders may have superadditive (or subadditive) valuations, i.e., when the bidder's valuation of the entire bundle is higher (or lower) than the sum of individual item valuations, as in the above example. We will refer to a bidding language as a set of allowable bid types (e.g., bundle bids or bids on individual items only) in an auction. If bidders can win multiple bids, this is referred to as an OR bidding language. If they can only win a single bid at most, then it is a XOR bidding language.

In simultaneous multi-object auctions where only individual item bids are allowed, bidders incur the risk that they may end up winning only a subset of items from their desired bundle, and that they may end up paying too much for this subset. This is called the *exposure problem* (Rothkopf et al. 1998). While bidding on bundles in combinatorial auctions solves this problem, the design of these auctions leads to several types of complexity. One type of complexity is computational complexity when determining an optimal allocation. Other types of complexity are strategic complexity for bidders, and communication complexity. Strategic complexity describes the difficulty for bidders to find an optimal bidding strategy, while communication complexity describes the number of messages (i.e., price announcements and bid submissions) which need to be exchanged between the auctioneer and the bidders in order to determine the efficient allocation. It has been shown that the communication complexity to find the efficient solution in combinatorial auctions is exponential with respect to the number of items (Nisan and Segal 2006).

Computational complexity is manageable in real-world applications with a low number of items, bidders, and submitted bids; e.g., a winner determination problem with 20-30 items and 10 bidders can typically be solved in seconds. In terms of strategic complexity, one possible solution is to use the Vickrey-Clarke-Groves (VCG) mechanism, which achieves efficiency in dominant strategies, i.e., bidders cannot increase their payoff by deviating from a truthful revelation of their valuations. Unfortunately, VCG is rarely used due to a number of practical problems (Ausubel and Milgrom 2006b). In particular, in many markets bidders are simply reluctant to reveal their true valuations to an auctioneer in a single-round sealed-bid auction, and they prefer an ascending (multi-round) auction format which is more transparent and conveys information about the competition in the market. In a recent paper, Levin and Skrzypacz (2014) write that dynamic auctions have an advantage in multi-item settings, because bidders can gradually find out how their demands fit together. This property is important but not necessarily given in all multi-item auction designs.

1.2. Inefficiency in ascending combinatorial auctions

As a result, much recent research has focused on ascending multi-object auctions, i.e., generalizations of the single-item English auction where bidders can outbid each other iteratively. However, it was recently shown that no ascending multi-object auction format can be incentive-compatible for general types of bidder valuations when modeled as a Bayesian game (Sano 2012, Goeree and Lien 2013). In other words, with sufficient prior information about other bidders and allowing any type of valuations, it is always possible that a bidder may profit from not bidding truthfully up to his valuation. Let's consider a simple example with two identical items and three bidders. One "global" bidder is only interested in the bundle of two items, while each of the two "local" bidders wants only one of the items. If the local bidders together are stronger than the global bidder (i.e., the sum of their one-item valuations is higher than the global bidder's two-item valuation), then they could always try to free-ride on each other. Suppose that the global bidder has a valuation of \$10 for the two-item bundle, and each local bidder has a valuation of \$8 for their item of interest. One local bidder could drop out at a price of \$2.5, such that the other local bidder is forced to bid up to \$7.5 to become winning. Sano (2012) has shown that, without complete information but having prior distributional information about bidder valuations in a Bayesian game, a local bidder might drop out too early resulting in the auction being inefficient in equilibirium. In any case, this and similar types of manipulations are only possible with sufficient prior information about other bidders' valuations.

In many real-world markets, the information set available to bidders is quite different from markets modeled under complete information or as Bayesian games with single-minded bidders. Bidders are interested in multiple packages, and it is unknown to a bidder which packages are of interest to his competitors. Also, the *common prior* assumption in Bayesian games, which has long been a concern in game theory (Wilson 1987), is particularly troublesome in combinatorial auctions with exponentially many possible packages a bidder can bid on. Bidders would need to have the same prior distributions for all possible packages, which is unrealistic in all but very small combinatorial auctions. In addition, in many auctions in procurement or on the Internet, the number of competitors is unknown, and there can always be a new bidder throughout the auction. Bid shading is less of a concern in such environments.

Even if bidders do not shade their bids due to a lack of prior information about others' valuations, this does not automatically lead to efficient outcomes because of the communication complexity of combinatorial auctions. If bidders do not bid on all bundles of positive value to them, but only a small subset thereof, then the auction may not end with an efficient outcome. This restricted bundle selection has been experimentally shown to be the biggest barrier to efficiency across auction formats (Scheffel et al. 2012).

Due to the exponential growth of possible bundles, even in combinatorial auctions with only 20 items bidders would not be able to reveal over a million possible bundle valuations, i.e., to submit all possible bundle bids. Recent combinatorial auctions used for spectrum sales had 100 licenses simultaneously on sale. It is clearly impossible to enumerate all the exponentially many bundles for a bidder. Finding promising bundles, i.e., bundles that stand a chance of becoming winning when combined with the bids of other bidders, becomes the central strategic problem of bidders in such auctions, which has largely been ignored in the game-theoretical literature on combinatorial auctions. This *coordination problem* requires different theoretical models.

1.3. Contributions and outline

The main contribution of this work is to propose an auction format that leverages the bidding information that the auctioneer collects throughout an ascending auction about losing, but high-revenue coalitions. We select such high-revenue coalitions and propose ask prices to the members of each coalition such that together they can outbid the current winning coalition. This new type of pricing rule is called "coalitional winning level (CWL)." The auctioneer can provide such prices based on the bids that he collected in past rounds, and he can distribute the additional amount needed to make the losing coalition winning in a fair manner using the cost-sharing rule based on the Shapley value (Dehez 2007). In particular, we show that such a cost-sharing rule not only satisfies fairness axioms, but also results in a cost sharing among the bidders in a coalition that is *in the core*, i.e., it does not create incentives to deviate for a subset of the bidder coalition.

In this work, we are focusing on markets where bidders typically do not have reliable prior information about other bidders' valuations or the number of their competitors. The coordination problem introduced earlier is the central strategic challenge for bidders in such markets. Bidders have an exponential number of packages to chose from, but want to coordinate on competitive equilibrium in a low number of rounds.

We introduce a stylized game-theoretical model of the strategic problem of bidders, which helps understand the role of the auctioneer aiding coordination in a CWL auction. In contrast to earlier game-theoretical models in this field, we allow bidders to be interested in multiple packages. Thus, the assumptions in our model highlight the coordination problem of bidders rather than the free-rider problem. We focus on markets without reliable prior distributional information, so that we can restrict our attention to the bidder's decision problem in a single bidding round. It can be shown that the auctioneer in this auction model acts like a third party in a correlated equilibrium. This analogy provides an explanation why rational bidders accept a CWL ask price for a package, even when the given package does not maximize absolute payoff in the given round based on minimum bid prices. In other words, acting on the information feedback provided by the auctioneer is a rational strategy for bidders even in a complete information model, where bidders have full information about other bidders' valuations which they could possibility use to manipulate. Also, in an online combinatorial auction in the field, without prior distributional valuation information and with hundreds of packages to bid on, the ask prices (CWLs) of the auctioneer provide a helpful recommendation how complementary bids of losing bidders can become winning in the next round.

Our experimental results show that the the CWL auction has significantly higher efficiency, and at the same time communication with the auctioneer is substantially reduced compared to ascending auction designs from prior literature. In our experiments we do not see free-riding behavior, i.e., the bidders indeed take advantage of ask prices to coordinate. This rapid convergence of the auction increases the practical applicability of the mechanism to a broad set of application settings.

The paper is structured as follows. In Section 2, we discuss related literature. In Section 3, we introduce the auction format and describe theoretical properties. Section 4 presents the experimental design, while Section 5 summarizes the results of the numerical simulations. These simulations provide an indicator for the outcome of such auctions with truthful bidders who bid on their payoff-maximizing bundles in each round. In Section 6 we summarize and discuss the findings of our lab experiments, before concluding the article with Section 7.

2. Related literature

Let us briefly survey the relevant literature in this section. As mentioned earlier, the well-known Vickrey-Clarke-Groves (VCG) mechanism achieves efficiency in dominant strategies. Its central limitation is that the auction outcome might not be *in the core* (Ausubel and Milgrom 2006b), i.e., the winning coalition of bidders might have to pay less than what a losing coalition of bidders was willing to pay. This is possible due to the Vickrey discount which the winning bidders are given (Goeree and Lien 2013). We provide a simple example with two items (A and B) and three bidders (1, 2, and 3) to make this apparent. Suppose bidder 1 only wants A for which he has bid his value of €7, bidder 2 only wants B for which he has bid his value of €8, and bidder 3 only wants the package AB with a value of €10. The auctioneer declares bidder 1 and 2 to be winners and the maximum of total valuations of the sale to be €15. In a VCG mechanism the winners get a discount, which incentivizes truthful bidding. As a result, the Vickrey payment for bidder 1 is €10 − €8 = €2 and that of bidder 2 is €10 − €7 = €3. Consequently, the auction revenue is €5, although bidder 3 was willing to pay €10. In many applications, such as high-stakes government auctions, such an outcome might be difficult to justify. Therefore, such high-stakes auctions are typically conducted as open-cry ascending auctions, rather than sealed-bid events. For these reasons, during the recent years there has been an increasing interest in core-selecting auctions (Day and Milgrom 2008), i.e., auctions where there cannot be a losing coalition of bidders that together could have outbid the winners based on their submitted bids.

iBundle (Parkes and Ungar 2000), the ascending proxy auction (APA) (Ausubel and Milgrom 2006a), and dVSV (de Vries et al. 2007) are examples of ascending core-selecting auction formats which provide allocatively efficient solutions when bidders follow a *straightforward* bidding strategy, i.e., when they truthfully bid on their payoff-maximizing bundle(s) in each round until the prices stop because a bidder becomes winning. If bidder valuations are *buyer submodular*, then this strategy is even an ex post Nash equilibrium, which is a strong solution concept where bidders do not need to reason about other bidders' valuations. Buyer submodularity requires that, if a bidder is added to a smaller coalition, then he adds more to the overall revenue than if added to a larger coalition with more bidders (Parkes 2006).

Note that super-additive valuations violate buyer submodularity. In the above example with three bidders, the third bidder has super-additive valuations since both items are complements for him (i.e., he does not want each item individually, only their combination), thus violating buyer submodularity. Also, it is easy to see that it is not an ex post Nash equilibrium strategy to bid truthfully until the price clock stops or the valuation is reached in this example. Bidder 1 might drop out before his price for one unit stops increasing for his one item and he might free-ride on bidder 2, who then needs to outbid bidder 3 in an ascending auction. In such cases, only a Bayesian Nash equilibrium is possible, which requires prior distributional information about other bidders' valuations. In this market, when bidders only have prior distributions about the valuations, the Bayes-Nash equilibrium strategy can even lead to non-bidding of the local bidders, and consequently, to inefficient outcomes (Sano 2012, Guler et al. 2016). We will refer to the family of efficient ascending multi-object auctions, which allow for an ex post Nash equilibrium at least for buyer submodularity, as *bidder-optimal ascending core-selecting* (BACS) auctions.

Adomavicius and Gupta (2005) introduce deadness (*DL*s) and winning levels (*WL*s) both as pricing rules and information feedback to bidders in combinatorial auctions and evaluate them in the lab (Adomavicius et al. 2013). Deadness levels are the lowest prices above which a bid can still potentially become winning in any future auction state (depending on the arrival of complementary bids from other bidders), winning levels are prices above which a bid would immediately become winning. Petrakis et al. (2013) showed that ascending combinatorial auctions with deadness levels as ask prices (the *DL* auction) belong to the above family of BACS auctions and share the same ex post Nash equilibrium strategy as BACS auctions.

BACS auctions can be thought of as algorithms designed to provide an exact solution to a hard computational problem. However, they typically lead to a huge number of auction rounds (Schneider et al. 2010), and lab experiments provide evidence that human bidders substantially deviate from straightforward bidding (Scheffel et al. 2011) so that efficiency is no longer guaranteed. More recent experimental research shows that restricted bundle selection due to the exponential growth of bundles is the main reason for inefficiency in combinatorial auctions (Scheffel et al. 2012, Bichler et al. 2013), while bid shading is much less of an issue. Rather than shading their bids optimally, bidders in such auctions are primarily concerned with finding the right bundle which, together with the bids of other bidders, will end up in a winning coalition. We will refer to this problem as the "coordination problem" and to markets with little or no distributional information about bidder valuations as "online" markets. We use the term *online markets* related to the concept of online algorithms or online mechanisms from the literature in computer science (Parkes 2007).

Our proposed combinatorial auction mechanism is different from the ones mentioned above. In particular, the auctioneer targets losing coalitions by proposing *coalitional winning levels* (*CWL*s) as ask prices to the members of these coalitions, which would allow them to jointly outbid the currently winning coalition. The semantics of *CWL*s is intuitive for bidders and provides guidance in what is arguably *the central problem* that bidders face in each round of a combinatorial auction with many items: the selection of promising bundles, which stand a chance of becoming winning together with the bids of other bidders.

3. The auctions

In what follows, we will briefly describe the *DL* auction as a representative of BACS (i.e., bidder-optimal ascending core-selecting) auctions. We will then introduce the proposed *CWL* auction and discuss some theoretical underpinnings of such an auction. Before we do this, we provide an example of different pricing rules as they have been discussed in the literature to better illustrate the different approaches.

3.1. An introductory example with different pricing rules

The following example extends the one used by Petrakis et al. (2013) to illustrate *DL*s. We compare *CWL*s with *iBundle*, *DL*s, *WL*s, and an auction format with linear ask prices, RAD (Kwasnica et al. 2005).

bundles	AB	BC	AC	B	C
bids	$22_1^*, 16_2$	24_3	20_4	7_5	8_6^*
DL	22_1, 16_2	24_3	20_4	7_5	8_6
WL	22_1, 22_2	30_3	23_4	10_5	8_6
iBundle	22_1, 17_2	25_3	21_4	8_5	8_6
RAD	22	24	14	16	8
CWL	22_1, 22_2	30_3	$\mathbf{21.5_4}$	$\mathbf{8.5_5}$	8_6

Table 1 Example with six bids and different ask prices.

The top two rows of Table 1 describe six bids from different bidders (i.e., bidders 1 to 6), submitted on subsets of three items (A, B, and C). In this example we will assume that at this point in the auction, bidders are only interested in those bundles for which they have submitted bids so far.

The bottom five rows of Table 1 describe bundle prices in different auction formats at this stage in the auction (i.e., after 6 bids have been submitted). Subscripts indicate bidders, i.e., 22_1 indicates a bid of €22 from bidder 1. Ask prices have subscripts only if they differ among bidders in this example. Asterisks denote the provisional winning bids. In this example, we assume a XOR bid language, where each bidder can win at most one bundle. For such languages, it is known that *DL*s and *WL*s for a given bundle may have different values for different bidders, i.e., their computation needs to be personalized (Petrakis et al. 2013). Losing bidders need to bid higher than these values by a minimum bid increment. As mentioned earlier, the *WL* for a given bundle describes the lowest bid price above which a submitted bid would instantly become winning, i.e., without needing any new complementary bids from other bidders. However, it is clear from the example that bidders 4 and 5 could possibly become winning even at lower prices than their current *WL*s, if they coordinate and form a coalition, indicated by prices in bold type. Ask prices in iBundle (Parkes and Ungar

2000) are in line with *DL*s, but they add a bid increment (€ 1) for losing bids. Linear programming-based heuristics for computing linear prices (i.e., where a bundle price is simply a sum of individual item prices) such as *RAD* (Kwasnica et al. 2005) are an alternative. Unfortunately, *RAD* prices can be lower than a losing bid (see the *RAD* ask price on *AC* for bidder 4) or unnecessarily much higher than the sufficient winning bid (see the *RAD* ask price on *B* for bidder 5) (Bichler et al. 2009).

In an online market, bidders typically start out bidding on their highest valued packages in order to find out if this package can become winning together with the bids of others. After the winner determination, the auctioneer can evaluate which losing bidders would, in combination, achieve high revenue and have a potential to outbid the current winning coalition. *CWL*s can be seen as a way to derive personalized and non-linear ask prices in-between *DL*s and *WL*s designed to quickly find a *competitive equilibrium*, i.e., a state where there is no coalition of bidders that can outbid the currently winning coalition of bidders at these ask prices, as defined below.

Definition 1 (Competitive Equilibrium, CE (Parkes 2006)). Prices α and allocation X^* are in competitive equilibrium if allocation X^* maximizes the payoff of every bidder and the auctioneer revenue given prices α. The allocation X^* is said to be *supported* by prices α in CE.

It has been shown that competitive equilibrium and the core, mentioned in Section 2, refer to the same concept in multi-object auctions (Bikhchandani and Ostroy 2002). *CWL*s are a way to find such prices such that, at the end of an ascending auction, there is no coalition of losing bids who could make themselves better off. *CWL*s leverage the information that is available about losing coalitions during the auction and provide tailor-made prices to bidders in these coalitions, i.e., proposals on how they can jointly outbid the currently winning coalition. The coalition of bidders 4 and 5 in our example would only need to increase their bids by a combined € 3 plus increment in order to become winning. Both bidders would become winning, if bidder 4 bids above € 21.5 and bidder 5 bids above € 8.5, for example. The proposed *CWL* feedback is designed to help coordinating bidders who form a high-revenue coalition, and it is particularly useful if bidders are interested in many packages. In what follows, we will describe *DL*s and *WL*s in a more formal way before we introduce *CWL*s and their properties.

3.2. The *DL* auction

We will first introduce the necessary notation and then describe the *DL* auction as an example of a BACS auction. There is set $\mathcal{K}$ of m indivisible items indexed with k, which are auctioned among set $\mathcal{I}$ of n bidders. Let $i, j \in \mathcal{I}$ denote the bidders and $v_i : 2^{\mathcal{K}} \to \mathbb{R}$ denote a value function of bidder i, which assigns a real value to every subset $S \subseteq \mathcal{K}$ of items. The bundle that is assigned to bidder i in allocation X is denoted as $X_i \subseteq \mathcal{K}$. We denote $X = (X_1, ..., X_n)$ as an allocation of the m items among bidders, with $X_i \cap X_j = \emptyset$ for every $i \neq j$, with $i, j \in \mathcal{I}$. A coalition is defined as a set of bidders whose bids constitute a feasible allocation. A winning coalition is the coalition of bidders whose bids constitute the revenue maximizing allocation, and a losing coalition is any coalition except the winning coalition. In other words, a bidder can be a member of the winning coalition and, at the same time, be a member of multiple losing coalitions, based on the bids he submitted. We denote a losing coalition L, and the set of all losing coalitions $\mathcal{L}$. Let Γ denote the set of all possible allocations, then $X^L \in \Gamma$ denotes an allocation of items among a losing coalition $L \in \mathcal{L}$.

The social welfare of an allocation $X = (X_1, ..., X_n)$ is $\sum_{i \in \mathcal{I}} v_i(X_i)$, and an efficient allocation X^* maximizes social welfare among all allocations X, i.e. $X^* \in \arg\max_X \sum_{i \in \mathcal{I}} v_i(X_i)$. The revenue maximizing allocation, $\overline{X}$, is such that $\overline{X} \in \arg\max_X \sum_{i \in \mathcal{I}} b_i(X_i)$, where b_i is the bid price of bidder i for the bundle assigned to him in allocation X.

We focus on ascending combinatorial auctions (CAs), which consist of different rounds and where an ask price $\alpha_i(S)$ is available for each bundle S and each bidder i in each round. A round defines a certain time period during which the auctioneer collects new bids from bidders and at the end of which a new allocation and new ask prices for the next round are computed. The *DL* auction uses the XOR bidding language, i.e., at most one bundle bid from a given bidder could be winning at any given time. Let B^t denote the bids submitted in round $t \in \mathbb{N}$, and $\overline{X}^t$ denote the revenue maximizing allocation after round t, based on the set of all bids B submitted in the auction so far. Note that in theory a round could close after each new bid submitted, such that t may also refer to a single bid. In our experiments each bidder can submit multiple bids in a round. B_l^t denotes the set of all losing bundle bids, even losing bids from a winning bidder (because a winning bidder can only win at most one bundle, but may well submit multiple bids in each round), after round t. ϵ describes a minimum bid increment per round. We next define deadness levels as ask prices:

Definition 2. The *deadness level*, *DL*, of a bundle S for bidder i at round t, $DL^t(i, S)$, is the minimal price that bidder i has to overbid to maintain a chance to win S at some future round $t' > t$.

So *DL*s are the highest prices at or below which a bid cannot become winning in any future auction state. Therefore, they constitute a lower bound for acceptable new bids. The *DL* auction uses only *DL* ask prices (i.e., DL + epsilon) and belongs to the family of BACS auctions, as was shown in Petrakis et al. (2013). Algorithm 1 outlines the *DL* auction.

As shown in Algorithm 1, the *DL* auction is conducted in a round-based ascending format. The auction begins with ask prices of 0. At the start of each subsequent round, each bidder is given the following information: ask prices on all bundles he has bid on so far in the auction (but can request ask prices of any other bundles on demand, if desired) and information whether he is currently winning any bundle he has previously bid on. This is the only information made available to each bidder, and he does not know anything else about other bidders, including the bundles they have bid on so far, or if they are winning some bundles. A bidder can then submit as many bids as he likes in this round. However, the bids need to be higher than or equal to the respective ask prices. A bid higher than the ask price is called a jump bid. When all bidders finish submitting new bids, the round closes and a new allocation with the current winning coalition as well as new ask prices are computed. The auction terminates when no new bid is submitted in a round, and winning bidders pay what they bid.

Algorithm 1: The *DL* auction algorithm

Result: $\overline{X}$ and bid prices $b_i(\overline{X}_i)$
Initialization
  for $i=1$ **to** n **do**
    foreach S **do** $\alpha_i(S) \leftarrow \epsilon$
    $X_i \leftarrow \emptyset$
  end
  termination $\leftarrow$ false
  $t \leftarrow 0$
  $B \leftarrow \emptyset$
while $(\neg$ *termination*$)$ **do**
  $t \leftarrow t+1$
  Bidders submit bids B^t where each $b_i(S) \in B^t$ satisfies $b_i(S) \geq \alpha_i(S)$
  if $(B^t = \emptyset)$ **then** *termination* $\leftarrow$ true
  else
    $B \leftarrow B \cup B^t$
    Compute $\overline{X}^t \in \arg\max_X \sum_{i \in \mathcal{I}} b_i(X_i)$
    foreach $b_i(S) \in B_l^t$ **do** $\alpha_i(S) \leftarrow DL^t(i,S) + \epsilon$
  end
end

We have also implemented two additional auction rules: an activity rule to incentivize bidders to stay active from the start, and the possibility for bidders to submit a so-called "*last-and-final bid*," which helps to avoid small efficiency losses due to bid increments. Both are described below and complete our description of the *DL* auction.

In general, ascending multi-object auctions can enforce different types of activity rules. In our experiments, we used an activity rule in line with earlier experiments in the combinatorial auction literature (Scheffel et al. 2012). If a currently losing bidder in round t does not submit any new bids in round $t+1$, then he is not allowed to bid in any future rounds $t' > t$. All his previous active bids will still be considered relevant for the auction, but he may not submit any new bid again for the auction. This activity rule does not apply to currently winning bidders, as their inactivity does not necessarily imply that they are not interested in the auction any more. We have used this soft activity rule in both the *DL* and the proposed *CWL* auction.

In addition, it is possible to have a situation, where the new ask price $\alpha_i(S)$ is too high for a bidder because the bid increment was too big. In this case, the *DL* auction allows for a *last-and-final bid* between these bounds (i.e., between the bidder's last bid on the package and the new ask price $\alpha_i(S)$) (Parkes 2006). Suppose bidder i has submitted a bid of €18 on bundle AB in round t. In the next round $t+1$, he sees that he has not won AB, and the new ask price for AB, based on the *DL* plus an increment, is €22. Assuming the bidder's true valuation for AB is €20, then he can now submit a last-and-final bid of €$20-\epsilon$, where ϵ is a profit margin he wants to achieve. However, after this round he would not be able to bid on AB anymore.

We described the key details of the *DL* auction, and will now turn to one of the main properties satisfied by the *DL* auction, as it is related to the *straightforward* bidding strategy.

Definition 3. A *straightforward* bidder i only bids ask prices on his demand set $D_i = \{S \subseteq \mathcal{K} : v_i(S) - \alpha_i(S) \geq v_i(S') - \alpha_i(S'), \forall S' \subseteq \mathcal{K}\}$ in each round, i.e., on those bundles which maximize his payoff, based on given ask prices.

Importantly, in the *DL* auction format, straightforward bidding is an ex post equilibrium if bidders' valuations are submodular (Parkes 2006, Petrakis et al. 2013). This is because with such valuations the auctions end up in VCG prices for the winners, and bidders do not have an incentive to shade their bids (Parkes 2006). Unfortunately, straightforward bidding also leads to a large number of auction rounds as *all* losing package valuations get elicited from all bidders via minimum bid increments in each round. While this process allows to prove efficiency of the allocation, the number of bids that need to be submitted by bidders is beyond what human bidders can be expected to do, except in auctions with only very few items. Schneider et al. (2010) have shown using numerical simulations that, with straightforward bidding, even small auctions with only 9 items can easily lead to 150 and more auction rounds. With 10 minutes per round this would lead to 25 hours, which would be unacceptable in most applications.

3.3. Coalitional winning levels

In addition to deadness levels (*DL*s), Adomavicius and Gupta (2005) also defined winning levels (*WL*s) as a form of information feedback to bidders. $WL + \epsilon$ is the minimum bid price for a bidder on a bundle, such that this bundle bid becomes winning at round $t+1$, if no other bid was submitted. As indicated in the introduction, *WL*s can be prohibitively high for small bidders in larger auctions with many items, since *WL*s reflect an amount a bidder needs to bid to become winning unilaterally, i.e., without the help of any new bids of other bidders.

The coalitional winning level (*CWL*) extends the concept of a *WL* ask price from an individual bidder to a group of losing bidders. It is an ask price that would make a losing coalition winning, if accepted by all members of the coalition. This is valuable feedback for overcoming coordination problems inherent to all combinatorial auctions, as illustrated in the following example.

EXAMPLE 1. Consider four small bidders, each one bidding €10 on a different single item, and a large bidder bidding €100 on the bundle containing all four of these items. The valuation of each small bidder for their respective single item is €50. By definition, the *WL* ask price faced by each small bidder is €70 $+\epsilon$ for the desired item, indicating the scenario where each small bidder competes with the large bidder individually. The *WL*-based ask price is higher than the small bidders' valuations; as a result, the small bidders would not bid anymore, and the efficient allocation is not achieved. For comparison, the *CWL* for the losing coalition is €100 in total. So if, for example, each small bidder in the coalition receives an individual *CWL* ask price of €$25+\epsilon$ and bids on it, the coalition would outbid the large bidder. Finally, the *DL* of each bidder would be €10 only, as each bidder could become winning at €10 if the other small bidders outbid the large bidder.

Example 1 illustrates that the spread between *DL*s (here €10) and *WL*s (here €70) can be very large. In examples with many items and bidders being interested in many bundles, *CWL*s can give bidders useful information about bundles for which complementary bids exist. In addition, they can help bidders focus on a few (rather than all) of their bundles with positive valuations (i.e., where the bidder valuation for a bundle is higher than its current ask price).

Let L denote a coalition of losing bidders, where bidder i desires the bundle S_i, with $S_i \cap S_j = \emptyset$ for all $i, j \in L$. Denote the collection of the desired bundles as $S^L = \bigcup_{i \in L} S_i$.

DEFINITION 4. The *coalitional winning level*, *CWL*, of coalition L for the desired bundles S^L at a particular round t is the minimal price that the coalition must bid in aggregate to win these bundles at auction state $t+1$: $CWL^t(L, S^L) = \min \Sigma_{i \in L} b_i(S_i)$, so that $S_i \in \overline{X}^{t+1} \forall i \in L$, assuming all new bids of this coalition come in the next round, i.e., $t+1$.

In this definition we assume that only the losing bidders $i \in L$ submit bids in round $t+1$, so that $\overline{X}^{t+1}$ describes the revenue maximizing allocation in round $t+1$. The *CWL* value for any losing coalition L can be computed as follows:

$$CWL^t(L, S^L) = CAP^t(\mathcal{K}) - CAP^t(\mathcal{K}, S^L). \tag{1}$$

$CAP^t(\mathcal{K})$ denotes the optimal value of the winner determination problem (CAP), and $CAP^t(\mathcal{K}, S^L)$ the optimal value of CAP in which each bidder $i \in L$ wins his desired bundle $S_i \in S^L$ for free. There is substantial literature on the computational hardness of CAP (Leyton-Brown et al. 2006), but instances of up to 20-30 items and 10 bidders can typically be solved in seconds. The computation of *CWL*s in Equation 1 can easily be derived from the proof for $WL^t(i, S) = CAP^t(\mathcal{K}) - CAP^t(\mathcal{K}, S_i)$ in Petrakis et al. (2013), where the desired bundles S^L of all bidders $i \in L$ are treated as if they were one single bundle S_i of one single losing bidder i. Similar computations have also been described by Adomavicius and Gupta (2005).

3.4. Computing individual CWLs

Once $CWL^t(L,S^L)$ is computed for losing coalition L, we still face the question of how to distribute this price among members of L. Let us denote $CWL_i^t(L,S_i)$ as the amount that transforms the $CWL^t(L,S^L)$ to individual ask prices for every member i of losing coalition L. Here, S_i describes the package assigned to bidder $i \in L$. There are different ways how bidders in a coalition can share the additional amount $\Delta^t = CWL^t(L,S^L) - \Sigma_{i\in L} b_i(S_i)$ that is needed to outbid the current winning coalition. One could think of many *cost sharing functions* $\Delta_i^t = g_i(\Delta^t)$ to distribute Δ^t among the bidders $i \in L$ such that $\Sigma_{i\in L}\Delta_i^t = \Delta^t$. For example,

- $\Delta_i^t = \Delta^t \times \frac{|S_i|}{|S^L|}$ based on the bundle size $|S_i|$ of a bundle S_i within a coalition;
- $\Delta_i^t = \Delta^t \times \frac{b_i(S)}{\Sigma_{i\in L} b_i(S)}$ based on the level of the bid prices $b_i(S)$ within a coalition;
- $\Delta_i^t = \Delta^t \times \frac{1}{|L|}$ based on the number of members in a coalition (a.k.a. uniform distribution).

These heuristic cost sharing functions are simple because their calculations require only a few simple arithmetic operations. Aside from computational simplicity, a *fair* division of Δ^t among the bidders $i \in L$ would be a natural design goal. The Shapley value is arguably the most well-known solution concept for coalitional games (Dehez 2007), and it is considered fair, as it satisfies a number of fairness axioms including symmetry and additivity (Shoham and Leyton-Brown 2009).

Let's briefly review the Shapley value. Let L be a coalition of $|L|$ bidders, and $M \subseteq L$ be some sub-coalition. Let $w(M)$ denote the coalitional value of M that needs to be distributed among its members. Coalitional value can also be the cost that a coalition has to bear. The Shapley value θ_i provides a unique distribution (among the players) of the value generated by the coalition of all bidders $i \in L$ and is defined as:

$$\theta_i = \sum_{M\subseteq L\setminus\{i\}} \frac{|M|!(|L|-|M|-1)!}{|L|!}(w(M\cup\{i\}) - w(M)) \tag{2}$$

Overall, the Shapley value has a number of general properties, which are desirable. For example, it distributes the total value of a coalition. Bidders with the same contribution to the coalitional value get the same Shapley value. Bidders who do not contribute to the coalitional value get a zero Shapley value. However, if designed appropriately as a convex game, there are two properties of our coalitional game that make the Shapley value particularly desirable.

In *super-additive* games with $w(L\cup L') \geq w(L) + w(L')$ and $L \cap L' = \emptyset$, where L and L' are two losing coalitions, the Shapley value guarantees each participant a payoff of at least the amount that he could achieve by not forming a coalition. An important subclass of super-additive games are *convex* games. A game is convex if $w(L\cup L') \geq w(L) + w(L') - w(L\cap L')$. For every convex game, the core is nonempty, and the Shapley value is also in the core for convex games. This means that, based on the Shapley value, there cannot be a losing sub-coalition of $M \subset L$ that can make themselves better off as compared to a situation where all members of the coalition L accepted the Shapley value. In other words, sub-coalitions do not have an incentive to deviate, and the coalition can be considered stable as neither individuals nor groups of bidders in L have an incentive to deviate.

Note that sharing a given $\Delta^t = CWL^t(L,S^L) - \Sigma_{i\in L} b_i(S_i)$ in a round among a losing coalition of bidders is neither a convex nor a super-additive game. However, instead of distributing Δ^t, one can distribute the overall savings that the coalition experiences compared to the sum of the winning levels of each bidder: $\Psi^t = \sum_{i\in L} WL^t(i,S) - CWL^t(L,S^L) = \sum_{i\in L} WL^t(i,S) - \sum_{i\in L} CWL^t_i(L,S_i)$. We require that $WL^t(i,S) \geq CWL^t_i(L,S_i)$ for all $i \in L$ for sharing functions, where this is not always satisfied. We can now use the Shapley value to derive Ψ^t_i from Ψ^t, and this game is super-additive and convex as the following results show.

Lemma 1. *The game of distributing $\Psi^t(L) = \sum_{i\in L} WL^t(i,S) - CWL^t(L,S^L)$ to individual bidders $i \in L$ is super-additive.*

Lemma 2. *The game of distributing $\Psi^t(L) = \sum_{i\in L} WL^t(i,S) - CWL^t(L,S^L)$ to individual bidders $i \in L$ is convex.*

The proofs to both lemmata can be found in Appendix A. Let's now define the Shapley value (SV) based computation of individual CWLs.

Definition 5. A Shapley value based $CWL^t_{i,\mathrm{SV}}(L,S_i)$ is defined as $WL^t(i,S) - \Psi^t_{i,\mathrm{SV}}$, where

$$\Psi^t_{i,\mathrm{SV}} = \sum_{M\subseteq L\setminus\{i\}} \frac{|M|!(|L|-|M|-1)!}{|L|!}(\Psi^t(M\cup\{i\}) - \Psi^t(M)) \tag{3}$$

In this definition, $\Psi^t(M)$ is the total coalitional value or savings of coalition M. This leads to the following proposition.

Proposition 1. *Consider only the members $i \in L$ of a losing coalition, who need to derive individual CWLs from $CWL^t(L,S^L)$. No sub-coalition $M \subset L$ can make itself better off as compared to when all members of the coalition accepted $CWL^t_{i,SV}(L,S_i)$ given that $v_i(S_i) \geq CWL^t_{i,SV}(L,S_i)$ for all $i \in \mathcal{I}$.*

Proof: Lemma 2 shows that the computation of $CWL^t_{i,\mathrm{SV}}(L,S^L)$ constitutes a convex coalitional game. Every convex game has a nonempty core, and in every convex game the Shapley value is in the core (Shoham and Leyton-Brown 2009, p. 394). Q.E.D.

Explaining the Shapley value and its properties to subjects in the lab takes a substantial amount of time. Since it is important in an economic experiment that subjects fully understand the mechanism, in our experiments, we have therefore decided to use a cost-sharing rule based on a uniform distribution for simplicity.

3.5. The *CWL* auction

Based on the definition of *CWL*s, we will now describe the *CWL* auction. The auction process is identical to Algorithm 1 and the *DL* auction, including the availability of last-and-final bids and the activity rule. However, the ask prices are different. Instead of the computation of the *DL*s on line 16 in Algorithm 1, *CWL*s are computed for some of the highest revenue coalitions which are currently losing.

In case some members of a losing coalition do not accept the *CWL*, then this coalition would also not become winning in the very next round, but the members (other than the ones who submitted a last-and-final bid on a relevant package) can always update their bids in a new round. All new bids from the previous round are taken into account at the end of the round, and they can be the foundation for new coalitions to be built. Therefore, this process implicitly supports the collaborative search for a competitive equilibrium.

There are some degrees of freedom in how the auctioneer selects losing coalitions. The auctioneer could only select one or also a few disjoint losing coalitions in each round from the list of those losing coalitions with high revenue. How many losing coalitions are selected in each round depends on the size of the auction. Another implementation choice for the auctioneer is whether bidders are required to respond to a *CWL* immediately (i.e., in the next round) or not. For example, in order to proactively discourage free-riding behavior, the auctioneer may choose to select one losing coalition in each round, and require a response from each member of this coalition. This strict rule was not used in our experiments.

If more coalitions are provided with a *CWL*, we find that this can further reduce the number of auction rounds. In our experiments, where we are restricted to smaller auctions, we computed a *CWL* for every losing $b_i(S) \in B_l^t$. This specific implementation leads to the fact that sometimes a given bid can be part of multiple coalitions. In order to determine minimal core prices and avoid coalitions having to pay too much, conservatively we selected the minimum across these ask prices across different coalitions: $\alpha_i(S_i) \leftarrow \min_{L\in\mathcal{L}} CWL_i^t(L, S_i) + \epsilon$.

It is interesting to point out that finding the minimum $\min_{L\in\mathcal{L}} CWL_i^t(L, S_i)$ can be directly computed as part of the computation of winning levels $WL^t(i, S) = CAP^t(\mathcal{K}) - CAP^t(\mathcal{K}, S_i)$. The result of $CAP_i^t(\mathcal{K}, S_i)$ returns the highest-revenue coalition with bidder i winning S_i. This provides all information necessary to compute the $\min_{L\in\mathcal{L}} CWL_i^t(L, S_i)$ for any of the cost sharing functions to compute individual *CWL*s described in the previous subsection.

Selecting the lowest possible CWL_i^t for each bidder comes at a cost. In some cases, a coalition might not win even if all members agree to the *CWL* ask prices. In other words, we intentionally avoid that bidders pay more than what would have been necessary to win at the potential expense of additional auction rounds. In contrast, if the highest *CWL* for a bundle across all coalitions, i.e.,

$\max_{L \in \mathcal{L}} CWL_i^t(L, S_i^L)$ was selected, then it could happen that members of the coalition pay more than what is necessary to become winning. In the simulations, the additional number of auction rounds caused by our proposed conservative pricing rule was very low, which is why we use it in our experiments.

3.6. Bidding strategy and bundle selection

A bidding strategy in an auction involves two decisions in each round: which packages to bid on and how high a bidder should bid on the selected packages. Straightforward bidding (see Definition 3) is one such strategy, were bidders always bid on the package maximizing (absolute) payoff at the ask prices. We have discussed that in a DL auction straightforward bidding is an ex post equilibrium at least for some types of valuations. Bidding straightforwardly would take hundreds of auction rounds (Schneider et al. 2010), and bidders with a positive cost for participating in a round are unlikely to bid straightforwardly. Moreover, straightforward bidding was not reported in earlier experiments (Scheffel et al. 2012, Adomavicius et al. 2013); in contrast, the authors describe jump bidding and different forms of bundle selection.

In a CWL auction, an auctioneer aids the coordination of bidders by adequate information feedback. Although this coordination avoids unnecessary auction rounds, a losing coalition needs to outbid the winning coalition such that the process still leads to a competitive equilibrium. First, let us provide an illustrative example on how the information feedback in a CWL auction can help reduce the number of auction rounds.

Example 2. Consider the sale of 18 pieces of land ($A-R$) on a shore line. One developer (bidder 1) needs three adjacent pieces of land for a small hotel, while the other developer (bidder 2) plans for a large resort and needs 15 adjacent pieces. Both compete against bidder 3, who is interested in all 18 pieces of land. Let's assume that, in the first round, bidder 1 submits XOR bids of €3 on bundles $A-C$, $D-F$, $G-I$, $J-L$, $M-O$ and $P-R$ for which he has the same preference, while bidder 2 bids on $A-O$ for €9, and bidder 3 on $A-R$ for €20. The CWL for bidder 1's bid on $P-R$ will be €7, while the CWL for bidder 2's bid will be €13 (using uniform distribution sharing rule for simplicity). In contrast, the CWL for all other bundle bids for bidder 1 will be €20. Therefore, the CWL information can serve as a signal that, right now, bidder 1 can focus on bundle $P-R$. In contrast, with only DLs available, bidder 1 could not see the difference of $P-R$ to his other five bundles of interest and might bid on other bundles (i.e., all DLs would be €3), which cannot become winning given the valuations. He could be trying to bid on these other packages over multiple rounds and increase the ask prices, but the allocation would not change. Furthermore, if WL information is available, the WL for $P-R$ on the other hand would be €11, which indicates the entire cost that is needed to outbid bidder 3 without taking into account possible coalitions.

In the absence of reliable prior distributional information about the valuations of competitors in an online market, a rational bidder will not submit a last-and-final bid below the CWL, because he does not have sufficient information to decide whether the bid is just high enough such that he becomes winning in expectation.

To further emphasize this issue, in this section we use a simple complete-information model that highlights the strategic problem of bidders within a single round. The coordination problem for bidders within a round is far from trivial if the auctioneer just provides DL prices, as we have shown in Example 2. We use our stylized model to highlight (and provide the intuition for) the key strategic difficulty that bidders face in these auctions, and the role that the auctioneer plays in a CWL auction in aiding coordination of losing bidders.

3.6.1. A complete information model: We model the strategic situation as a complete-information coordination game (Cooper 1999),[1] where bidders need to bid on complementary packages that together are high enough to outbid the current set of winners. In other words, bidders can realize mutual gains, but only by making mutually consistent decisions. As is typical in the auction literature, we assume a trusted auctioneer and refer to this environment as the *complete information CWL auction model*. Example 3 with two bidders and three items helps us illustrate the model.

Example 3. Suppose that the auctioneer in a CWL auction selects a single losing coalition L with two losing bidders 1 and 2 (row and column players, respectively, in Table 2) and three items (A, B, C). Both bidders are symmetric in that they are interested in the same two packages AB and C, and have identical valuations for them: €8 for AB and €4 for C. The losing bidders are interested to become winning in the next round. The two bidders have both bid on AB for €4 and C for €1 in previous rounds, but have been outbid by bidder 3 with a bid in round t on ABC for €8, which is his valuation. In this auction the losing bidders can either submit a minimum bid, which is their last bid on a package plus a bid increment (i.e., the DL auction ask price), or they bid on the CWL recommended by the auctioneer. With a bid increment of €0.5, both bidders need to bid at a minimum €4.5 for AB and €1.5 for C, which would not make them winning in the next round.

Table 2 shows the payoff matrix of the normal-form game with complete information in round t. If the two losing bidders do not outbid the winning coalition with their bids, the payoff in the next round is zero. If both bidders increase their bid by €2 there are two Nash equilibria with a positive payoff. Without coordination, there is a mixed Nash equilibrium in this round, in which each player

[1] Multi-object auctions are often modeled as complete information games to understand the special case where a bidder has all possible information to manipulate auction outcomes (Bernheim and Whinston 1986, Ausubel 2006, Day and Milgrom 2008).

	$b(AB)=6$	$b(C)=3$	$b(AB)=5$	$b(C)=2$	$\cdots$
$b(AB)=6$	0,0	2,1*	0,0	0,0	
$b(C)=3$	1,2	0,0	0,0	0,0	
$b(AB)=5$	0,0	0,0	0,0	0,0	
$b(C)=2$	0,0	0,0	0,0	0,0	
$\cdots$					

Table 2 Payoff matrix of a normal-form coordination game with two losing bidders and two packages in round t.

bids on AB with a probability of 2/3 and C with a probability of 1/3. The expected payoff for each bidder is €2/3 as a result. Note that if the row bidder selects a strategy of one equilibrium and the column bidder the strategy of another equilibrium, then the bids will not constitute an equilibrium.

The auctioneer in a CWL auction resolves this equilibrium selection problem. For example, he provides a CWL price (including the increment of €0.5) of €6 for AB to bidder 1 (the row player), and €3 for C for bidder 2 (the column player). The cell is marked with "*" in Table 2. So, in this coordination game, the auctioneer randomizes over both pure Nash equilibria and selects one equilibrium from the set of all equilibria. If both players accept the CWL, then the payoff of the row player is €2 and the payoff of the column player is €1, which is higher for both players than their payoff in the mixed Nash equilibrium without the auctioneer. Therefore, it is in the interest of bidder 2 to bid on C, although his absolute payoff would be higher in the equilibrium with him bidding on AB. Of course, there can also be situations where there is a unique Nash equilibrium such that this equilibrium maximizes payoff for all participants. In those cases it is even easier to propose a CWL to a losing coalition.

The example only shows a part of the payoff matrix assuming bidders are only interested in two packages and two prices. However, the payoff matrix would grow very large for any but small auctions. The number of packages a bidder can bid on grows exponentially, and a bidder could submit many prices for each of these packages starting with the DL ask price. This would lead to a huge equilibrium selection problem, which further illustrates the difficulty of coordination in ascending combinatorial auctions with DLs only. McLennan and Berg (2005) showed that the mean number of all Nash equilibria in a bi-matrix game with only two players and z pure strategies for each player grows exponentially in z. With more bidders and items and no prior knowledge about the competitors, the likelihood of coordinating with all other bidders in a round goes to zero with a growing number of bidders and items. If bidders want to become winning in the next round, accepting a CWL maximizes their chances, because there are complementary bids of others.

3.6.2. Expected payoff maximization and coordinated equilibria: We will now formalize the insights from Example 3 and show that the example describes a *correlated equilibrium*, a solution concept introduced by Aumann (1987), where the auctioneer fulfills the role of a trusted party.

Definition 6 (Correlated Equilibrium). A *correlated equilibrium* is a probability distribution $\{p_s\}$ on the space of strategy profiles that obeys the following conditions: For each player i, and any two different strategies b, b' of i, conditioned on the event that a strategy profile with b as a strategy was drawn from the distribution, the expected utility of playing b is no smaller than that of playing b':

$$\sum_{s \in S_{-i}} (\pi^i_{sb} - \pi^i_{sb'}) p_{sb} \geq 0.$$

A strategy profile is a vector of strategies (i.e., bids) of all players. By S_{-i} we denote the set of strategy profiles of all players except for i. If $s \in S_{-i}$, sb denotes the strategy profile in which player i plays b and all other players play s. The inequalities show that if a strategy profile is drawn from the distribution $\{p_s\}$ and each player is told, privately, his or her own component of the outcome, and if furthermore all players assume that the others will follow the signal, then the expected profit of player i cannot be increased by switching to a different strategy b'.

In the *complete information CWL auction model* the auctioneer is a trusted party who randomizes over the pure Nash equilibria, i.e., draws from the distribution $\{p_s\}$ in a correlated equilibrium. For instance, this probability distribution could be 0.5 for each of the two Nash equilibria described in Example 3. We draw on the correlated equilibrium concept in our model to show that a CWL maximizes expected payoff for a bidder in this model (see Appendix A for the proof).

Proposition 2. *Suppose a bidder wants to become a winner in a given round of the complete information CWL auction model, then accepting a CWL for a package maximizes expected payoff in a given round.*

There might be packages with higher absolute payoff for a bidder in a round based on the minimum bid price (i.e., based on the standard DL ask price), but the likelihood of winning them is very low due to exponentially many packages and numerous possible prices for each package the bidders can chose from.

The complete-information model is an abstraction to highlight the central strategic problem, and is not meant to fully describe CWL auctions in reality. In the lab or in the field, the auctioneer does not have complete information. However, the auctioneer can select high-revenue coalitions based on estimates from the bid history, and historical bids often provide auctioneers with good signals about the preferences bidders have for different bundles. Also, in our experiments, we implemented the auctioneer to select more than a single coalition in each round to further speed up the auction. Still, the correlated equilibrium is a useful analogy when thinking about the role of an auctioneer and the value of the information he provides to the bidders throughout in a CWL auction.

As outlined in the introduction, straightforward bidders are also able to coordinate in a DL auction, and a competitive equilibrium arises. However, such a competitive equilibrium comes at the cost of a huge number of auction rounds. In DL auctions with straightforward bidders, all package values of losing bidders need to be revealed, and this might be an unrealistic assumption given the exponential number of packages bidders can bid on. In contrast, CWL auctions can lead to high efficiency in markets with many more objects, because bidders are able to coordinate more effectively.

In the next section, we provide experimental results which demonstrate that bidders accept *CWL*s and, as a consequence, the number of auction rounds is reduced substantially, while efficiency is significantly higher than in a DL auction in the lab.

4. Experimental Design

Several ascending combinatorial auction formats have been analyzed in the past (see discussion in Section 2). An experimental comparison with all of these formats would be beyond the scope of a single paper. Because BACS auctions satisfy a strong solution concept (at least for a restricted set of bidder preferences) and, in particular, because the *DL* auction is a BACS auction that has been the focus of much recent research in IS, the *DL* auction represents a natural candidate to compare against. In addition, the proposed *CWL* auction is an extension of the literature on *DL* auctions (Adomavicius and Gupta 2005, Petrakis et al. 2013, Adomavicius et al. 2013).

In what follows, we will introduce three different bidder value models for which we compare the *DL* and *CWL* auction formats. Another set of value models and the respective simulations are described in Online Appendix D. They yield the same results and, due to space constraints, in the main paper we only included those value models for which we conducted both computational simulation and lab experiments with human participants.

4.1. Value Models

We use three different value models (VMs) in our experiments. These are the Threshold (**Thr**) VM, the **Mix** VM, and the Symmetry (**Sym**) VM. The Sym VM is based on an earlier work by Adomavicius et al. (2013) in their experimental studies on *DL* and *WL*. We added the **Thr** VM and the **Mix** VM in order to understand if the results carry over to other environments. The **Thr** VM models a threshold problem with a single global bidder and several local bidders only interested in small packages. Such environments have received much attention, as they could lead to free-riding behavior as outlined in the introduction.

In addition to the value models described in this section, in Online Appendix D we provide numerical simulations with three additional value models, two with 18 and one with 9 items, which resemble the ones used in lab experiments by Scheffel et al. (2011) and Goeree and Holt (2010).

These value models are modeled after spectrum auction markets with regional licenses and real-estate markets. There is no significant difference in efficiency the simulations, while the efficiency of the CWL auctions is significantly higher than that of the DL auctions in the lab experiments. In the lab and in simulations the number of bids and the number of auction rounds in CWL auctions are substantially reduced.

4.1.1. The Threshold Value Model In this value model, we consider a market with a single global bidder and two local bidders facing a threshold problem. They compete for 6 items labelled A to F. The global bidder is defined to be single-minded and has interest only in the bundle containing all 6 items. Each of the local bidders is interested in various bundles of smaller sizes, but the experimental subjects did not know the specific bundles that were of interest to other bidders. Bidders also did not have distributional information about the other bidders' valuations before the auction. They only knew that two local bidders were competing against a global bidder. The bidder valuations for the individual bundles are drawn from uniform distributions based on pre-specified intervals, which was not known by the bidders. Table 3 represents the basic bidder preferences and value distributions for all bundles. This model is designed in such a way that the local bidders could potentially overcome the threshold posed by the global bidder if they could coordinate and form a coalition containing either the bundles $ABCD$ from bidder 1 and EF from bidder 2, or the bundles $CDEF$ from bidder 1 and AB from bidder 2. Other combinations of package bids from bidder 1 and bidder 2 did not stand a chance of winning if the global bidder bids up to his true valuation.

<table>
<tr><th>Items</th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>Global Bidder</td><td colspan="6">[60,65]</td></tr>
<tr><td rowspan="5">Local Bidder 1</td><td colspan="5">[50,55]</td><td></td></tr>
<tr><td></td><td colspan="5">[50,55]</td></tr>
<tr><td colspan="4">[40,50]</td><td></td><td></td></tr>
<tr><td></td><td colspan="4">[40,50]</td><td></td></tr>
<tr><td></td><td></td><td colspan="4">[40,50]</td></tr>
<tr><td rowspan="5">Local Bidder 2</td><td colspan="2">[20,30]</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="2">[20,30]</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td colspan="2">[20,30]</td><td></td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td colspan="2">[20,30]</td></tr>
</table>

Table 3 Preference structure of the Threshold VM. Global bidder is interested in 1 bundle only, while local bidder 1 and local bidder 2 are both interested in 5 different bundles each.

4.1.2. The Mix Value Model Similarly to the Threshold model, the Mix value model was designed to analyze the ability of two local bidders to enter into a successful winning coalition that can outbid the global bidder. Compared to the Threshold value model, the two local bidders have more bundles of interest, making a successful coalition more challenging.

In particular, we again have 6 items labelled A to F. The global bidder is only interested in the bundle containing all 6 items. The local bidders are interested in all 6 items and all bundles of size up to 4 items. For each local bidder, there exists a "preferred item" (which is chosen randomly) that has a higher value for that bidder than other items. We introduce local complementarities by implementing an additional (i.e., bonus) value of 10% for each adjacent item in a bundle.

The values for the individual bundles are randomly determined from different intervals, depending on the baseline draws for single-item valuations. In particular, for each bidder, we first determined the "preferred item", for which the baseline draw is uniform from the range [90,110]. The adjacent items to this preferred item then had valuations drawn uniformly in the range [40,60]. Their neighboring items next had valuations drawn uniformly in the range [20,30], while the last remaining item, i.e., the item with the greatest distance from the preferred item, had valuation drawn uniformly from [7,17]. Once these single-item valuations were drawn for each bidder, the bundle valuations were computed, and a list of all valuations was provided to the subjects privately. Figure 1 shows an example of baseline draws for a local bidder in this setting. In this example, the bidder has a valuation of $(90+45+20)*1.2 = 155*1.2 = 186$ for the bundle BCD, as this bundle contains 2 adjacent neighbors and, therefore, gets 20% bonus. The number of bidders and the fact that there are two local and a global bidder were common knowledge among bidders. However, bidders did not know the preferred item for the other local bidders.

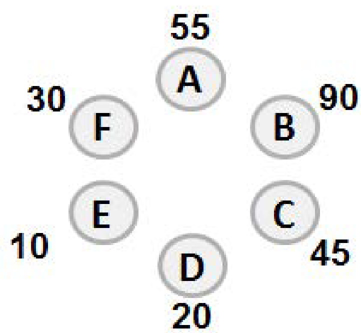

Figure 1 **An example for the item valuations for a local bidder in the Mix VM**

4.1.3. The Symmetry Value Model This was one of the value models (Setup 1) used by Adomavicius et al. (2013) to test and evaluate the impact of deadness and winning levels as price feedback in the lab. It allows us to compare our result to theirs as it does not include random draws. In the Sym VM, bidders have equal strength in their valuations and there is no threshold problem as in Thr VM and Mix VM.

There are again 6 items labelled A to F, which need to be auctioned among three bidders. A distinct item, designated the "preferred item", is identified for each bidder participating in the auction. This item has the highest value (100 monetary units) for the bidder, with the value of each remaining item decreasing by 50% the further the item is from the preferred item. There are complementarities among items by creating superadditive valuations for bundles with adjoining items in them. This is accomplished by adding 10% to the additive valuation of the items for each adjoining item in the bundle, as in the Mix VM. Adomavicius et al. (2013) motivated this setting with the real-world scenario of real-estate properties around a lake, where local complementarities arise. To preserve the symmetry between bidders, we picked the items A, C and E as preferred items for each of the three bidders respectively, as in Adomavicius et al. (2013).

The identity of the preferred item is private information to each bidder. Bidders were not told how many other participants were in their specific auction, as in Adomavicius et al. (2013). Furthermore, while the rules for generating the valuations of the items were common knowledge, and each bidder in an auction knew the distribution of his own values, participants had no explicit knowledge of the valuations of other bidders as they did not know the respective preferred items of other bidders. Figure 2 shows an example for the private valuations for all bidders. In this example, bidder 1 has a valuation of $(100+50)*1.1+25=190$ for the bundle ABE, as the bundle contains only 1 adjacent neighbor, and therefore gets 10% bonus.

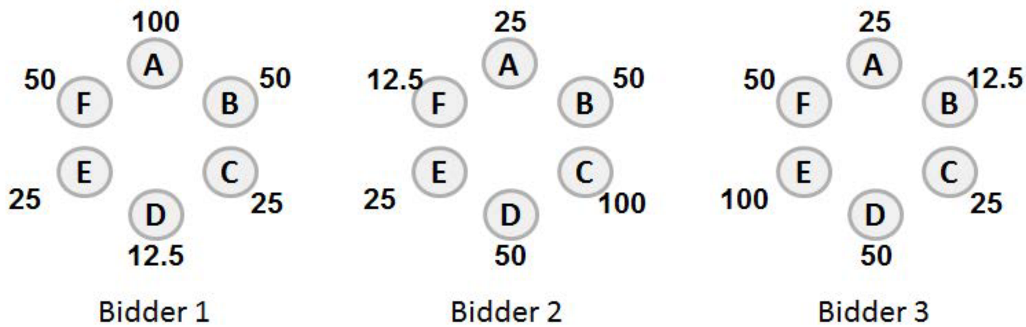

Figure 2 An example for the item valuations for all bidders in the Symmetry VM, where preferred item for bidder 1 is A, for bidder 2 is C, and for bidder 3 is E

4.2. Treatments

We have used a fully factorial design with the two treatment variables, the auction format and the value model. Six sessions were conducted for each auction format in the Threshold, Symmetry, and Mix value model. Every session only used one auction format, and every session consisted of two waves with different value draws. During a session, the two waves were run in parallel with two groups of students. Each wave consisted of three auctions, one for each of the value models (i.e., Thr, Sym, and Mix). These three auctions were conducted sequentially, and in each wave we used a different sequential order of these three auctions to level out learning aspects. We used the same 12 waves for the first 6 sessions testing the *DL* auctions and for the last 6 sessions testing the *CWL* auction to allow for better comparison. Table 4 provides an overview of the sessions and waves in the experiments. In this table we consider the three VMs (Thr, Sym and Mix) together, as the respective auctions are always conducted together in a wave. Overall, there were 12 auctions per treatment combination (72 auctions in 6 treatments) and 72 total participants, each of which having participated in 3 auctions.

Treatment	AF	VM	# Sessions	Auctions per Session	# Auctions	# Participants per Auction	# Participants
1 2 3	DL DL DL	Thr Sym Mix	6	6	36	3	36
4 5 6	CWL CWL CWL	Thr Sym Mix	6	6	36	3	36
					72		**72**

Table 4 Treatments, sessions, and participants for the different test combinations.

The *DL* and *CWL* auction formats have been implemented as described in the previous section. In all experiments we used the same user interface and round-based auction process. We displayed *WL*s in the *DL* auction as additional information feedback (i.e., in addition to the ask prices based on *DL*s) in order to allow for better comparison to Adomavicius et al. (2013). We used a round-based auction process and did not determine the winners after every single submitted bid. Also, we used an XOR bid language throughout, because this bid language is able to express any valuations, i.e., it is fully expressive compared to an OR bid language (Nisan 2006). The bid increment was 15 Francs per item in the Mix and Symmetry VMs, and 3 Francs per item in the Threshold VM. We used Francs as a name for our experimental currency. A bid increment of 3 Francs per item means a bundle increment for a bundle containing three items is 9 Francs. We used a per-item bid increment rather than a bundle bid increment because a per-item bid increment takes bundle sizes into account when raising prices. Therefore, a per-item bid increment can help bidders more effectively to focus on smaller and, thus, more coalition-prone bundles during an auction. Also, as valuations in the Threshold VM are generally smaller than those in the other two VMs, we scaled down the increment appropriately.

4.3. Procedures for human subject experiments

All experiments were conducted from November 2012 to July 2013 with students at a major European university. Each session started with a presentation in which we explained the auction format to be used, the pricing rules, and the different value models in detail. This presentation was also provided to students as a hand-out. Then subjects participated in one training auction to get to know the auction environment, the software, and the user interface. Afterwards, we repeated the main rules, and subjects were asked questions in a quiz to make sure they understood all rules and were familiar with the auction procedure.

The number of auctions was announced in advance. The first auction round was not time-restricted and only ended when every bidder announced they were ready to enter into the second round. In all subsequent rounds, bidders had at most 5 minutes to place their bids. This was perceived to be sufficient by the participants. Bidder roles (e.g., global vs. local) were randomly assigned for each auction in order to alleviate earning differences across different bidders. Earnings were calculated by converting the experimental payoff amounts to EUR by 3:1, i.e., bidders were paid on their economic performance in the auctions. The resulting earnings were between the minimum of €5 (i.e., show-up fee) and the maximum of €50 per subject across all waves. Average earnings were €29, and the average duration of a wave was 1 hour and 19 minutes without the introductory part and without the training auction. This includes the time for breaks between auctions. In Online Appendix E, we provide screenshots of the Web-based system used for the auction.

5. Simulation results

In this section we present the results of numerical simulations based on two different bidding strategies: (i) straightforward bidders (s), as described earlier in the paper, and (ii) heuristic bidders (h). Heuristic bidders bid on 5 bundles in each round, where these bundles are randomly chosen from among their 10 best (i.e., payoff-maximizing) bundles at that time. Heuristic bidders model bidder behavior that is based on observations from lab experiments reported in prior literature (Scheffel et al. 2012). It models a "trembling hand", where bidders want to bid on their best bundles by payoff, but they make small mistakes. We have chosen 10 bundles, because in Scheffel et al. (2012) it was shown that bidders typically focus on a small set of bundles, independent of the number of possible bundles in an auction. We have analyzed variations of these bidders, e.g., bidders who bid on their best three packages, but the differences were minor. For our analysis, these artificial bidders serve as a baseline. In the lab experiments in the next section, we analyze whether the differences between DL and CWL auctions carry over to the lab, where bidders are heterogeneous and follow different strategies on how they select packages or how they use jump bids.

Throughout, we use ***allocative efficiency*** E as a primary aggregate measure for comparing different auction mechanisms.[2] In addition, we measure ***auctioneer's revenue share*** R, which shows how the resulting total surplus is distributed between the auctioneer and the bidders.[3]

Optimal surplus describes the resulting revenue of the winner-determination problem if all valuations of all bidders were available, while actual surplus considers the true valuations for those packages of bidders selected by the auction. In contrast, auctioneer's revenue used in the revenue distribution describes the sum of the bids selected by the auction, not their underlying valuations.

Table 5 shows the efficiency results, averaged over all auction instances used in the lab experiments. In particular, we simulated the two different agent bidding strategies described above for each auction instance which was later used in the lab. The average of these simulated values is then compared to results of the lab experiments using the three value models.

Efficiency	AF	Strategy	Threshold VM	Mix VM	Symmetry VM
Simulation Results	DL	s	100.0%	100%	100.0%
	DL	h	100.0%	99.1%	100.0%
	CWL	s	100.0%	99.8%	100.0%
	CWL	h	100.0%	98.9%	100.0%
Lab Results	DL		96.3%	97.2%	98.3%
	CWL		100.0%	98.1%	100.0%

Table 5 Average efficiency achieved for all simulated bidder behavior and lab results.

Overall, efficiency was very high in both the simulations and the lab experiments. Only the average efficiency in the Mix VM simulation was lower, as bidders had up to 56 bundles of interest, which makes the coordination problem harder. The Symmetry and the Threshold VM had 100% efficiency throughout, with the single exception of the *DL* auction in which the efficiency was significantly worse in the lab for the Threshold and the Mix VMs, as compared to the simulations, which might be due to the bundle selection and the jump bidding of bidders in the lab, which will be described below.

Table 5 provides an initial comparison of efficiency results between all simulated agent behaviors and real bidder behaviors in the lab. Although there are differences between the simulation and the lab, these differences are minor at an aggregate level. As we will show, at an individual level, bidder behavior with respect to bundle selection and jump bidding exhibit some differences compared to the simulations. Table 6 shows a comparison of all aggregate simulation results including the auctioneer revenue share, the number of rounds, and the number of bids submitted.

[2] We measure efficiency as $E = \frac{\text{actual surplus}}{\text{optimal surplus}} \times 100\%$

[3] We measure measure auction revenue share as $R = \frac{\text{auctioneer's revenue}}{\text{optimal surplus}} \times 100\%$

VM	AF	Strategy	Efficiency	Revenue Share	No. of Rounds	No. of Bids
all	DL	s	100%	97.3%	100%	100%
all	CWL	s	99.9%	98.1%	37.0%	45.3%
all	DL	h	99.6%	96.4%	28.3%	93.6%
all	CWL	h	99.5%	97.8%	16.2%	44.3%
Threshold	DL	s	100%	99.1%	100%	100%
Threshold	CWL	s	100%	99.7%	40.9%	48.4%
Threshold	DL	h	100%	98.5%	32.0%	98.1%
Threshold	CWL	h	100%	99.1%	23.8%	58.5%
Mix	DL	s	100%	97.4%	100%	100%
Mix	CWL	s	99.8%	98.0%	31.8%	42.9%
Mix	DL	h	99.1%	96.5%	22.6%	90.1%
Mix	CWL	h	98.9%	97.8%	8.3%	30.6%
Symmetry	DL	s	100%	74.3%	100%	100%
Symmetry	CWL	s	100%	79.6%	50.8%	35.7%
Symmetry	DL	h	100%	71.3%	51.7%	82.2%
Symmetry	CWL	h	100%	82.6%	19.2%	38.9%

Table 6 Average measures of auction performance: aggregate and for each of the three value models Threshold, Mix and Symmetry

The differences in efficiency between DL and CWL auctions across all simulations were insignificant (t-test, $\alpha = 0.05$). Differences in revenue were mostly significant but small. Most importantly, however, although there are only small differences in efficiency and revenue between the *DL* and *CWL* auctions, the communication between the auctioneer and the bidders in the *CWL* auction is substantially reduced. The average number of rounds is significantly smaller in the *CWL* auction, and so is the number of bids submitted throughout the auction. We normalized the numbers so that the *DL* auction with straightforward bidders describes 100% of the rounds and number of bids. Using these results as a conjecture, in the next section we investigate whether the same pattern of auction outcomes emerges in the lab experiments. Online Appendix D presents the results of a number of additional numerical simulations on larger value models, which are in line with the results presented in this section.

6. Experimental Results

Next we will present our results of the lab experiments. First, we will look at the results at an aggregate level, concentrating on the comparison of the two auction formats *DL* and *CWL*, using metrics such as allocative efficiency and auctioneer's revenue share as well as the number of bids submitted and rounds required by the bidders. For the pairwise comparisons of aggregate metrics we use the Wilcoxon rank sum test. In the second phase, we will analyze the bidders' bundle selection and jump bidding behavior during the auctions.

Result 1: (High efficiency of the proposed CWL auctions) *The allocative efficiency of the CWL auction is significantly higher than that of the DL auction.*

It is interesting to see that the efficiency of the *CWL* auction tends to be even higher than the already high efficiency of the *DL* auction. We have fitted a regression model with efficiency as the dependent variable and control for auction format, session, and value model. The coefficient for the *DL* auction format is actually significant (p-value 0.003) and negative, while the different value models did not have a significant influence on efficiency.

While we did not find signs of fatigue among the subjects in their responses after the auction or in the bid data, an influence of fatigue on the results can always be an issue. Fatigue could help explain lower efficiency in the *DL* auction, but the long auction durations in the *DL* auction are actually also a concern for applications in the field.

The auctioneer's revenue share of the *CWL* auction equals that of the *DL* auction in the Thr VM, but is lower than the *DL* auction for the Mix and Sym VM. In the Sym VM bidders did not have to outbid a large bidder and the preferred items were disjoint, such that they could coordinate faster at lower prices. The aggregate results are presented in Table 7. Figure 3 provides box plots of the efficiency and revenue share.

The auctioneer's revenue share is higher in the numerical simulations compared to those in the lab. This is due to the fact that, in the simulation, straightforward bidders reveal all valuations of all losing bundle bids truthfully, often using last-and-final bids. Also, heuristic bidders in simulation reveal their preferences to a large degree. In the lab, subjects typically want to get a payoff and sometimes drop out of the bidding process for a bundle before they reveal their true valuation.

Note that the revenue share in the Sym VM is lower than in the other two value models. In the Thr and Mix VMs the smaller bidders need to outbid the global bidder, who drives up prices, which leads to a higher revenue share. In contrast, in the Sym VM, it can happen that bidders coordinate very early leading to low prices and consequently a low revenue for the auctioneer. This is also a reason for the higher variance in revenue in the Sym VM.

We have included the efficiency and revenue results from Setup 1 (Sym VM) in the experiments by Adomavicius et al. (2013). Note that even though we used the same value model, the experiments are not fully comparable because Adomavicius et al. (2013) used a continuous auction and a bid increment of 1 monetary unit, i.e., not a round-based format with a bid increment of 15 monetary units and last-and-final bids. They also used an OR bidding language instead of an XOR bid language, but this should not matter in the experiments as the valuations in the Sym VM are super-additive. We conjecture that the round-based auction process has advantages for convergence, because bidders do not get updates in the prices and allocations continuously. Instead, they receive new prices after each round. As this information does not change until the round is over, this might

AF	VM	E	R	Avg. no. of all bids	Avg. bundles	Avg. bid improvements	Avg. time (min.)	Avg. no. of rounds
DL	Thr	96.3%	80.8%	30.0	4.9	8.5	14.7	8.3
CWL	Thr	100.0%	80.8%	19.2	4.9	3.1	10.6	5.4
DL	Mix	97.2%	89.8%	114.0	29.3	25.7	37.0	15.3
CWL	Mix	98.1%	88.4%	83.8	29.3	10.9	25.3	9.4
DL	Sym	98.3%	78.4%	166.6	31.4	22.6	45.8	13.6
CWL	Sym	100.0%	63.8%	116.6	29.7	8.8	28.7	10.5
DL^A	Sym	93.5%	65.3%	n.a.	n.a.	n.a.	n.a.	n.a.

Table 7 Average aggregate measures of auction performance in all eight combinations of auction formats and value models. Superscript A refers to results from Adomavicius et al. (2013)

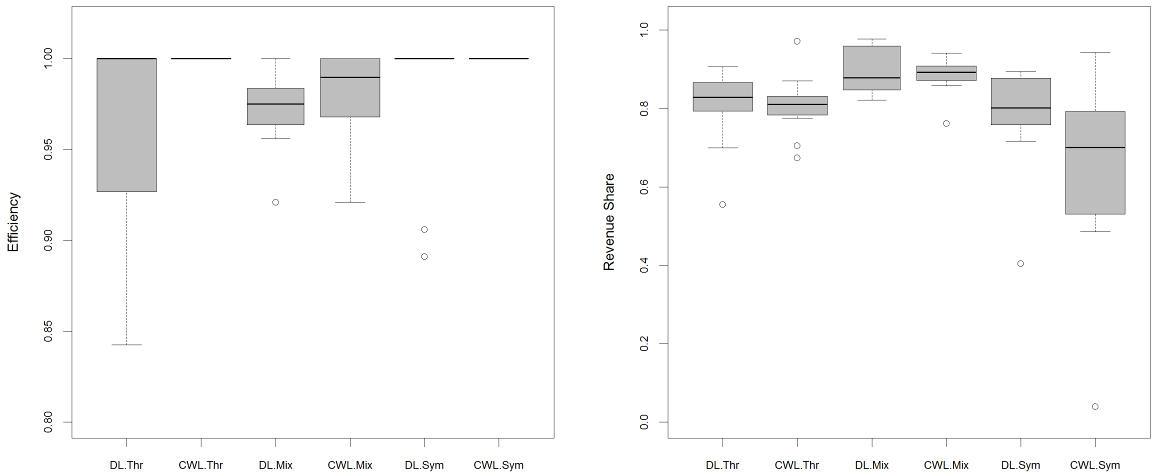

Figure 3 Left: Distribution of allocative efficiency in different treatment groups; Right: Distribution of revenue share in different treatment groups.

lead to a more structured way of decision making. Adomavicius et al. (2013) analyzed an online environment where bidders join throughout the auction and such a round-based mechanism might not be possible. Detailed results of individual auctions can be found in Online Appendix B.

Result 2: (Fast convergence of the proposed CWL auctions) *In the CWL auction, the number of bids submitted, the time to finish the auction, and the number of rounds was substantially lower than in the DL auction.*

As in the numerical simulations, significantly less communication was required to achieve high efficiency. We provide the average number of all bids per auction in Table 7. In addition, we provide the average number of bundles that the small bidders bid on (Avg. Bundles), and their average number of bundle bids improving a starting bundle bid throughout the auction (Avg. Bid

Improvements). We exclude the big bidder, as he is only interested in one bundle and, thus, only bids on and improves this one bundle. Interestingly, the number of Avg. Bundles is identical or similar in all value models, meaning that in both auction formats small bidders bid on a similar number of new bundles, mostly in the first rounds of the auctions. In contrast, the number of Avg. Bid Improvements was much lower in the *CWL* auction than in the *DL* auction across all three value models, suggesting that the *CWL* format enabled small bidders to improve their bundle bids in a much more structured and focused way. Finally, we report the average auction duration time in minutes and the average number of bidding rounds required – these numbers are significantly higher in the *DL* auction. It is particularly interesting to see that the savings in the average time increase with the complexity of the value models; i.e., the *CWL* format saves 4.1 minutes on average in the Thr VM, 11.7 minutes in the more complex Mix VM, and 17.1 minutes in the Sym VM.

In the simulations, the number of bids in the *CWL* auctions was mostly less than half of those submitted in the *DL* auction. The number of bids was also reduced substantially in the *CWL* auctions in the lab, and *CWL* auctions took 64-74% of the bids in the *DL* auction. Differences from the simulation can be explained by the use of jump bids in the *DL* auction and the deviations from straightforward bidding – these characteristics are representative of human bidding behavior. In both the numerical simulations and the lab experiments we used the same bid increments, and last-and-final bids were available to mitigate the potential efficiency losses due to bid increments.

Finally, we look at the threshold problem posed by the big bidder in the value models Thr and Mix.

Result 3: *In the Thr VM, small bidders always find the efficient allocation in a CWL auction in those auctions where a coalition of small bidders is efficient. In contrast, in the DL auction 40% of these auctions are not efficient, i.e., in 40% of the DL auctions where a coalition of small bidders should have won in the efficient allocation, the big bidder won instead. In the Mix VM, there was no significant difference among the auction formats.*

In Figure 4 we show the percentage of auctions where an efficient allocation favored a coalition of small bidders. Of those auctions we show the percentage where a coalition of small bidders won with the 100% efficient allocation or with an allocation that was not 100% efficient, and where the threshold bidder ("big" bidder) won instead. There was a large number of possible allocations with the small bidders winning in the Mix VM compared to the Thr VM. This is the reason why the auctions never resulted in the 100% optimal solution in the Mix VM, but only in the solutions that were close to optimal.

We will also report on bundle selection and jump bidding, because both describe bidder strategies and both can influence the efficiency and revenue of an auction. Bundle selection in the lab varied

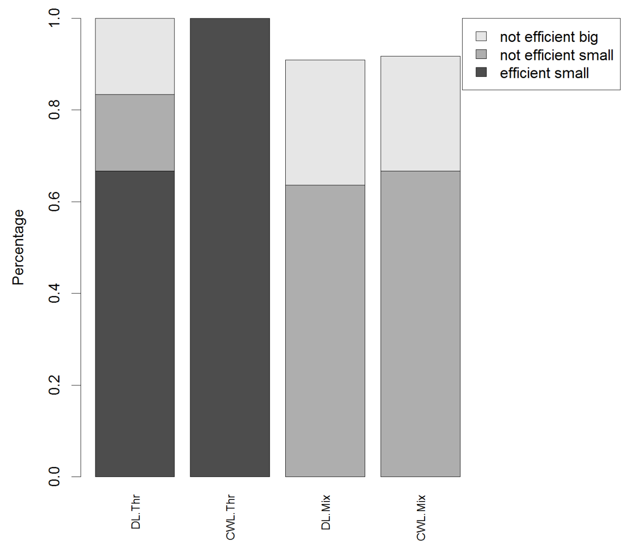

Figure 4 **Auctions where the small bidders won in a 100% efficient or an inefficient allocation, or they lost to the large bidder, as percentages of the number of auctions where the small bidders should have won in an efficient allocation.**

a lot across bidders and was different from straightforward or heuristic bidder strategies used in the simulations.

Result 4: *In the Mix and Sym VMs with more than 50 or 60 bundles of interest resp., bidders submitted bids on 20 to 25 packages in the auctions (median). The number of packages varied significantly across bidders. A high payoff and a high valuation of a package relative to other bundles have a positive impact on the likelihood of a bidder selecting relevant bundles in both auction formats.*

Figure 5 shows the number of bundles on which bidders submitted bids in all value models. While in the Thr VM there are only five bundles of interest for the small bidders, and they typically bid on all of them, the number of possible bundles of interest for small bidders in the Mix VM is 56, and in the Sym VM it is 63 for all bidders, and bidders typically bid only on a subset of possible bundles.

We also analyzed significant covariates for bundle selection using a logistic regression. For this, we generated a table of all bundles with a positive payoff (i.e., relevant bundles) that a bidder could have bid on in each round. The dependent variable describes whether a given relevant bundle has been selected by the bidder or not (i.e., whether the bidder submitted a bid on a given bundle with positive payoff). The model includes covariates such as the auction format, rank of a bundle by valuation, and rank of a bundle by payoff in a round. We used bidder ID dummy variables to control

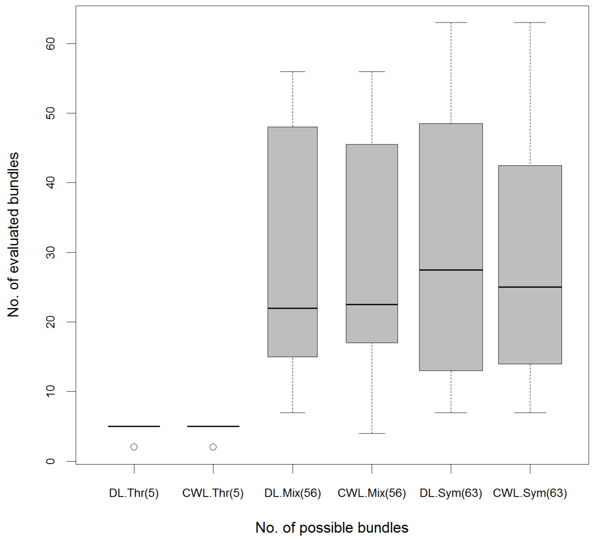

Figure 5 **Average number of bundles evaluated by small bidders relative to all possible numbers of bundles for them, indicated by numbers in parentheses.**

for fixed effects. We also control for the round of the auction, the number of the auction within an experimental session, and the value model. All covariates were significant. The probability of a bundle being selected decreases with a lower rank by payoff or a lower rank by valuation (Table 8). In the Thr VM bidders have a higher likelihood to bid on a bundle, while in the Sym VM bidders have a lower likelihood, compared to the Mix VM. This is induced by the comparatively low number of bundles of interest (i.e., 5) in the Thr VM and vice versa in the Sym VM. Most importantly, there are also significant bidder-specific idiosyncrasies, as the dummy variables for Bidder ID revealed. Overall, payoff influenced the bid selection, but the analysis indicates that pure straightforward bidding cannot fully explain the bidding behavior, and bidder idiosyncrasies matter. This is in line with earlier experimental work on bidder idiosyncrasies in the bundle selection in BACS auctions and the various factors influencing this selection (Scheffel et al. 2012).

Result 5: *Bidders in the CWL auction explore more bundles in the initial rounds of the auctions, but they submit fewer bids in later rounds.*

Table 7 includes statistics on the average number of bundles that small bidders bid on (Avg. bundles) and the number of bundle bids improving a starting bundle bid throughout the auction (Avg. bid improvements). Figure 6 shows the average submissions of new bids and improved bids for all three value models over time. This figure shows a consistent progress for bid submissions both for the new bundles and the improved bids on the previous bundles. In all value models, new bids are mostly submitted during the first 10-20% of the auction process, with bidders generally

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-0.011	0.151	-0.075	0.940
DL auction	-0.747	0.197	-3.786	0.000
Rank by value	-0.018	0.001	-17.344	0.000
Rank by payoff	-0.010	0.001	-7.396	0.000
Auction round	-0.219	0.005	-40.596	0.000
Auction no. in session	-0.073	0.027	-2.704	0.007
Sym VM	-0.231	0.045	-5.152	0.000
Thr VM	1.055	0.077	13.664	0.000
Bidder ID	...	...	...	...
Null deviance:	35533	on 68386	deg. of freedom	
Residual deviance:	29451	on 68303	deg. of freedom	
AIC:	29619			

Table 8 Logistic regression of the bidder's likelihood to bid on a bundle.

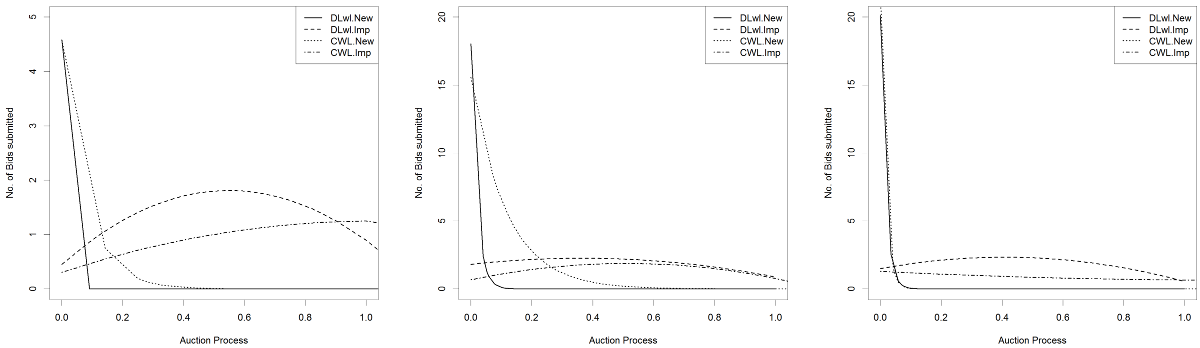

Figure 6 Average bid submission progress during the auction in the Thr, Mix and Sym VM.

submitting new bids for more rounds in the *CWL* auction compared to the *DL* auction. Much of the coordination takes place in these initial rounds. In contrast, bidders in the *CWL* auction need to submit fewer bids on bundles that they have already bid on (i.e., improved bids "Imp") and bidding is more focused.

Bidders were allowed to submit bids that are higher than the ask price in both auctions. Such jump bids represent a possibility to signal about high-valued packages, which can become part of a coalition of winning bidders. We wanted to understand how bidders use jump bidding (because high jump bids could impact revenue and also efficiency) and whether we can find differences between the two auction formats.

Result 6: *Bidders submitted most jump bids in the first half of the auction rounds. There was no significant difference in the proportion of jumps in both auction formats across the value models.*

The bar chart in Figure 7 provides an overview regarding what proportion of jump bids was submitted in the first, second, third, and fourth quarter of the auctions on average. It shows that in both auction formats the majority of jump bids is submitted in the first quarter of the auction.

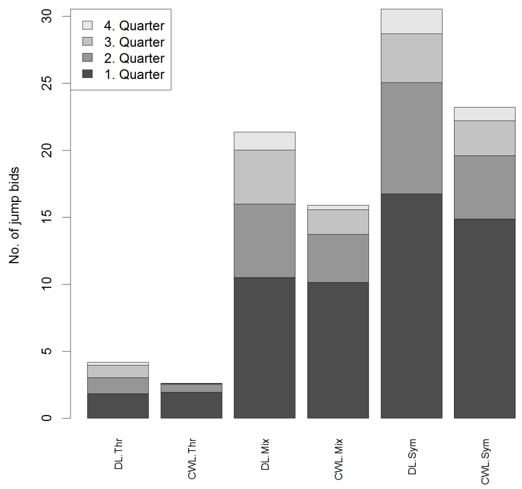

Figure 7 Number of jumps in the the first to fourth quarter of the auction rounds.

We conjecture that bidders try to signal packages of high value in the early rounds and find bidders interested in complementary packages this way. They submit fewer jump bids towards the end, where bidders only focus on a few bundles that they are trying to win.

Overall, there was heterogeneity in the number of package bids a bidder selected in each round and in the number of jump bids. The high efficiency in the auctions can be seen as an indicator for robustness against these differences in bidding behavior.

We also ran some lab experiments with instances of a large value model with 18 items and found qualitatively similar results. Efficiency was higher than 99% for the DL and CWL auctions, but the number of bids and rounds in the latter were substantially reduced. Results are reported in Online Appendix C.

7. Conclusions

The design of efficient multi-object auctions is a fundamental problem with many applications in e-sourcing and other domains. Ascending combinatorial auctions do not require bidders to reveal valuations on exponentially many packages as in sealed-bid auctions, but rather the allow bidders to discover winning packages iteratively, which is preferred to sealed-bid auctions (such as the Vickrey-Clarke-Groves mechanism) in many online markets. The main strategic challenge in ascending combinatorial auctions is coordination: How can bidders find the right packages from a large set of possible alternative packages, which together with bids of other bidders could become a winning coalition? This type of coordination among losing bidders has largely been ignored, but it is arguably a pivotal problem for bidders in larger online combinatorial auctions.

Our approach leverages information about losing coalitions, which can be collected by the auctioneer throughout the auction. Coalitional winning levels are provided in each auction round to help losing bidders coordinate implicitly and outbid a coalition of standing winners. They provide an implicit proposal on how much to bid in order to become winning jointly. Aside from a theoretical characterization of this auction format using intuitions from a complete information model and the correlated equilibrium concept, the results of numerical simulations and corresponding lab experiments with realistic value models indicate substantial savings in the number of auction rounds and bids, and even higher efficiency in the lab. This type of information feedback helps bidders coordinate with much less communication, which makes combinatorial auctions a viable mechanism in many more practical applications. The results provide substantial contributions to a recent stream of IS literature focusing on information feedback design in multi-object auctions.

We would like to note that the proposed auction mechanism has been designed with online markets in mind, i.e., markets with little or no distributional information available publicly about other bidders valuations and where rational bidders typically do not have incentives to deviate from truthful bidding. The empirical performance of this auction format in numerous experiments with real bidders provides evidence that the proposed coalition-based pricing mechanism indeed facilitates the intended performance improvements, e.g., substantially accelerated convergence with high efficiency. However, it has been well-documented that strategic bidder behavior does occur in numerous settings (free-riding, jump bids, sniping, etc.), especially where some information about valuations of other bidders is available or can be learned through repeated interactions. For example, if bidders had precise information about the preferences of their competitors, then strategic bidders have incentives to manipulate (unless the auctioneer uses a Vickrey-Clarke-Groves mechanism). However, in a combinatorial auction with exponentially many packages and many bidders, this would presume the availability of a lot of information. In summary, it is important for an auctioneer to understand the information available to bidders in a market before deciding on a specific auction format.

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Online Appendices

Appendix A: Proofs

Lemma 1: *The game of distributing $\Psi^t = \sum_{i\in L} WL^t(i,S) - CWL^t(L,S^L)$ to individual bidders $i\in L$ is super-additive.*

Proof: Let $K = CAP^t(\mathcal{K})$ be the revenue achieved by the winning coalition at auction state t, i.e., the threshold for any losing coalition at state t. The winning level of a single-minded bidder $i\in L$ is $WL^t(i,S) = K - \Sigma_{j\in L\setminus i} b_j$, where b_j are the bids. The coalitional winning level (CWL) of any losing sub-coalition $L'\subseteq L$ is $CWL^t(L',S^{L'}) = K - \Sigma_{j\in L\setminus L'} b_j$.

Super-additivity is defined as $\Psi(L) \geq \Psi(L') + \Psi(L'')\ \forall L',L''\subseteq L,\ L'\cap L'' = \emptyset$. We dropped the superscript t for brevity.

Now we have

$$\Psi(L') = \sum_{i\in L'} WL^t(i,S) - CWL^t(L',S^{L'}) \tag{4}$$
$$= \sum_{i\in L'}(K - \sum_{j\in L\setminus i} b_j) - (K - \sum_{j\in L\setminus L'} b_j) \tag{5}$$
$$= K(|L'|-1) - \sum_{i\in L'}\sum_{j\in L\setminus i} b_j + \sum_{j\in L\setminus L'} b_j \tag{6}$$
$$= K(|L'|-1) - (|L'|\sum_{j\in L} b_j - \sum_{j\in L'} b_j) + \sum_{j\in L\setminus L'} b_j \tag{7}$$
$$= K(|L'|-1) - |L'|\sum_{j\in L} b_j + \sum_{j\in L} b_j \tag{8}$$
$$= (|L'|-1)(K - \sum_{j\in L} b_j) \tag{9}$$

Note that $(K - \sum_{j\in L} b_j) \geq 0$ as L is a losing coalition, and it is the same for $\Psi(L)$, $\Psi(L'')$, and $\Psi(L')$. It remains to show that $|L| \geq |L'| + |L''| - 1$ which is true for all subcoalitions $L',L''\subseteq L$, because L' and L'' are disjoint. Q.E.D.

Lemma 2: *The game of distributing $\Psi^t = \sum_{i\in L} WL^t(i,S) - CWL^t(L,S^L)$ to individual bidders $i\in L$ is convex.*

Proof: Convexity is defined as $\Psi(L) \geq \Psi(L') + \Psi(L'') - \Psi(L'\cap L'')$, $\forall L',L''\subseteq L$. If $L'\cap L'' = \emptyset$, then $\Psi(L'\cap L'') = 0$ and convexity reduces to super-additivity. Else assume $|L'\cap L''| > 0$. Based on Lemma 1 it remains to show that $(|L|-1) \geq (|L'|-1) + (|L''|-1) - (|L'\cap L''|-1)$, which is equivalent to $|L| \geq |L'| + |L''| - |L'\cap L''| = |L'\cup L''|$ which is true for all $L',L''\subseteq L$.

Proposition 2: *Suppose a bidder wants to become a winner in a given round of the complete information CWL auction model, then accepting a CWL for a package maximizes expected payoff in a given round.*

Proof: The complete information CWL auction model can be described as a single-stage coordination game $\Gamma = (\mathcal{I},\Sigma,\pi)$, where bidders $i\in\mathcal{I}$ can submit bids for different bid amounts on exponentially many packages. Σ are possible bids of the bidders for different packages, and π are the resulting payoffs. Suppose an auctioneer selects a losing coalition $L\subset\mathcal{I}$ and provides a CWL to the members of this coalition $i\in L$, i.e., he recommends bidder i with value $v_i(X_i)$ for the package to submit a bid $b = b_i(X_i)$ such that $v_i(X_i) - b = \pi^i_{sb}$. The auctioneer computes these equilibrium bids according to a probability $\{p_s\}$ such that

$$\sum_{s \in S_{-i}} (\pi^i_{sb} - \pi^i_{sb'}) p_{sb} \geq 0.$$

This is equivalent to a correlated equilibrium, and consequently the expected payoff of a bidder bidding $b_i(X_i)$ in this round is higher than that of any other bid $b' = b'_i(X'_i)$. If there is one payoff-dominant Nash equilibrium, then the auctioneer selects this equilibrium with certainty such that there is also no incentive for bidders to deviate from this equilibrium. Q.E.D.

Appendix B: Detailed Results of Lab Experiments with the Three Main Value Models

B.1. Auction results in the Threshold VM

The detailed results for the Threshold VM can be found in Table 9.

AF.VM	Session	Wave	$E(X)$	$R(X)$	No. of Bids	New Bids	Improved Bids	Time (min.)	Rounds
DL.Thr	1	1	100%	88.8%	27	5.0	7.0	22.5	8
DL.Thr	1	2	100%	83.7%	42	5.0	14.5	14.4	13
DL.Thr	2	1	100%	70.0%	33	5.0	9.5	13.3	8
DL.Thr	2	2	93.4%	77.3%	12	5.0	0.5	5.2	3
DL.Thr	3	1	84.2%	81.5%	29	5.0	8.5	11.3	5
DL.Thr	3	2	100%	84.8%	25	3.5	7.5	18.2	9
DL.Thr	4	1	100%	87.0%	41	5.0	14.0	26.0	9
DL.Thr	4	2	86.4%	82.0%	23	5.0	4.5	14.1	6
DL.Thr	5	1	91.9%	55.5%	29	5.0	8.0	8.2	7
DL.Thr	5	2	100%	81.5%	39	5.0	12.5	11.1	10
DL.Thr	6	1	100%	86.4%	26	5.0	5.5	18.5	12
DL.Thr	6	2	100%	90.6%	34	5.0	10.0	13.9	9
CWL.Thr	7	1	100%	80.9%	19	5.0	3.5	8.2	5
CWL.Thr	7	2	100%	82.3%	26	5.0	6.0	13.9	6
CWL.Thr	8	1	100%	82.4%	13	3.5	1.5	5.2	4
CWL.Thr	8	2	100%	80.2%	21	5.0	3.5	12.0	9
CWL.Thr	9	1	100%	97.1%	15	5.0	1.5	8.9	3
CWL.Thr	9	2	100%	77.5%	25	5.0	6.5	15.8	5
CWL.Thr	10	1	100%	70.5%	17	5.0	2.5	5.3	4
CWL.Thr	10	2	100%	67.4%	18	5.0	0.5	11.7	4
CWL.Thr	11	1	100%	81.3%	15	5.0	1.0	5.0	5
CWL.Thr	11	2	100%	87.0%	21	5.0	4.0	10.3	6
CWL.Thr	12	1	100%	84.0%	21	5.0	4.0	17.2	8
CWL.Thr	12	2	100%	79.3%	19	5.0	3.0	13.4	6

Table 9 Auction results for the Threshold VM

B.2. Auction results in the Mix VM

The detailed results for the Mix VM can be found it Table 10. We omitted the results of Session 6, Wave 1, Mix VM in this table due to errors in bidding behavior (bidder inadvertently submitted bids which were much higher than their valuations).

AF.VM	Session	Wave	$E(X)$	$R(X)$	No. of Bids	New Bids	Improved Bids	Time (min.)	Rounds
DL.Mix	1	1	98.5%	97.7%	126	20.0	40.5	53.4	27
DL.Mix	1	2	96.5%	85.5%	190	50.0	43.5	41.4	21
DL.Mix	2	1	98.1%	90.8%	64	13.0	17.0	34.9	8
DL.Mix	2	2	97.5%	95.9%	121	51.5	8.0	17.9	11
DL.Mix	3	1	100%	82.7%	47	13.0	8.5	26.9	10
DL.Mix	3	2	100%	97.4%	138	37.5	28.5	66.8	28
DL.Mix	4	1	95.6%	87.5%	101	17.5	31.0	30.7	12
DL.Mix	4	2	98.3%	87.9%	31	12.0	2.5	19.9	6
DL.Mix	5	1	96.9%	82.2%	134	39.0	26.0	27.8	11
DL.Mix	5	2	92.1%	84.1%	87	21.0	20.5	37.4	15
DL.Mix	6	2	96.2%	95.9%	215	47.5	56.5	50.0	19
CWL.Mix	7	1	99.4%	92.3%	136	39.5	27.0	39.2	13
CWL.Mix	7	2	100%	90.7%	81	32.0	7.0	14.0	5
CWL.Mix	8	1	97.4%	88.6%	60	14.5	13.5	25.1	14
CWL.Mix	8	2	97.5%	89.3%	99	43.5	5.0	31.6	16
CWL.Mix	9	1	100%	85.9%	96	26.5	20.0	24.8	6
CWL.Mix	9	2	100%	89.6%	58	23.0	4.5	39.4	10
CWL.Mix	10	1	95.6%	76.2%	31	11.5	2.0	13.4	7
CWL.Mix	10	2	100%	88.4%	48	19.0	4.5	13.6	6
CWL.Mix	11	1	98.5%	90.9%	50	19.0	1.5	17.1	9
CWL.Mix	11	2	92.1%	89.2%	48	19.5	3.0	17.9	5
CWL.Mix	12	1	99.9%	94.2%	126	50.0	11.5	22.8	6
CWL.Mix	12	2	96.2%	85.9%	172	53.5	31.0	44.6	16

Table 10 Auction results for the Mix VM

B.3. Auction results in the Symmetry VM

The detailed results for the Symmetry VM can be found in Table 11.

Appendix C: Detailed Results of Lab Experiments with an Additional 18-item Value Model

Finally, we also ran another lab experiment with the large 18-item global-synergy value model (18GSVM) with 7 bidders, which is based on a value model by Goeree and Holt (2010) used in their experiments. The experimental design is consistent with the ones in the Section D.1 in the online appendix. Such experiments are costly and it is difficult to determine the bid increments such that the *DL* auction actually terminates in a reasonable time frame in the experiment. This is why we only report on a single experiment. Again, consistent with our initial results, we found that, when comparing the *DL* with the *CWL* auction format, the number of auction rounds was reduced substantially from 46 to 21, and efficiency was greater than 99% in both cases. The Table 12 shows detailed results for this value model.

AF.VM	Session	Wave	$E(X)$	$R(X)$	No. of Bids	New Bids	Improved Bids	Time (min.)	Rounds
DL.Sym	1	1	100%	84.2%	98	20.0	12.3	48.5	11
DL.Sym	1	2	100%	79.2%	242	48.0	32.0	61.5	16
DL.Sym	2	1	100%	71.7%	80	18.0	8.7	25.8	15
DL.Sym	2	2	100%	89.5%	343	56.3	42.7	68.8	13
DL.Sym	3	1	100%	74.5%	111	20.7	16.3	43.3	10
DL.Sym	3	2	100%	40.4%	174	50.0	8.0	37.4	7
DL.Sym	4	1	100%	88.0%	140	14.0	32.7	29.3	8
DL.Sym	4	2	100%	87.5%	148	28.3	20.3	47.6	13
DL.Sym	5	1	89.1%	77.2%	220	34.3	38.7	33.2	7
DL.Sym	5	2	100%	79.6%	191	23.3	40.3	60.4	31
DL.Sym	6	1	90.6%	80.7%	114	34.3	3.7	51.3	19
DL.Sym	6	2	100%	88.3%	138	29.0	15.0	42.4	13
CWL.Sym	7	1	100%	72.7%	81	19.7	7.3	18.1	3
CWL.Sym	7	2	100%	67.5%	121	31.3	9.0	35.0	12
CWL.Sym	8	1	100%	79.2%	94	21.7	9.3	30.4	26
CWL.Sym	8	2	100%	4.0%	36	9.7	2.3	15.9	11
CWL.Sym	9	1	100%	73.9%	136	35.3	9.7	24.7	6
CWL.Sym	9	2	100%	94.3%	130	28.0	15.3	25.3	6
CWL.Sym	10	1	100%	79.3%	128	23.3	19.3	40.4	9
CWL.Sym	10	2	100%	48.6%	165	44.0	8.0	40.2	7
CWL.Sym	11	1	100%	51.8%	49	13.0	3.3	19.9	14
CWL.Sym	11	2	100%	57.1%	114	32.3	5.3	21.6	7
CWL.Sym	12	1	100%	54.5%	154	54.3	9.3	34.3	6
CWL.Sym	12	2	100%	83.5%	191	44.0	7.3	38.2	19

Table 11 Auction results for the Symmetry VM

AF.VM	Session	Wave	$E(X)$	$R(X)$	No. of Bids	New Bids	Improved Bids	Time (min.)	Rounds
DL.18GSVM	1	1	99.3%	65.5%	713	41.4	40.1	148.3	46
CWL.18GSVM	1	2	99.8%	79.7%	586	43.3	28.0	73.7	21

Table 12 Auction results for the 18-item global-synergy value model

Appendix D: Simulation Studies with Three Additional Complex Value Models

In this online appendix, we provide the results of numerical simulations with three additional value models which were different to those in the main part of the paper. Two value models have 18 items, one has 9 items. The two global-synergy value models (with 18 and 9 items respectively) are based on an earlier work by Goeree and Holt (2010) and use global complementarities (GSVM). The last value model uses local complementarities between 18 items (LSVM), modeled after a real-estate auction. We simulated 64 instances of the GSVM and 160 of the LSVM.

D.1. The 18-Item Global-Synergy Value Model (18GSVM)

The global-SVM is based on the experiments in Goeree and Holt (2010) and involves seven bidders and 18 items. Figure 8 represents the bidders' preferences. The six regional bidders (labeled 1 through 6) are each interested in four adjacent items of the national circle (consisting of items A through L) and two items of the regional circle (consisting of items M through R) while for the national bidder (labeled 7) the twelve items of the national circle are relevant. This information was common knowledge, but it was not known which bidders were interested in a particular item. For example experimental subjects did not know that besides bidder 7 also 3 and 4 were interested in item H.

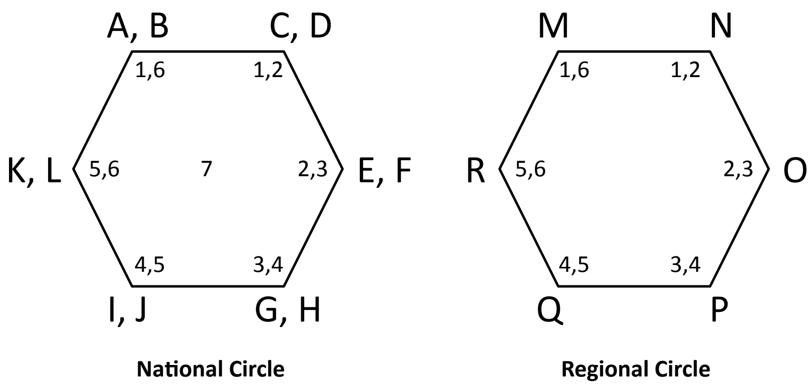

Figure 8 **Competition structure of the Global-SVM. Regional bidders 1-6 are interested in four items from the national circle and two items from the regional circle. National bidder 7 is interested in all twelve items from the national circle (Goeree and Holt 2010).**

The values for the individual items are randomly determined. For the national bidder the baseline draw distributions are uniform from the range $[0,10]$ for items A-D and I-L and uniform from the range $[0,20]$ for items E-H. For regional bidders the baseline draw distributions are uniform from the range $[0,20]$ for items A-D, I-L and M-R and uniform from the range $[0,40]$ for items E-H. These value distributions (not the actual draws) were common knowledge among the experimental subjects. For comparison, we use the same draws as in Goeree and Holt (2010), which the authors have kindly provided. For both bidder types the value of items in a package increases by 20% (with two items), 40% (with three items), 60% (with four items), etc. and by 220% for the package containing twelve items. For the computation of the complementarities the identity of the items does not matter. For example, a bidders' valuation of a package of items increases by the same percentage independent of the adjacency of the items; this means, this is a value model in which global complementarities apply.

Activity and purchase limits are such that regional bidders can bid on and acquire at most four items in a single round, while the national bidder is able to acquire all his twelve items of interest. The number of $2^{18}-1=262,143$ possible packages for the national bidder did not allow for a pure straightforward strategy in each round of the simulation. Therefore, the bidders were restricted with respect to the packages they could select. From the national circle they only included packages containing both items on a vertex. This means for example that a bidder only bids on packages including, say, both A and B, but not packages with A alone, as both A and B are on the same vertex of the national circle. As a result, even straightforward bidders in the DL auction could not achieve full efficiency. In order to be able to simulate a straightforward bidder, we designed a version of this value model with 9 items, wich allows us to analyze all possible packages in each round in the 9-item global-synergy value model described next.

D.2. The 9-Item Global-Synergy Value Model (9GSVM)

Figure 9 represents the bidders' preferences. The three regional bidders (labeled 1 through 3) are each interested in four adjacent items of the national circle (consisting of items A through F) and one item of the regional circle (consisting of items G through I) while for the national bidder (labeled 4) all six items of the national circle are relevant.

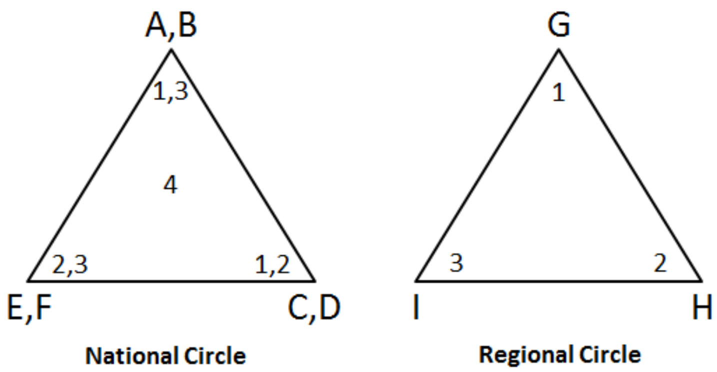

Figure 9 Competition structure of the Global-SVM. Regional bidders 1-3 are interested in four items from the national circle and one item from the regional circle. National bidder 4 is interested in all six items from the national circle.

The values for the individual items are randomly determined. For the national bidder the baseline draw distributions are uniform from the range $[0, 15]$ for items A,B,E,F and uniform from the range $[0, 30]$ for items C,D. For regional bidders the baseline draw distributions are uniform from the range $[0, 20]$ for items A,B,E,F and G,I,H and uniform from the range $[0, 40]$ for items C,D. For both bidder types the value of items in a package increases every time by 20% with each additional item. For the calculation of the complementarities the identity of the items does not matter, e.g. a bidders' valuation of a package of items increases by the same percentage independent of the adjacency of the items; i.e, this is a value model in which global complementarities apply. Activity and purchase limits are such that regional bidders can bid on and acquire at most three items in a single round, while the national bidder is able to acquire all his six items of interest.

D.3. The 18-Item Local-Synergy Value Model (18LSVM)

The local-synergy value model consists of 18 items arranged quadratically and considers the scenario in which complementarities are gained from spatial proximity. In this value model items are placed on a quadratic map. The arrangement of items matters for the calculation of the complementarities, which only arise if the items are neighboring.

This model also contains two different bidder types: one national bidder, interested in all bundles consisting of at least 7 items, and three regional bidders. Each regional bidder is interested in a randomly determined preferred item and all horizontal and vertical adjacent items. This means that a regional bidder is interested in three to five items. Examples are shown in Table 13, in which the preferred item of a regional bidder is Q or K, and all gray shaded items in the proximity of the preferred item have a positive valuation. For each bidder i we draw the valuation $v_i(k)$ for each item k in the proximity of the preferred item from a uniform distribution. Item valuations for the national bidder are from the range $[3,9]$ and for regional bidders from the range $[3,20]$.

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q*	R

A	B	C	D	E	F
G	H	I	J	K*	L
M	N	O	P	Q	R

Table 13 Local-SVM with the preferred items Q and K of two regional bidders. All their positive valued items are shaded.

We assume that bidders experience only low complementarities on small packages, but complementarities increase heavily with a certain amount of adjacent items. We further assume that adding items to already large packages do not increase the complementarities anymore. The rationale for these assumptions is the lack of economies of scale with small packages and a saturation of this effect with larger packages. Therefore, complementarities are modeled based on a logistic function, which assigns a higher value to larger packages than to smaller ones. Valuations used in the simulations are available on request.

D.4. Efficiency and Revenue

As in the main text of our paper, we analyze straightforward and heuristic bidding stragegies: two types of straightforward bidders (s_{DL} and s_{CWL}) and two types of heuristic bidders, who randomly select 5 from their best 10 bundles (h_{DL} and h_{CWL}). Table 14 summarizes the results of the simulations. The first observation is that with last-and-final bids all auctions with the value models 9GSVM and 18LSVM were fully efficient. This is not surprising for s_{DL}, where there are proofs for full efficiency. It is, however, interesting to see that even those auctions where the *CWL*s were used as ask prices yielded full efficiency. Differences in revenue were significant (t-test, $\alpha = 0.05$) but small.

As indicated earlier, in the 18GSVM value model the number of packages explored in each round was reduced to keep the package selection tractable, which led to an efficiency loss in all auctions. However, the differences in efficiency and revenue are hard to interpret due to the resulting randomness in the package selection. Still the number of auction rounds and the number of bids was reduced substantially with *CWL* auctions as compared to *DL* auctions.

Appendix E: Screenshots of Bid Submission

In this appendix we will provide selected screenshots to illustrate how a bidder in the experiments interacted with the auction and what information feedback he received. Figure 10 shows the start of an auction, where a bidder sees a list of all bundles for which he has positive valuations together with the valuations.

VM	AF	Strategy	Efficiency	Revenue	Rounds	Number of Bids
9GSVM	DL	s_{DL}	100%	77.5%	100%	100%
9GSVM	CWL	s_{CWL}	100%	68.2%	81.0%	14.7%
9GSVM	DL	h_{DL}	100%	61.2%	178.8%	77.2%
9GSVM	CWL	h_{CWL}	100%	64.7%	33.3%	18.6%
18GSVM	DL	s_{DL}	97.87%	84.28%	100%	100%
18GSVM	CWL	s_{CWL}	95.66%	76.71%	40.33%	39.02%
18GSVM	DL	h_{DL}	98.00%	72.16%	36.37%	78.22%
18GSVM	CWL	h_{CWL}	96.95%	76.64%	14.72%	33.99%
18LSVM	DL	s_{DL}	100%	83.9%	100%	100%
18LSVM	CWL	s_{CWL}	100%	83.5%	51.5%	25.7%
18LSVM	DL	h_{DL}	100%	82.7%	108.3%	100.6%
18LSVM	CWL	h_{CWL}	100%	84.7%	35.6%	36.5%

Table 14 Average measures of auction performance.

Figure 10 List of bundles with positive valuations for a bidder.

If a bidder decides to submit a bid and clicks “Submit Bid” for a bundle (in our example for the bundle AB) in Figure 10, a window pops up, shown in Figure 11, where the bidder can submit a bid price on the selected bundle. In the same window, he is shown whether he has already submitted a bid on this bundle in a previous auction round and, if so, his last active bid price; his private valuation for the bundle; the ask price (in bold) and the lower boundary for a last-and-final bid price if he chooses to submit one (also in bold).

Now let us assume the bidder has submitted two new bids in the first round of the auction, on the bundles A and AB respectively. After he submitted these two bids a new list of active bundles is created for him which is shown in Figure 12. There he sees for each of his bundles the bundle; his bidding status on this bundle (“new” if he is bidding for the first time on this bundle in the current round, indicated in orange; “winning” if his bid on a bundle in one of the previous rounds is winning in the current round, indicated in green; or “losing” if his bid on a bundle in one of the previous rounds is losing in the current round, indicated in red.), his active bid price, ask prices, his potential payoff after paying the ask price and a fix increment, and finally his private valuation for this bundle.

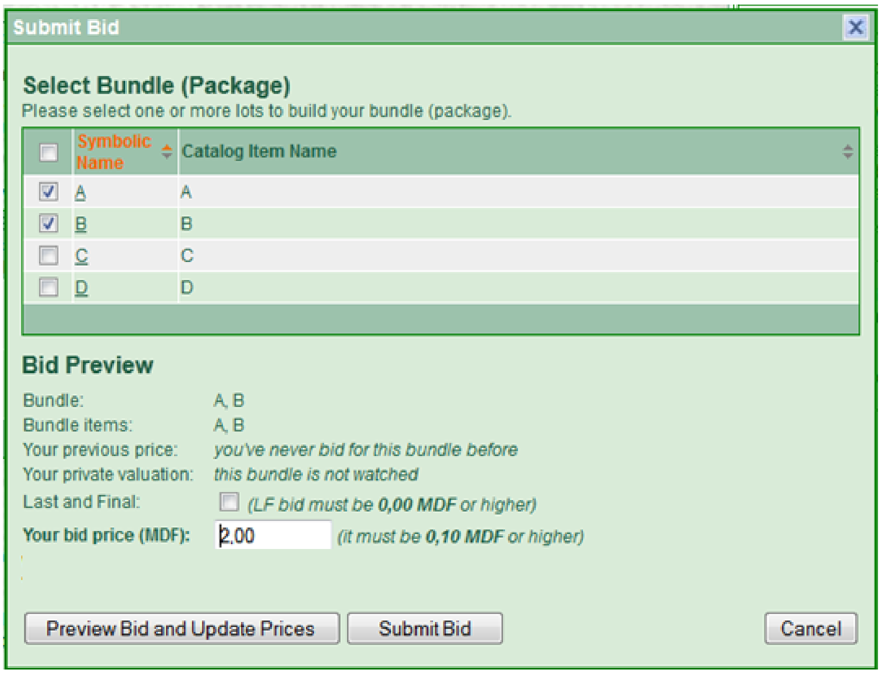

Figure 11 **Window for bid submission on a package.**

Figure 12 **List of bundles a bidder has bid on in previous rounds or a bidder has bid on for the first time in the current round.**

Figure 13 then shows a screenshot to illustrate the next round. Our bidder has submitted two bids in the first round, one for the bundle A and one for the bundle AB, both with prices at 1. In the current round 2, both active bundles are shown together with the information, that his bundle bid on A is winning, while his bundle bid on AB is losing. Additionally, the CWL prices are shown as well as his potential gain when winning at the current prices, which equals his private valuation minus CWL prices and the increment.

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Figure 13 **The updated list of active bundles and bids of a bidder after he submitted some bids in the previous round.**