# AN ANALYSIS OF LINEAR PRICES IN ITERATIVE COMBINATORIAL AUCTIONS

**Pavlo Shabalin, Alexander Pikovsky, Martin Bichler**

*Internet-based Information Systems Roland Berger & $O_2$ Germany Chair, TU Munich*

$\{$*shabalin, pikovsky, bichler*$\}$*@in.tum.de*

**Abstract:** *Ask prices are the most essential form of information feedback in Iterative Combinatorial Auctions (ICA) and vital in achieving an efficient auction outcome. Different pricing concepts have been analyzed in the literature, including linear, non-linear and discriminative non-linear prices. Linear prices, i.e. per-item prices, are intuitive and easy to understand for bidders, however, under certain conditions they can hinder auction efficiency. In this paper we analyze ways how approximate linear prices are calculated, identify problems of existing approaches and suggest alternative rules for calculating linear prices and stopping the auction. We use computer simulations as a tool to benchmark different auction designs, and compare their results based on allocative efficiency.*

**Keyword:** *combinatorial auction, efficiency, linear prices, discrete event simulation*

## 1 Introduction

Combinatorial auctions (CA) have become a popular research topic in Economics, Operations Research, and Computer Science throughout the past few years [CSS06]. CAs are those auctions where bidders can place bids on combinations of items, called ”packages” or ”bundles” rather than just individual items. These types of auctions have found application in various domains ranging from transportation to industrial procurement and the allocation of spectrum licenses for wireless communication services in the US. In addition, researchers have shown that CAs can achieve high levels of allocative efficiency in the presence of complex bidder valuations [KLPD05, ACM06]. In comparison to sealed-bid designs, Iterative Combinatorial Auctions (ICA) have been selected in many practical applications, since they help bidders express their preferences by providing feedback, such as provisional pricing and allocation information in each round.

In this work we focus on so called linear prices, where each item in the auction is assigned an individual price, and the price of a package of items is simply the sum of the item prices. They are easy to understand for bidders in comparison to non-linear prices, where the number of prices that has to be communicated in each round is exponential [BO06, Par06]. Even though linear prices are often impossible to calculate exactly, they can be approximated to provide a guideline for bidders to improve their bid. Kwasnica, Ledyard, Porter, and DeMartini have proposed a respective auction design, called Resource Allocation Design (RAD) [KLPD05].

In this paper we analyze ways how approximate linear prices are calculated, identify problems of existing approaches and suggest alternative rules for calculating linear prices and stopping the auction. We use computer simulations as a tool to benchmark different auction designs, and compare their results based on allocative efficiency.

In Section 2 we provide an overview of iterative combinatorial auctions and describe relevant concepts and terms. Section 3 discusses approaches to calculate linear prices, while

Section 4 deals with auction stopping rules. In Section 5, we describe the results of computer simulations. Finally, in Section 6 we will draw conclusions and provide an outlook on future research.

## 2 Iterative Combinatorial Auctions

In this section, we provide an overview of iterative combinatorial auctions and describe relevant concepts and terms.[1] We first introduce some necessary notation: $\mathcal{K} = \{1, \dots, m\}$ denotes the set of items indexed by $k$ and $\mathcal{I} = \{1, \dots, n\}$ indexed by $i$ denotes a set of bidders with private valuations $v_i(S) \geq 0$ for $S \subseteq \mathcal{K}$. A typical goal in auction design is to achieve an *efficient allocation* $X^*$ which can be obtained by solving the *Combinatorial Allocation Problem* (CAP) (also called the Winner Determination Problem, WDP). The CAP is known to be NP-complete [LMS06], and has a straight-forward integer programming formulation using binary decision variables $x_i(S)$ to indicate whether the bid of bidder $i$ for bundle $S$ is in the allocation or not:

$$
\begin{aligned}
&\max_{x_i(S)} \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) v_i(S) \\
&s.t. \\
&\qquad \sum_{S \subseteq \mathcal{K}} x_i(S) \;\leq\; 1 \qquad \forall i \in \mathcal{I} \\
&\sum_{S: k \in S} \sum_{i \in \mathcal{I}} x_i(S) \;\leq\; 1 \qquad \forall k \in \mathcal{K} \\
&\qquad\qquad x_i(S) \;\in\; \{0, 1\} \qquad \forall i, S
\end{aligned}
\qquad \textbf{(CAP)}
$$

which requires that $X^*$ is a combination of bundles which maximizes the sum of valuations. The first set of constraints guarantees that any bidder can win at most one bundle, which is only relevant for XOR-bidding. The second set of constraints ensures that each item is only sold once with $k \in \mathcal{K}$ being the set of items to be sold. Note that at this point it is not defined how much bidders pay for the goods, i.e. what the ask prices are. Package bids as they are used in CAs lead to a number of hard problems.

- The ***Winner Determination Problem*** (WDP), introduced above, has been attracting intense research efforts, and polynomial-time solutions for restricted cases are known [RP98].

- The ***Preference Elicitation Problem*** (PEP) of a bidder, which is again $\mathcal{NP}$-complete in the number of auction items, is much less well understood. Compared to the WDP, which is solved centrally by the auctioneer and is precisely defined mathematically, preference elicitation is done by each bidder individually [SB06]. The problem can be further separated into the *bundle selection* and the *bundle evaluation* problem. In addition, the *strategy problem* of calculating bid prices in various auction formats has been a main focus in the classic game-theoretic auction literature, but turns out to be difficult to solve for iterative combinatorial auctions.

- ***Communication Complexity*** is related to the PEP and describes the problem of having to transfer an exponential number of valuations from the bidder to the auctioneer [Nis00].

PEP "has emerged as perhaps the key bottleneck in the real-world application of combinatorial auctions. Advanced clearing algorithms are worthless if one cannot simplify the bidding problem facing bidders" [Par06]. ICAs are up to date the most promising way of addressing

[1] We refer the reader to [Par06] for a more detailed introduction into Iterative Combinatorial Auctions.

problems of preference elicitation and communication complexity. Bidders do not need to conduct complete preference elicitation for all possible bundles in one step, but can use feedback obtained from the auctioneer during the auction to significantly reduce the space of interesting bundles. Further interesting benefits of ICA include *transparency* (and therefore empirical fairness), *privacy* and *distribution of computations*(see also [Par06]).

## 2.1 Pricing in ICAs

Because of computational requirements, ICAs are usually round-based rather than continuous. Bidders can submit their bids whilst the round is open, and after each round the auctioneer publishes feedback about the current market status. ICAs typically provide feedback in form of ask prices. Different pricing schemes have been discussed in the literature (see [PB05] for a more detailed discussion):

**Definition 1** *A set of prices $p_i(S), i \in \mathcal{I}, S \subseteq \mathcal{K}$ is called:*

- ***linear** (or **additive**), if*

$$\forall i, S : p_i(S) = \sum_{k \in S} p_i(k)$$

- ***anonymous**, if*

$$\forall i, j, S : p_i(S) = p_j(S)$$

In other words, prices are *linear* if the price of a bundle is equal to the sum of prices of its items, and prices are *anonymous* if prices of the same bundle are equal for every bidder. Non-anonymous prices are also called *discriminatory* prices. By combining the introduced notions the following three sets of ask prices can be derived after each auction round:[2]

1. a set of linear anonymous prices $\mathcal{P} = \{p_i(k)\}$
2. a set of non-linear anonymous prices $\mathcal{P} = \{p(S)\}$
3. a set of non-linear discriminatory prices $\mathcal{P} = \{p_i(S)\}$

For a bidder $i$, a set of prices $\mathcal{P}$ and a bundle $S$ let $\pi_i(S, \mathcal{P}) = v_i(S) - p_i(S)$ denote the bidder's payoff and $\Pi(S, \mathcal{P}) = \sum_{i \in \mathcal{I}} p_i(S)$ denote the auctioneer's revenue from bundle $S$ at prices $\mathcal{P}$. In addition, let $\Gamma$ denote the set of all possible allocations with $X^* \subseteq \Gamma$. A important guideline in constructing efficient price-based auctions is the economic theory of *competitive equilibrium*, where supply equals demand.

**Definition 2 (Competitive Equilibrium, CE)** *Prices $\mathcal{P}$ and allocation $X^* = \{S_1^*, \ldots, S_n^*\}$ are in competitive equilibrium if:*

$$\pi_i(S_i^*, \mathcal{P}) = \max_{S \subseteq \mathcal{K}} [v_i(S) - p_i(S), 0] \quad \forall i \in \mathcal{I}$$
$$\Pi(X^*, \mathcal{P}) = \max_{S \in \Gamma} \sum_{i \in \mathcal{I}} p_i(S)$$

In the CE the payoff of every bidder (and the auctioneer) is maximized and the auction will effectively end because bidders will not want to change the allocation by submitting any further bids. The allocation $X^*$ is said to be *supported* by prices $\mathcal{P}$ in CE.

Bikhchandani and Ostroy [BO06] show that $X^*$ is supported in CE by some set of prices $\mathcal{P}$ if and only if $X^*$ is an efficient allocation. This fact allows for construction of ICAs, which update prices in the direction of CE prices until there are no new bids. Such ICA will normally converge to a minimal CE price set, which is usually the desired design goal.

[2]To our knowledge linear discriminatory prices have not been discussed in the literature.

**Definition 3 (Minimal CE Prices)** *Minimal CE prices minimize the auctioneer revenue $\Pi_S(X^*, \mathcal{P})$ on the efficient allocation $X^*$ across all CE prices possible for this allocation.*

Simple examples illustrate that both linear and anonymous prices do not exist for a general CAs, in other words, for certain types of bidder valuations it is impossible to find linear prices which support the efficient allocation $X^*$ [PB05] . On the other hand, Bikhchandani and Ostroy have shown that non-linear discriminatory competitive equilibrium prices do always exist and support the efficient allocation [BO02]. However, discriminatory pricing introduces additional complexity by the sheer volume or prices and is often considered unfair by bidders.

It is interesting to characterize types of valuations, where linear CE prices are possible. The sufficient and almost necessary condition for existence of linear CE prices is the *goods are substitutes* property [Par06]. Intuitively this implies that the bidder will continue to demand items which do not change in price, even when prices on other items are increasing. The *goods are substitutes* condition is very restrictive as most known practical applications of combinatorial auctions rather deal with complementary goods.

Although, the existence of linear CE prices is limited they are of high practical value. Having obtained linear price from the auctioneer, a bidder can easily determine prices for different bundles. For non-linear prices, this would require a special *request-for-quotes* protocol, since the transmission of all bundle prices is usually impractical. These arguments advocate application of approximative linear prices for the design of ICA mechanisms.

### 2.2 Auction Designs based on Linear Prices

The *Resource Allocation Design* (RAD) [KLPD05], the *combinatorial clock auction* and the *clock-proxy auction* [ACM06] have been proposed as iterative combinatorial auction designs based on linear prices. Whereas the combinatorial clock auction and the clock-proxy auction do not allow submitting bid prices in each round and use a simple incremental price update rule, RAD utilizes the submitted bid prices to approximate linear prices for the next round. This allows fine tuning of the price calculation procedure to produce prices most appropriate to serve as a guideline for the bidders. Since RAD has been shown to produce high levels of allocative efficiency and a detailed comparison of the combinatorial clock and clock-proxy auction to RAD is beyond the scope of this paper, we will restrict our attention to RAD-based auction designs in the following.

## 3 Calculating Linear Prices based on Bid Prices

In this section we introduce related terminology and discuss some desirable properties of approximated linear ask prices based on bid prices. Based on this, we discuss some pitfalls in calculating linear prices and suggest an improved price calculation algorithm which avoids these problems.

### 3.1 Desirable Properties and the Overall Procedure

Though linear prices is a very attractive instrument due to their intuitive meaning to the bidder, their calculation requires significant computational effort, as we will see later in this section. Since linear CE prices do not always exist [BO02], linear ask prices often do not lead the auction to the efficient allocation. Moreover, as will be shown later, even the existence of *compatible linear prices* in each iteration is not guaranteed. Therefore we try to approximate the desired bundle prices by linear prices to provide a good enough guideline for the bidders. There are different ways of doing this, and as of now there is little research on the effects of those pricing heuristics.

In general there are three obvious negative effects of the price approximation: (a) if the price of some bundle is approximated too high, this can keep a bidder from submitting a potentially winning bid, (b) if the price of some bundle is approximated too low, a new bid might have little chances of winning [3], and (c) since there are several feasible price sets, an unfavorable price selection can worsen the threshold problem.

At this point we need some additional notation. Let $t = 1, 2, 3, \ldots$ denote the current auction round, and $B^t$ be the set of all bids submitted in the round $t$ with $b \in B^t$ denoting a single bid. A bid $b = b_i(S)$ represents the bid price submitted by the bidder $i$ on the bundle $S$. Furthermore, for the current provisional allocation $X^t$ let $W^t \subseteq B^t$ and $L^t \subset B^t$ be the currently provisionally winning respectively the provisionally losing bids, with $W^t \bigcap L^t = \emptyset$, $W^t \bigcup L^t = B^t$. In other words, $b = b_i(S) \in W^t \Leftrightarrow x_i^t(S) = 1$. In the following we will often omit the round index $t$ always indicating with $B, W, L, X$ the provisional allocation in the current round $t$ and with $\mathcal{P}$ the prices to be calculated for the next round $t + 1$.

**Definition 4** *A set of prices $\mathcal{P} = \{p_i(S)\}$ is called* ***compatible***[4] *with the allocation $X$ and bids $B = \{b_i(S)\}$, iff*

$$\forall b \in B : b \in W \Leftrightarrow p_i(S) \leq b_i(S) \textit{ and } b \in L \Leftrightarrow p_i(S) > b_i(S)$$

The interpretation is quite intuitive: the set of prices is compatible with the given allocation and bids if all winning bids are higher than or equal to the prices and all losing bids are lower than the prices. In other words, compatible prices should explain the winners, why they won, and the losing bidders, why they lost.

Inspired by RAD, we try to fulfill the following three properties, that seem important to weaken the negative effects of the price approximation described above, in the following preference order:

1. The prices for the next round should be compatible with the current provisional allocation and submitted bids. If such prices do not exist, they should be approximated as close as possible.

2. The prices should be balanced across items to be perceived as fair and to weaken the threshold problem.

3. The prices should be minimal enabling bidders to submit bids as long as they can.

RAD's price calculation rules try to satisfy the first two properties by solving a series of linear programs (LPs), however, the RAD pricing rules are not sufficient to achieve these properties. In the following we propose an extended RAD-based price calculation algorithm, which fulfills all the three properties and always produces a unique[5] price set. We will also illustrate the leaks of RAD by examples and explain how our algorithm avoids these pitfalls.

The overall approach can be schematically described as follows:

$$\begin{aligned}
&\min_{p(k),\delta_b} \quad \{max\{\delta_b\}, max\{p(k)\}\} \\
&\text{subject to:} \\
&\sum_{k\in S} p(k) = b_i(S) && \forall\, b = b_i(S) \in W \\
&\sum_{k\in S} p(k) + \delta_b \geq b_i(S) && \forall\, b = b_i(S) \in L \\
&\delta_b \geq 0 && \forall\, b \in L \\
&p(k) \geq 0 && \forall\, k \in \mathcal{K}
\end{aligned} \tag{1}$$

[3] This scenario can delay the auction for several more rounds, but does not necessarily cause inefficiency.
[4] We introduced this concept in [PB05].
[5] By unique we mean independent from the LP solver implementation.

The first condition sets the bid prices of the winning bids equal to the ask prices which satisfies the first compatibility requirement.[6] The second condition tries to satisfy the second compatibility requirement as close as possible, whereby the distortions $\delta_b$ represent the deviations from the ideal. The schematically defined objective function $min\,\{max\{\delta_b\}, max\{p(k)\}\}$ stands for balanced minimizing all distortions $\delta_b$ and then balanced minimizing the prices, which can be done by solving a series of LPs. The next two sections describe these steps in full detail.

### 3.2 Minimizing Distortions

In this step we sequentially lower all distortions while trying to keep them balanced. We first minimize the maximum of all distortions, then fix those distortions that can not be further improved and repeat. Let $\hat{L}$ denote the set of all bids $b$, for which $\delta_b$ can not be improved any more, and initialize it with $\hat{L} = \emptyset$. Then solve the following linear program (2):

$$\begin{array}{rcll} \min\limits_{p(k),Z,\delta_b} & Z & & \\ \text{subject to:} & & & \\ \sum\limits_{k\in S} p(k) & = & b_i(S) & \forall\, b = b_i(S) \in W \\ \sum\limits_{k\in S} p(k) + \hat{\delta}_b & = & b_i(S) & \forall\, b = b_i(S) \in \hat{L} \\ \sum\limits_{k\in S} p(k) + \delta_b & \geq & b_i(S) & \forall\, b = b_i(S) \in L \setminus \hat{L} \\ 0 \leq \delta_b & \leq & Z & \forall\, b \in L \setminus \hat{L} \\ p(k) & \geq & 0 & \forall\, k \in \mathcal{K} \end{array} \tag{2}$$

Let $Z^*, \delta^*, \mathcal{P}^*$ be the solution of (2) and let $L^* := \{b : \delta_b^* = Z^*\}$. If $Z^* = 0$ we are done. Otherwise RAD would fix the distortions for all bids in $L^*$ and proceed. However, if $L^*$ contains more than one element, some of these distortions may still be improved. Moreover, if the Simplex optimization algorithm [NW88] is used, we will very likely get some $\delta_b^* = Z^*$ since it always finds some vertex of the feasible polyhedra. This makes additional steps necessarily. We now bound the distortions by $Z^*$ and minimize the sum of all distortions in $L^*$ as follows:

$$\begin{array}{rcll} \min\limits_{p(k),\delta_b} & \sum\limits_{b\in L^*} \delta_b & & \\ \text{subject to:} & & & \\ \sum\limits_{k\in S} p(k) & = & b_i(S) & \forall\, b = b_i(S) \in W \\ \sum\limits_{k\in S} p(k) + \hat{\delta}_b & = & b_i(S) & \forall\, b = b_i(S) \in \hat{L} \\ \sum\limits_{k\in S} p(k) + \delta_b & \geq & b_i(S) & \forall\, b = b_i(S) \in L \setminus \hat{L} \\ 0 \leq \delta_b & \leq & Z^* & \forall\, b \in L \setminus \hat{L} \\ p(k) & \geq & 0 & \forall\, k \in \mathcal{K} \end{array} \tag{3}$$

If at least one of the distortions in $L^*$ can be improved, this will be done by (3). We now remove all bids with improved distortions from $L^*$ and repeat with (3) until no more distortions can be improved. At this point we set $\hat{L} := \hat{L} \cup L^*$, fix all non-improvable distortions ($\forall b \in L^*$ set $\hat{\delta}_b := \delta_b^*$), and continue with (2).

[6]Though this is a little bit more restrictive than the corresponding condition in the price compatibility requirement, it looks reasonable and guarantees the uniqueness of the prices. We follow the RAD approach at this point.

### 3.3 Balancing Prices

After the set of all positive distortions $\hat{L}$ is identified and all those distortions are minimized and fixed to $\{\hat{\delta}_b\}$, the prices may still not be unique. For example in the ideal case we get $\hat{L} = \emptyset$ and we still have a lot of freedom in setting prices. We now balance prices similar to minimizing distortions in the previous step. We first minimize the maximum of all prices, then fix those prices that can not be further lowered and repeat. Let $\hat{\mathcal{K}}$ denote the set of all items which prices can not be lowered any more, and initialize it with $\hat{\mathcal{K}} = \emptyset$. Then solve the following linear program (4):

$$\begin{array}{rcll} \min\limits_{p(k),Y} & Y & & \\ \text{subject to:} & & & \\ \sum\limits_{k\in S} p(k) & = & b_i(S) & \forall\, b = b_i(S) \in W \\ \sum\limits_{k\in S} p(k) + \hat{\delta}_b & = & b_i(S) & \forall\, b = b_i(S) \in \hat{L} \\ \sum\limits_{k\in S} p(k) & \geq & b_i(S) & \forall\, b = b_i(S) \in L \setminus \hat{L} \\ p(k) & = & \hat{p}(k) & \forall\, k \in \hat{\mathcal{K}} \\ 0 \leq p(k) & \leq & Y & \forall\, k \in \mathcal{K} \setminus \hat{\mathcal{K}} \end{array} \tag{4}$$

Let $Y^*, \mathcal{P}^*$ be the solution of (4) and let $\mathcal{K}^* := \{b : p^*(k) = Y^*\}$. Now RAD would fix the prices for all bids in $\mathcal{K}^*$ and proceed. But again, if $\mathcal{K}^*$ contains more than one element, some of these prices may still be lowered and this is very likely to happen when using a Simplex-based LP solver. This can be illustrated by the following examples.

Consider an auction with three items ***A***, ***B***, ***C*** and four currently active bids from different bidders $b_1(A) = 55, b_2(C) = 55, b_3(AB) = 40, b_4(BC) = 40$. Obviously the provisional winners are $1$ and $3$ and $\hat{L} = \emptyset$. After removing redundant inequalities the linear program (4) looks like:

$$\begin{array}{rcl} \min\limits_{p(B),Y} & Y & \\ \text{subject to:} & & \\ p(A) & = & 55 \\ p(C) & = & 55 \\ 55 & \leq & Y \\ 0 \leq p(B) & \leq & Y \end{array}$$

We can get two possible solutions of this problem when using a simplex-based LP solver: $\{p^*(B) = 55, Y^* = 55\}$ or $\{p^*(B) = 0, Y^* = 55\}$. In the first case RAD would fix all prices to $55$, which would completely distort the bidder's understanding of the current demand for the item $B$.

Another important point is the the balancing method used. RAD proposes maximizing minimal price instead of minimizing maximum price. However, if the solver finds the second solution, RAD would fix $\hat{p}(A) = 55$ and $\hat{p}(C) = 55$ and then yield $p^*(B) = \infty$ in the next iteration.

Now consider another auction with three items ***A***, ***B***, ***C*** and two currently active bids $b_1(ABC) = 160, b_2(A) = 70$, where the provisional winner is $1$, and, again, $\hat{L} = \emptyset$. The linear program (4) looks like:

$$\begin{array}{rcl} & \min\limits_{p(A),p(B),p(C),Y} & Y \\ \text{subject to:} & & \\ p(A)+P(B)+P(C) & = & 160 \\ p(A) & \geq & 70 \\ 0 \leq p(A),p(B),P(C) & \leq & Y \end{array}$$

With a simplex-based solver this would yield one of two possible solutions: $\{p^*(A) = 70, p^*(B) = 20, p^*(C) = 70, Y^* = 70\}$ or $\{p^*(A) = 70, p^*(B) = 70, p^*(C) = 20, Y^* = 70\}$. In both cases RAD would stop with this solution. There were no reason, why the prices for the items $B$ and $C$ are different.

To avoid the pitfalls illustrated in the above examples we continue by bounding the prices by $Y^*$ and minimize the sum of all prices in $\mathcal{K}^*$ as follows:

$$\begin{array}{rcll} & \min\limits_{p(k)} & \sum\limits_{k\in\mathcal{K}^*} p(k) & \\ \text{subject to:} & & & \\ \sum\limits_{k\in S} p(k) & = & b_i(S) & \forall\, b = b_i(S) \in W \\ \sum\limits_{k\in S} p(k) + \hat{\delta}_b & = & b_i(S) & \forall\, b = b_i(S) \in \hat{L} \\ \sum\limits_{k\in S} p(k) & \geq & b_i(S) & \forall\, b = b_i(S) \in L \setminus \hat{L} \\ p(k) & = & \hat{p}(k) & \forall\, k \in \hat{\mathcal{K}} \\ 0 \leq p(k) & \leq & Y^* & \forall\, k \in \mathcal{K} \setminus \hat{\mathcal{K}} \end{array} \tag{5}$$

If at least one of the prices in $\mathcal{K}^*$ can be lowered, this will be done by (5). We now remove all items with lowered prices from $\mathcal{K}^*$ and repeat with (5) until no more prices can be improved. At this point we set $\hat{\mathcal{K}} := \hat{\mathcal{K}} \cup \mathcal{K}^*$, fix all non-improvable prices ($\forall k \in \mathcal{K}^*$ set $\hat{p}(k) := p^*(k)$), and continue with (4), unless $\mathcal{K} \setminus \hat{\mathcal{K}} = \emptyset$.

# 4 Auction Termination Condition

An important decision which obviously can affect the auction efficiency is when to close it. The RAD design [KLPD05] has an eligibility based stopping rule and forces a minimum bid increment. As illustrated below this is not always sufficient to ensure auction termination. One of the other stopping rules, defined in the RAD auction design, is an identical provisional allocation in two consecutive rounds . However, the approximative nature of liner prices in RAD in combination with this stopping rule can result in inefficient allocations.

Consider an example auction with the following valuations $v_i(S)$.

| Item | A | B | C | AB | AC | BC | ABC |
|---|---|---|---|---|---|---|---|
| Bidder1 | 10 | | | | | | 35 |
| Bidder2 | | | | 32 | | 32 | |

The efficient outcome would be to sell $A$ to the *Bidder1* and $\{B,C\}$ to the *Bidder2*. Let the following bids be active at some point during the auction, and the minimum bid increment be 2 monetary units (MU)[7].

[7] In RAD minimum increment must be added to ask price of every item in bundle, and not to the bundle as a whole

| Item | A | B | C | AB | AC | BC | ABC |
|---|---|---|---|---|---|---|---|
| Bidder1 | | | | | | | 30.5 |
| Bidder2 | | | | 23 | | | |

According to the price calculation described in the previous section, the resulting ask prices $p_i(S)$ are:

| Item | A | B | C |
|---|---|---|---|
| Price | 11.5 | 11.5 | 7.5 |

The *Bidder2* must choose between bidding 27 for $\{A,B\}$ or 23 for $\{B,C\}$. As he has equal valuations for both combinations, the second alternative is chosen. The next round bids are:

| Item | A | B | C | AB | AC | BC | ABC |
|---|---|---|---|---|---|---|---|
| Bidder1 | | | | | | | 30.5 |
| Bidder2 | | | | | | 23 | |

This is the second round with the same provisional allocation, and consequently the auction will be terminated with *Bidder1* receiving all three items. Obviously, this is not an efficient outcome. For *Bidder2* the auction termination comes as a surprise as form his point of view, he had submitted competitive bid prices, and was still ready to submit higher bids.

A naïve approach of removing this termination rule and relying only on the eligibility-based principle [KLPD05] causes other problems. Continuing the above example, prices of the new round will change to:

| Item | A | B | C |
|---|---|---|---|
| Price | 7.5 | 11.5 | 11.5 |

At this point the *Bidder2* can again bid 23 MU on the package $\{A,B\}$, and the auction might loops without stopping at all. The reason of this infinite loop is the possibility for prices to fall. Prices in RAD can go down from one round to another, because bidders are able to retract loosing bids from previous rounds.

In order to avoid this type of undesired behavior, we suggest to omit the auction stopping rule based on two successive identical allocations and introduce two alternative new rules to prevent auctions from looping:

- Increase the minimum increment with each equal allocation, but reset the minimum increment to the original value if the allocation changes.
- Request every bidder to outbid own bids, which were submitted in previous rounds on same bundles.

If the loosing bidder's valuation is high enough, both rules will eventually cause the allocation to change. Otherwise the loosing bidder will at some point stop bidding and the auction will end according to the eligibility rule.

## 5 Simulation

In this section we summarize the results of discrete event simulations of ICAs using linear prices as described in the previous sections. We focus on two aspects:

- What is the impact of complementarity among items in bidder valuations. Typically, the stronger this complementarity is, the more approximative linear prices deviate from CE prices, which can negatively affect auction efficiency.
- How do the enhanced rules described in Section 3.3 compare to the original RAD with respect to allocative efficiency.

### 5.1 Setup

The simulations are run against a slightly modified version of the *pairwise synergy* value model described in An et al. [AEK05]. It is defined by valuations for individual items $v_1, v_2, \ldots, v_m$ and a matrix of pairwise synergies between items $syn_{k,l} : k, l \in \mathcal{K}, k \neq l, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0$. Valuation for a bundle $S$ is calculated as

$$v(S) = \sum_{k=1}^{|S|} v_k + \frac{2}{|S|-1} \sum_{k=1}^{|S|} \sum_{l=1}^{|S|} syn_{k,l}$$

Synergy values of 0 correspond to independent goods, positive synergy values define a market with super-additive valuations, and negative synergies define sub-additive valuations. Parameters for the value model are the absolute interval for randomly generated item values, and relative to the item price interval for synergy values.

To analyze the role of ask prices on the auction outcome we need a competitive market with different but similar valuations. Such a scenario is modeled by generating private value models for each bidder using different parameters, taken from the same narrow distribution. Simulated bidders are myopic best-response agents which are restricted on the maximum bundle size they need (and consequently bid on), and with enough computational resources to evaluate all possible combinations. There is no budget constraint. The auction format is RAD with modifications which take into account problems described in the Section 3. The minimum increment is set very small in order not to impact the result.

We measure auction efficiency following [KLPD05] as the ratio of the total valuations of the resulting allocation $X'$ in the simulation to the total valuation of the efficient allocation.

$$E(X') = \frac{\sum\limits_{S \in X'} \sum\limits_{i \in \mathcal{I}} x'_i(S) v_i(S)}{\max\limits_{x_i(S)} \sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i(S) v_i(S)}$$

### 5.2 Impact of Item Complementarity on Auction Efficiency

Looking at the types of valuations, where linear CE prices are possible (Section 2.1), one can assume that auction efficiency will decline as complementarities among items increase, because approximated linear prices deviate further from CE prices.

In order to quantify the efficiency loss, we ran a series of 300 simulation rounds with different synergy values in the set of valuations. The simulated auction scenario included 5 items, 7 bidders restricted to maximum bundle size of 3, and synergy values varying in small increments from -2 to 4. The allocative efficiency related to synergy values is shown in the Figure 1.

As it was expected, auction efficiency is at 100% for negative synergy values (sub-additive valuations) and a synergy value of 0 (independent goods). With growing super-additivity in the

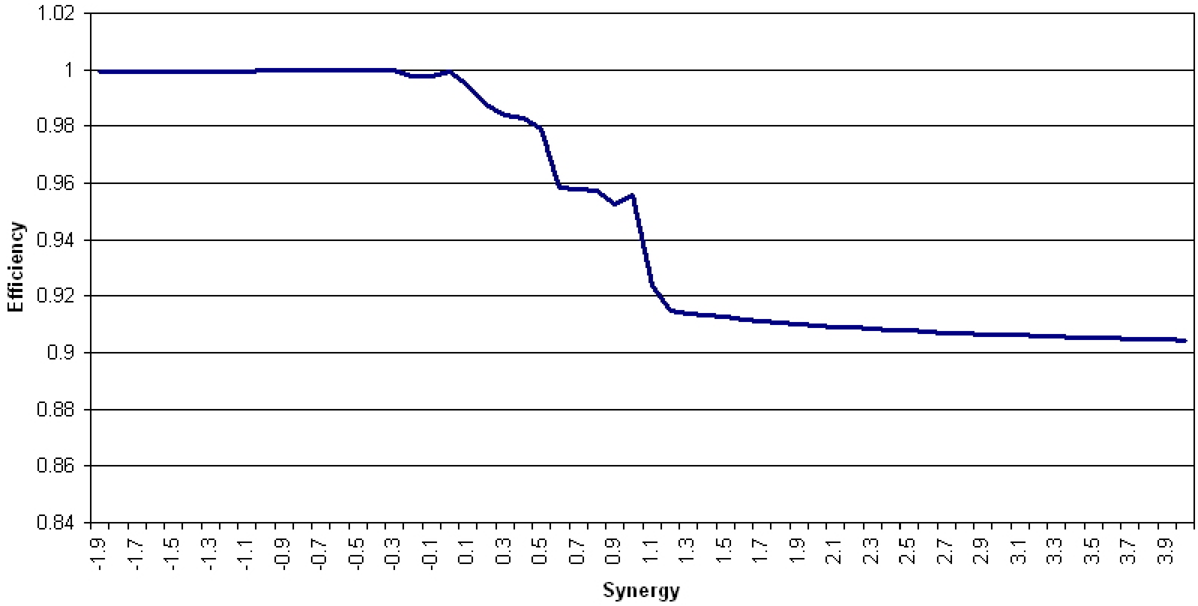


Figure 1: Linear Price ICA Efficiency depending on Relative Synergy

value model, linear prices deviate further from CE ask prices, which are necessary to guide bidders to an efficient auction outcome.

The results suggest that linear ask prices can be successfully applied for value models with low or negative complementarity between items. In case of complementarities, a certain efficiency loss can be attributed to the use of linear prices. In practice, there will be a tradeoff between the efficiency loss in ICAs with discriminative non-linear prices that is due to the increased complexity and the large number of ask prices that have to be communicated, and the efficiency loss due to linear prices. One needs to mention, that the results are limited to the pairwise synergy value model and cannot easily be transferred to other types of valuations.

### 5.3 Impact of different Price Calculation Methods

The second set of simulations was set up to compare the updated pricing procedure described in Section 3.3 with the original RAD rules. We have created 28 auction setups with 5 items, 7 bidders, and relative synergy values from 0.5 to 2.0. Each setup was run using all of the three pricing algorithms, and average efficiency was measured.

| Pricing method | Original RAD | Advanced price calculations (see section 3) |
|---|---|---|
| Efficiency | 0.94486 | 0.94812 |

In the current setup, the advanced price calculations performed consistently better than RAD, but the gains were limited. We expect the difference to be more significant when some bidders submit jump bids and ask prices change not monotonically. The impact of balanced prices vs. unbalanced prices in laboratory settings might also be higher, which is, however, subject of future research.

## 6 Conclusion

Iterative combinatorial auctions provide an efficient way to solve complex negotiation problems on multiple items in the presence of complementarities. Economists typically use game theory and laboratory experiments to analyze market mechanisms. It turns out, however, that computing for example Nash equilibria of iterative combinatorial auctions is computationally hard because the space of bidding strategies can be very large. Alternatively, laboratory experiments are extremely helpful, but limited in the number of different treatment variables one can analyze in a single experiment (e.g., number of items, synergies among items, pricing rules, stopping rules, number of bidders).

In recent years, computation has become the third research methodology, complementing theory and experiment in many sciences (aka computational science). Computer simulation makes it possible to investigate regimes that are beyond current experimental capabilities and to study phenomena that cannot be replicated in laboratories easily.

In this paper, we have used computer simulations to analyze linear pricing procedures in iterative combinatorial auctions for myopic best-response bidders in different settings. Based on these simulations we could identify design problems in RAD, including unbalanced ask prices and inefficiencies due to the RAD auction stopping rules. We have suggested modifications in the original design and compared these with the original design. In our future research, we plan to extend this approach and provide detailed sensitivity analyses of iterative combinatorial auction formats (using linear and non-linear prices) under various parameter settings.

## 7 Glossary

| Variable | Explanation |
|---|---|
| $\mathcal{K} = \{1, \ldots, m\}$ | set of items |
| $k \in \mathcal{K}$, also $l$ | index for items |
| $\mathcal{I} = \{1, \ldots, n\}$ | set of bidders |
| $i \in \mathcal{I}$, also $j$ | index for bidders |
| $S \subseteq \mathcal{K}$ | bundle (package) |
| $v_i(S)$ | valuation of the bidder $i$ for the bundle $S$ |
| $p_i(S)$ | discriminatory bundle ask price of the bundle $S$ for the bidder $i$ |
| $p(S)$ | anonymous bundle ask price of the bundle $S$ |
| $p(k)$ | anonymous linear ask price of the item $k$ |
| $\mathcal{P} = \{p_i(S)\}$ or $\mathcal{P} = \{p(S)\}$ or $\mathcal{P} = \{p(k)\}$ | set of ask prices |
| $\pi_i(S, \mathcal{P}) = v_i(S) - p_i(S)$ | payoff of the bidder $i$ for the bundle $S$ |
| $D_i(\mathcal{P}) = \{S : \pi_i(S, \mathcal{P}) = \max_{T \subseteq \mathcal{K}} \pi_i(T, \mathcal{P}),$ $\pi_i(S, \mathcal{P}) \geq 0, S \subseteq \mathcal{K}\}$ | maximum demand set |
| $b_i^t(S) \geq 0$ | bid (bid price) of the bidder $i$ for the bundle $S$ in the round $t$ |
| $B^t = \{b_i^t(S)\}$ | set of bids (bid prices) in the round $t$ |
| $b \in B^t$ | index for bids |
| $W^t \subseteq B^t$ | set of provisionally winning bids in the round $t$ |
| $L^t \subseteq B^t$ | set of provisionally losing bids in the round $t$ |
| ... $X$ | allocation |
| ... $X^*$ | efficient allocation |
| ... $\Pi(X, \mathcal{P}) = \sum_{i \in \mathcal{I}} p_i(S_i)$ | auctioneer's revenue from allocation $S$ |

| Acronym | Explanation |
|---|---|
| CA | Combinatorial Auction |
| CAP | Combinatorial Allocation Problem |
| CE | Competitive Equilibrium |
| ICA | Iterative Combinatorial Auction |
| LP | Linear Program |
| MU | Monetary Unit |
| PEP | Preference Elicitation Problem |
| RAD | Resource Allocation Design |
| WDP | Winner Determination Problem |

## 8 Acknowledgements

The financial support from the Deutsche Forschungsgemeinschaft (DFG) (BI 1057/1-1) is gratefully acknowledged.

## 9 Biography

PAVLO SHABALIN studied Informatics at the Lviv State University, Ukraine, and at the TU Munich, Germany. Currently he is PhD student in the group of Prof. BICHLER. His main research interests are electronic markets and information security.

ALEXANDER PIKOVSKY first studied applied mathematics at the Kharkiv National University, Ukraine, than he continued his studies at the TU Munich, Germany, where he received his masters degree in financial mathematics. Since April 2003 he is working as a research assistant in the group of Prof. BICHLER.

MARTIN BICHLER is full professor of Internet-based Information Systems at the TU Munich, Germany. He received his PhD and his habilitation from the Vienna University of Economics and Business Administration, Austria. His research interests are in the area of electronic markets, business intelligence, and service management.

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