Combinatorial First-Price Auctions: Theory and Experiments

Gian-Marco Kokott, Martin Bichler,* Per Paulsen

Technical University of Munich, Germany†

Ex-post split-award auctions are a widespread form of combinatorial procurement auctions in which the demand for some quantity to be procured is split into multiple shares. Markets with diseconomies of scale are wide-spread, but strategically challenging. We show that, unlike in single-object auctions, first-price sealed-bid and the Dutch combinatorial auction formats are not strategically equivalent. While the former exhibits a coordination problem for bidders, the Dutch formats have only efficient equilibria. The price information revealed during the Dutch auction formats avoids equilibrium selection problems and helps bidders coordinate. Also, the theoretical predictions explain bidder behavior in the lab remarkably well.

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1. Introduction

Combinatorial auctions in which bidders can submit bids on packages of objects have found wide-spread application, because they allow bidders to express synergistic valuations for different objects (Cramton et al. 2006). In particular, the simple first-price sealed-bid (FPSB) auction format has found numerous applications in industrial procurement (Bichler et al. 2006), logistics (Caplice

* Corresponding author: Boltzmannstr. 3, 85748 Garching/Munich, Germany, Phone: +49-89-289-17500, E-Mail: martin.bichler@mytum.de

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2007), and for the sale of spectrum licenses.[1] Unfortunately, equilibrium bidding strategies in FPSB combinatorial auctions are not yet well understood. In a widely cited paper, Bernheim and Whinston (1986) studied the equilibrium of an FPSB combinatorial auction under complete information. However, incomplete information analysis has turned out to be challenging. The seminal article by Anton and Yao (1992), which provides a Bayesian Nash equilibrium analysis for a procurement market with two objects, two bidders, and diseconomies of scale provided an important step forward in understanding first-price sealed-bid combinatorial auctions. Their analysis yields an equilibrium selection problem for bidders, suggesting that such auctions could be inefficient in practice. The assumption that bidders know the economies of scale in the market is realistic in many procurement markets, and it allows for the characterization of equilibrium bidding strategies in first-price auctions. It is unclear, however, whether the 2-bidder case generalizes, because bidders can veto the split award unilateraly. In this paper, we also focus on markets with diseconomies of scale, but analyze the 2-bidder and the n-bidder case for different types of first-price combinatorial auctions. We provide a comprehensive theoretical analysis of FPSB and Dutch auction formats and also results of lab experiments, which provide further evidence that different versions of the Dutch combinatorial auction formats have remarkable properties.

1.1. Motivation

In split-award auctions a procurement manager splits his demand for a larger quantity of a good into two (or more) shares (aka. lots), such as a 30% and a 70% share or two 50% shares. In an ex-post split-award auction, the buyer allows suppliers to submit bids on individual shares as well as on 100% of the order, which differentiates ex-post split-award auctions from their ex-ante counterpart (Bichler et al. 2015, Chaturvedi et al. 2016).[2] We limit our attention to the case with two shares only and diseconomies of scale, and assume that the suppliers know the scale economies in the market, but the buyer does not.[3] The assumption of diseconomies of scale is practically relevant, for example, if suppliers face capacity limits or stepwise fixed costs. Moreover, it is strategically interesting, because suppliers must coordinate to obtain an efficient solution.

Split-award auctions are often used for multi-sourcing in industrial procurement. Companies such as Sun and HP, for example, procure products worth hundreds of millions of dollars using different

[1] The regulators in Norway (2013) and France (2011) used a FPSB combinatorial auction.

[2] For the sake of simplicity, we reduce the term ex-post split-award auctions to split-award auctions in the rest of this paper. If we mention the ex-ante format, it is denoted explicitly.

[3] These assumptions are similar to Anton and Yao (1992). Note that ex-post split-award auctions differ from share auctions for a perfectly divisible object in which bidders submit continuous bid schedules for different shares of the object (Wilson 1979, Ausubel and Cramton 1998).

types of multiple sourcing auctions (Elmaghraby 2007). For risk considerations, such firms often want to have more than one supplier, but they are also particularly interested when the dual source solution is the cost-minimal solution. This work is partly motivated by procurement practices of a large European engineering and electronics multinational. Within one year (April 2015 to March 2016) more than one third of its procurement auctions were split-award auctions.[4] 81% of these were ex-ante and 19% of these were ex-post split-award auctions, with 88% of the split-award auctions allocated two shares only. 14% of the split-award auctions were FPSB auctions, in which suppliers submitted bids on individual shares and on the overall package; the others were run sequentially using a Dutch ascending auction followed by an FPSB auction, or using two Dutch auctions in a sequence, in which the winner of the first share was allowed to quote for the remaining share(s) as well in the ex-post formats.

The choice of a proper auction design depends on the prevailing scale economies in the market: with economies of scale (and no risk-premium), it is efficient to select a single supplier (sole source award), and the procurement manager can also employ a single-object auction. However, the solution with two suppliers (split award) is efficient with diseconomies of scale and when a buyer can expect savings from an ex-ante split-award auction. A buyer who knew the scale economies in the market, could use the appropriate auction design for each efficiency scenario. However, while it is reasonable to assume that bidders know the scale efficiencies of their product, this is often not true for the buyer. Thus, an auctioneer often wants to employ an (ex-post) split-award auction, in which both a sole source and a split award are possible outcomes. In these auctions, the suppliers can submit bids on shares of the business as well as on the whole business, i.e., the package of both shares. The buyer then selects the cost minimizing combination of bids, i.e., the decision whether to split the award or to select a single supplier is endogenous.

Anton and Yao (1992) characterize Bayesian Nash equilibria for FPSB split-award auctions in procurement markets with diseconomies of scale, two shares, and two bidders. They assume that the efficiency setting in the market is common knowledge among the suppliers and is such that an outcome with both bidders winning is efficient. The authors show that coordination on a split award is a Bayesian Nash equilibrium in their model, which is referred to hereafter as a σ equilibrium. However, they show that there is also a winner-takes-all (WTA) equilibrium in which both bidders compete for only the package.

This environment modeled by Anton and Yao (1992) is interesting for a number of reasons. First, in the efficient σ equilibrium bidders need to coordinate, which is strategically challenging. Second,

[4] Only auctions with a total spend beyond a minimum level were recorded. The spend for the individual auctions ranged from 250,000 to 175 million Euros.

environments with diseconomies of scale are relevant for a large number of procurement events because of the risk premiums often associated with such environments. In around 75% of the ex-post split-award auctions overseen by the above mentioned engineering and electronics manufacturer, procurement managers determined a risk premium for the sole source allocation a priori; this made them frequently select the dual-source solution even in situations in which the sole source solution was cheaper. In fact, the setting with diseconomies of scale is strategically equivalent to one in which the buyer defines a constant risk premium for the sole source award and bidders benefit from economies of scale, as we will discuss later.

1.2. Contributions

The contributions of this paper are twofold. First, we derive bidding strategies in various first-price combinatorial auction formats. We extend the results of Anton and Yao (1992) for the FPSB auction to $n > 2$ bidders and provide Bayesian Nash strategies as closed-form solutions in such an environment. As in the setting with two bidders, there is a coordination problem for the bidders, because multiple equilibria emerge. Additionally, alternative auction formats such as the Dutch split-award auction and a combination of Dutch and FPSB auction in a second phase are analyzed. We are not aware of a game-theoretical treatment of these Dutch auctions, in spite of their wide-spread use in procurement practice, nor are we aware of an analysis with more than two bidders. We show that both auction formats reduce the strategic complexity for the bidders considerably compared to the FPSB auction, because only efficient equilibria exist.

While the Dutch and the Dutch-FPSB split-award auctions are cost equivalent, this is only true for the FPSB split-award auction when bidders choose the payoff-dominant efficient equilibrium. By contrast, the Dutch and FPSB auctions are strategically equivalent in single-object auctions. Overall, the fine differences among the information revealed to bidders during the FPSB, Dutch-FPSB, and Dutch formats lead to interesting and non-obvious differences in the equilibrium bidding strategies of multi-object auctions.

Second, we provide an experimental analysis of the three first-price auction formats and show that the theoretical models explain important empirical regularities in the lab. As predicted the two Dutch auction formats are more efficient than their sealed-bid counterpart. We found evidence for tacit collusion in all auction formats with 2 bidders as suggested by theory. In the competitive three-bidder setting the bidding behavior in the Dutch auction does not significantly differ from the equilibrium strategy. This is remarkable, because bidder behavior in single-object first-price auctions typically deviates substantially from the risk-neutral Bayesian Nash equilibrium strategies (Cox et al. 1983, Filiz-Ozbay and Ozbay 2007, Kirchkamp and Rei 2011, Bichler et al. 2015).

The price information from the first phase of the Dutch combinatorial auction formats provides an excellent signal helping bidders to coordinate on the efficient equilibrium. In contrast, the equilibrium selection problem makes it very hard for bidders to coordinate in the FPSB auction with two bidders. The experimental results show that bidders in a FPSB auction with two bidders select both types of equilibria described by Anton and Yao (1992), and we find 55% inefficient allocations.

As theory suggests, the Dutch auction is much more efficient than the FPSB auction in the two-bidders environment, at a higher cost. In two-bidder markets, the Dutch-FPSB auction appears as an interesting and practically simple alternative that yields the highest share of efficient allocations (82%) of all three mechanisms and low procurement cost. The sealed-bid auction in the second phase of the Dutch-FPSB format allows for a broader set of equilibrium bid prices and leads to lower bid prices in the first phase of the experiments. This avoids coordination problems that arise in the Dutch auction, in which bidders sometimes overbid the unique equilibrium bid price in an attempt to achieve a higher payoff, making it attractive to win the package. This phenomenon, actually leads to a higher number of inefficient sole source allocations in the Dutch, compared to the Dutch-FPSB auction. Furthermore, we find evidence for pooling and tacit collusion, as bidders, who succeeded to coordinate on the split, achieved high profits in all three auction formats.

Interestingly, the addition of just one more bidder levels the differences among the three auction formats and almost always results in the selection of an efficient split. This is also the case for FPSB split-award auctions, although this format still has an inefficient winner-takes-all equilibrium even with more than two bidders. Addition of a third bidder also has a substantial effect on the total procurement costs, which drop by 42% in the FPSB and Dutch-FPSB auctions and by 49% in the Dutch auction, because a high pooling price cannot be maintained anymore in equilibrium. Overall, we find surprisingly high levels of efficiency in simple combinatorial first-price auctions, except for the FPSB auction with two bidders only, in which the equilibrium selection problem and the power of bidders to veto a split award unilaterally, lead to a high share of inefficient allocations. Overall, with more than two bidders first-price combinatorial auctions achieve high levels of efficiency in our procurement environment, which is in contrast to what one would expect from the analysis of the FPSB auction with two bidders only.

2. The Model

Before describing the auctions discussed in this paper, we first provide some necessary notation and terminology.

2.1. Notation and Terminology

A buyer conducts a split-award auction in order to award a business among $n \geq 2$ ex-ante symmetric, risk-neutral, and profit-maximizing suppliers.[5] We focus on a simple setting in which bidders can win either a contract for 50 or 100% of the business, which makes it technically a combinatorial (reverse) auction with two identical units and the package up for auction. The possibility to submit all-or-nothing package bids makes this type of auction different from multi-unit auctions with multi-unit demand as discussed by Chakraborty (2006). Bidder i's (with $i \in \{1, 2, ..., n\}$) costs for 100% of the business, k_i^s, are determined by a private cost parameter Θ_i. The cost type is independently drawn from an arbitrary distribution function $F(\cdot)$ with support $[\underline{\Theta}, \overline{\Theta}]$ $(0 < \underline{\Theta} < \overline{\Theta})$ with the density f positive and continuous. A constant efficiency parameter $0 < C < 1$, which is equivalent for and known to all suppliers, determines the costs for 50% of the business, $k_i^\sigma = C\Theta_i$. Costs for no award are zero. Furthermore, the buyer does not know the efficiency parameter C. Hereafter, the i-th lowest order statistic out of n different cost types is denoted by $\Theta_{i:n}$.

Each bidder i either submits or accepts prices for 100 and/or 50% of the business, $p^s(\cdot)$ and $p^\sigma(\cdot)$ respectively. Bidders are assumed to be individually rational, which means that all submitted bids, $p^s(\cdot)$ and $p^\sigma(\cdot)$, must be at least as high as the supplier's costs for the respective allocation. The auctioneer is ex-ante indifferent between awarding 100% of the business to a single supplier (sole source award) and awarding 50% of the business each to two different suppliers (split award). Hence, the winner determination in a split-award auction must satisfy the auctioneer's indifference condition.

A split-award auction, which implements the buyer's indifference condition, must assure that the sole source award (split award) is selected, if and only if $\min_{i \in \{1,2,...,n\}} \{p^s(\Theta_i)\} < (\geq) \min_{i \neq j;\ i,j \in \{1,...,n\}} \{p^\sigma(\Theta_i) + p^\sigma(\Theta_j)\}$. Similar to Anton and Yao (1992), we focus on markets with strong diseconomies of scale in which suppliers must coordinate in the efficient solution. Dual Source Efficiency (DSE) describes a setting in which it is always efficient for the buyer to award 50% of the business to each of two different suppliers. Anton and Yao (1992) show that with $n = 2$ bidders independent of the two draws of the bidders' cost types, the split is always the efficient award if the efficiency parameter C lies below $\frac{\Theta}{\underline{\Theta} + \overline{\Theta}}$. This can be easily transfered to a setting with $n > 2$ bidders.

An ex-ante defined risk-premium by the procurement manager extends the scope of DSE. The same types of equilibria emerge in a setting with a constant risk premium r for the sole source

[5] We apply notation in line with Anton and Yao (1992). The terms auctioneer and buyer as well as bidder and supplier are used interchangeably.

award and with $C < \frac{\underline{\Theta}+r}{\underline{\Theta}+\overline{\Theta}}$, which also allows values for C of greater than 0.5, e.g., a setting with $C = 0.52$, $\Theta \in [100, 140]$, and $r = 25$ in which an equilibrium with pooling prices and split awards exists even though suppliers have economies of scale.

2.2. The Auctions

We next describe the auction formats analyzed in this paper. As the FPSB split-award auction is simple and well-known from Anton and Yao (1992), we only introduce the Dutch and Dutch-FPSB auction.

2.2.1. The Dutch Split-Award Auction The Dutch split-award auction can be divided into two stages or phases. In the first phase, bidders simultaneously compete for the split as well as the sole source award. After one of the bidders accepts the price for 100% of the business, the auction ends. In the case in which one of the bidders approves a counteroffer for the 50% share, phase 2 starts.

Phase 1: In each round r of the auction, bidders simultaneously receive counteroffers[6] for 50%, c_r^σ, and 100% respectively, of the business, c_r^s. The starting prices for both counteroffers should be at least lower than or equal to the minimal costs for each share, i.e. $c_1^\sigma \leq C\underline{\Theta}$ and $c_1^s \leq \underline{\Theta}$. The auctioneer can also start close to zero. Subsequently, both price functions are raised continuously by the buyer such that in each round r $c_r^s = 2c_r^\sigma$. The auctioneer must stick to this pricing rule in every round to assure that the outcome of the auction satisfies his indifference condition.[7] The buyer awards the business on a first-come-first-served basis.

In each round, a bidder i has three options: he can approve the counteroffer for 50 or 100% of the business, or he can reject both. The following three scenarios are possible:

(i) If bidder i is first to accept a counteroffer for the 50% share in round r, the split is awarded to supplier i at a price of c_r^σ and phase 1 is over;

(ii) If bidder i is first to accept c_r^s in round r, this supplier i wins the sole source award and the auction terminates immediately;

(iii) If a bidder rejects both counteroffers in round r, he risks losing the whole or at least a share of the business.

[6] Typically, there is a request for quotation (RfQ) before the final awarding, in which suppliers are asked by the buyer to submit first offers for the business. Hence, the auctioneer's offers in the final Dutch are normally called counteroffers in procurement practice.

[7] Suppose that the buyer would choose a pricing rule with $c_r^s < 2c_r^\sigma$. If one bidder accepts the counteroffer for 50% in round t of phase 1 and a different bidder is willing to pay the same price in phase 2, the auction ends with the buyer awarding the split at a price $2c_t^\sigma$. However, the buyer does not know, if a supplier would have accepted 100% of the business for a counteroffer c_r^s, such that $c_t^s < c_r^s < 2c_t^\sigma$. With such a deviation from the pricing rule proposed above, a buyer would risk higher purchasing costs. Similar reasoning applies if the buyer commits to a pricing rule such that $c_r^s > 2c_r^\sigma$.

Phase 2: The second phase is only relevant, when the split award has been awarded to a single supplier in phase 1. In this case, the remaining 50% of the business is auctioned off to all suppliers in a regular single-unit Dutch auction. Regardless of the price for the 50% share in phase 1, the starting price in phase 2 is c_1^σ (the same as in phase 1); this is necessary to allow efficient equilibrium bidding strategies (Gretschko et al. 2014). The first bidder to approve a counteroffer wins the remaining half of the business and the auction is over.

Because all bidders participate in the auction in phase 2, the bidders are now asymmetric. The winner, w, of phase 1 faces costs of Θ_w if he wins the second auction, while the other bidders, $l \neq w$, have costs of $C\Theta_l$. If the winner of phase 1 wins the remaining 50% share in phase 2 as well, he gets paid the sum of both split prices. Individual rationality, however, dictates that his costs for 100% of the business have to be covered by the sum of both split prices.

2.2.2. The Dutch-FPSB Split-Award Auction The Dutch-FPSB split-award auction is a hybrid format containing elements from both the Dutch and the FPSB split-award auction formats. It can also be divided into two phases, with phase 1 following the same rules as in the Dutch format. Phase 2 becomes relevant if the split is awarded to a bidder in phase 1. However, the remaining 50% of the business is auctioned off by an FPSB in phase 2. All bidders including the winner of phase 1 are submitting bids for the remaining 50% share, and the supplier with the lowest price wins.

3. Equilibrium Analysis

We start with an introduction of the different equilibrium types, which can emerge in the split-award model. Subsequently, we give a short recap of the results on the FPSB format of Anton and Yao (1992) for two bidders and derive equilibria in the Dutch and the Dutch-FPSB split-award auction.

3.1. Equilibrium Types

The FPSB split-award model can be characterized as a one-stage game with incomplete information, for which we consider symmetric Bayesian Nash equilibria in pure strategies, similar to Anton and Yao (1992). Such an equilibrium consists of an equilibrium bidding strategy for the sole source and the split award $(p_e^s(\cdot), p_e^\sigma(\cdot))$. Both functions are non-decreasing in cost types Θ and continuous.

In equilibrium, there must not be a deviating strategy for any cost type $\hat{\Theta} \in [\underline{\Theta}, \overline{\Theta}]$ that yields a higher expected payoff than the equilibrium payoff. Hence, three different types of deviations have to be excluded in equilibrium: a sole source deviation in which a bidder focuses on winning the entire business, a split deviation in which a bidder concentrates on winning 50% of the business and a hybrid deviation which can result in both allocations with positive probability. By applying a sole source or split deviation a bidder submits a competitive price for his target award and a high bid-to-lose price for the other award in order to implement the desired allocation. This must not apply for hybrid deviations, in which no award is excluded completely. Furthermore, bidders can even unilaterally exclude the split allocation by using a so-called veto strategy $(p^s(\cdot), p^\sigma(\cdot)) = (p^s(\cdot), p^s(\cdot) - C\underline{\Theta})$ in case of $n = 2$ bidders. If bidder i plays a veto strategy, then the auctioneer always choses the sole source allocation independently of the strategy of supplier j, as the sum of both split prices can never be lower than their minimum. Obviously, such a unilateral veto strategy is not possible for sole source awards, as these will always be attractive for a buyer as a result of competitive bidding by the other supplier, or in case of $n > 2$ bidders.

The Dutch and the Dutch-FPSB split-award auction are modeled as two-stage games with observed actions and incomplete information, for which we analyze perfect Bayesian equilibria. Thus, the equilibrium (S, μ) is characterized as a set S of strategies, which have to be sequentially rational given a system of beliefs $\mu = \{\mu_w, \mu_l\}$. In phase 2, μ_w characterize the beliefs of a loser from the first phase about the type of the winner of phase 1, whereas μ_l are the beliefs of any bidder in phase 2 about the type of a loser of phase 1. Additionally, the system of beliefs μ has to be consistent given S for every possible information set. A strategy in the Dutch auction is a price function $p_e^{s1}(\cdot)$ for 100% of the business in phase 1 or a pair of price functions $p_e^{\sigma 1}(\cdot)$ and $p_e^{\sigma 2}(\cdot)$ for 50% of the business in phases 1 and 2, respectively, for which a bidder accepts the respective counteroffers c_r^s or c_r^σ in equilibrium. In the Dutch-FPSB format, the function $p_e^{\sigma 2}(\cdot)$ represents the submitted prices in the sealed-bid stage for the remaining 50% share. The set of beliefs μ is defined by probability distributions over the possible types of the opponents' before phase 1 and phase 2.

In the Dutch and Dutch-FPSB auctions, a sole source deviation can be realized either by accepting the counteroffer for 100% in phase 1 or by trying to win the 50% offer in phases 1 and 2; a split deviation is possible in both phases. There are no veto strategies in these (partly) ascending split-award variants, as suppliers can only accept the counteroffers presented by the auctioneer, who sticks to his indifference condition. There is no unilateral strategy, which fully excludes a specific allocation from being awarded. Furthermore, no hybrid deviations have to be considered, because a bidder cannot accept a split and a sole source award simultaneously.

Similar to Anton and Yao (1992), three different general equilibrium types can be distinguished: In a WTA equilibrium, the sole source award is chosen by the auctioneer with probability 1 independently of the cost draws of the suppliers. The same applies for the split award in a σ equilibrium.

A hybrid equilibrium is a bidding strategy that can result in both a split and a sole source award depending on the cost draws of the suppliers.

3.2. The 2-Bidder Model

First, we analyze equilibrium bidding behavior in split-award auctions with only two bidders, for which bidders can veto the split outcome. This is a specific environment, which needs be analyzed differently. However, it provides a basis for our analysis of markets with more bidders. We start with a short recap of the results on the FPSB format of Anton and Yao (1992). Subsequently, we derive equilibria in the Dutch and the Dutch-FPSB split-award auction.

3.2.1. The FPSB Split-Award Auction For the sake of convenience we shortly summarize the results of Anton and Yao (1992) and cite the most important propositions in order to make our article self-contained. Anton and Yao (1992) analyze equilibrium bidding behavior in a FPSB split-award auction with two bidders and DSE, demonstrating both WTA and a σ equilibria. An important result of this work is that constant pooling prices for 50% of the business are necessary in order to derive a σ equilibrium; they show that various σ equilibria with different pooling prices $p_e^\sigma \in [\overline{\Theta}C, (1-C)\underline{\Theta}]$ can exist. In such cases, bidders submit high sole source prices that support the equilibrium and must not be higher than a given boundary $G(p_e^\sigma, \Theta)$ in order to avoid profitable deviations for the sole source award. As bidders are individual rational, σ equilibria can only exist, when the boundary $G(p_e^\sigma, \Theta)$ allows for sole source prices above costs for 100% of the business.

Proposition 1 *(Anton and Yao 1992) In the FPSB split-award auction with $n = 2$ risk-neutral bidders and DSE, a σ equilibrium $(p_e^s(\Theta), p_e^\sigma(\Theta))$ is given by*

$$p_e^s(\Theta) = \Theta$$
$$p_e^\sigma(\Theta) = p_e^\sigma \in [\overline{\Theta}C, (1-C)\underline{\Theta}],$$

if $\Theta \geq G(p_e^\sigma, \Theta)$ for all $\Theta \in [\underline{\Theta}, \overline{\Theta}]$.

An inefficient WTA equilibrium exists as well, as bidders can strategically veto the split allocation with high bid-to-lose prices for 50% of the business. The sole source price of such a strategy equals the price in a single-unit auction, as this is the profit-maximizing strategy of a bidder, when the probability to win the split award is zero in equilibrium.

Proposition 2 *(Anton and Yao 1992) In the FPSB split-award auction model with $n=2$ risk-neutral bidders and DSE, a WTA equilibrium $(p_e^s(\Theta), p_e^\sigma(\Theta))$ with*

$$p_e^s(\Theta) = \Theta + \frac{\int_\Theta^{\overline{\Theta}} (1-F(t))^{n-1} dt}{(1-F(\Theta))^{n-1}}$$
$$p_e^\sigma(\Theta) = p_e^s(\Theta) - C\underline{\Theta}$$

exists.

Hybrid equilibria were introduced for settings with uncertain economies of scale in (Anton et al. 2010). These type of equilibria are described by a strategic cost type τ, for which bidders change their equilibrium bidding strategy: low-cost bidders with $\Theta < \tau$ focus on winning the sole source award, whereas bidders with high cost types try to win the split award. It is interesting to note that the same type of hybrid equilibria as presented in proposition 1 of Anton et al. (2010) also exist in settings with DSE. Because the split is the efficient award for all cost draws under DSE, τ is not restricted to a specific interval as with uncertain economies of scale. Hence, hybrid equilibria with $\tau \in (\underline{\Theta}, \overline{\Theta})$ can exist as long as individual rationality is given by condition 1. The proof of corollary 1 is a straightforward extension.

Corollary 1. *In the FPSB Split-Award Auction Model with $n=2$ risk-neutral bidders and DSE, a constant parameter $\tau \in (\underline{\Theta}, \overline{\Theta})$ and DSE, a hybrid equilibrium with*

$$(p_e^s(\Theta), p_e^\sigma(\Theta)) = \begin{cases} (\min\{2[\tau - C\tau], \Theta\}\ ,\ \tau(1-C)) & \text{if } \Theta \geq \tau \\ (\Theta + \tau(1-2C)\frac{1-F(\tau)}{1-F(\Theta)} + \int_\Theta^\tau \frac{1-F(x)}{1-F(\Theta)} dx\ ,\ \tau(1-C)) & \text{if } \Theta < \tau \end{cases}.$$

exists, if

$$2\tau(1-C) > \overline{\Theta} \tag{1}$$

applies.

The fact that beyond the WTA and the σ equilibrium also hybrid equilibria exist, underscores that there is a veritable equilibrium selection problem. This will be discussed further in section 4.2, in which we analyze conditions for payoff dominance of the efficient σ equilibrium.

3.2.2. The Dutch Split-Award Auction In this section, we analyze bidding behavior in the Dutch split-award auction, for which perfect Bayesian equilibria are applied as solution concept. Thus, an equilibrium strategy in the Dutch split-award auction defines prices, for which a supplier accepts either the split or the sole source award, as well as a system of beliefs μ. Unlike the FPSB split-award auction, only the σ equilibrium with the highest pooling price is possible in the Dutch split-award auction model.

COROLLARY 2. *Consider the Dutch split-award auction model with $n = 2$ risk-neutral bidders and DSE. Then, if a σ equilibrium exists, the split price must be constant and equal to $p_e^\sigma = \underline{\Theta}(1-C)$ for all bidders with cost types $\Theta \in [\underline{\Theta}, \overline{\Theta}]$.*

All proofs can be found in the Appendix A. As in the FPSB auction, the split price in a σ equilibrium must be constant in the Dutch split-award auction. Otherwise, it would be always more profitable for the supplier with the lower price for 50% to accept the same counteroffer as his opponent. However, only a σ equilibrium with a split price $p_e^\sigma(\Theta) = \underline{\Theta}(1-C)$ can emerge, not multiple efficient equilibria as in the FPSB format.

The main difference between the FPSB and the Dutch split-award auction is the information provided about the opponent's behavior. Whereas the Dutch split-award auction is similar to a two-stage game, in which the winner immediately observes a deviation from a σ equilibrium, this information is provided ex-post in the FPSB split-award auction. If bidder A is the winner of 50% of the business for a price p^σ in phase 1 of a Dutch split-award auction then it must be a possible threat for A to accept the offer for the remaining share at a price of $p^\sigma + \varepsilon$ (with $\varepsilon > 0$) as soon as it becomes obvious that his opponent deviates from equilibrium.

In a two-stage game such a threat is only credible if bidder A makes at least as much payoff as already achieved in phase 1, i.e., if at least bidder A's additional costs for providing 100% of the business, $(1-C)\Theta_A$, are covered. Therefore, $p_e^\sigma = (1-C)\underline{\Theta}$ remains as the only possible split price because for lower split prices, a profitable split deviation as described above cannot be prevented by the winner of phase 1. In the next proposition, we will provide conditions for which a pure σ equilibrium exists.

Proposition 3 *In the Dutch split-award auction model with $n = 2$ risk-neutral bidders and DSE, there is a unique and efficient σ equilibrium with a strategy profile $(p_e^{\sigma 1}(\cdot), p_e^{\sigma 2}(\cdot))$ and beliefs $\mu(\cdot \mid p_e^{\sigma 1}(\cdot)) = \{\mu_w(\cdot \mid p_e^{\sigma 1}(\cdot)), \mu_l(\cdot \mid p_e^{\sigma 1}(\cdot))\}$ after the loser of phase 1 observes $p_e^{\sigma 1}(\cdot)$, if for all $x \in (\underline{\Theta}, \overline{\Theta}]$*

$$\Delta^{\Pi}(x, \overline{\Theta}) = (x(1-C) - C\overline{\Theta})(1 - F(x)) - \underline{\Theta}(1-C) + C\overline{\Theta} < 0 \tag{2}$$

applies.

A bidder accepts the 50% share in phase 1 for a price $p_e^{\sigma 1}(\Theta) = (1-C)\underline{\Theta}$ independent on his cost type Θ.

The second-phase strategy is different for both bidders: The winner of phase 1 with cost type Θ_w threatens to accept the remaining share for a price of $p_e^{\sigma 2w}(\Theta_w) = \Theta_w(1-C)$, while the loser of phase 1 with cost type Θ_l, $l \neq w$, accepts the split award again at a price of $p_e^{\sigma 2l}(\Theta_l) = (1-C)\underline{\Theta}$.

The beliefs of the loser of phase 1 about the cost type of the winner of phase 1 are the same as ex-ante. Hence, no updating has to be considered and $\mu_w(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = F(\Theta)$ for $\Theta \in [\underline{\Theta}, \overline{\Theta}]$.

The same applies for the beliefs of the winner of phase 1 about the type of the loser, i.e. $\mu_l(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = F(\Theta)$ for $\Theta \in [\underline{\Theta}, \overline{\Theta}]$.

Sole source deviations can be ignored, as for all possible cost types Θ the payoff for the split award is a pair of two higher than the payoff for the sole source award in every round $q < r$ with counteroffers $c_q^s < c_r^s < 2\underline{\Theta}(1-C)$. Because the buyer sticks to his indifference condition, such a deviation cannot be realized unilaterally. The proof of corollary 2 shows that split deviations for the remaining share are difficult to exclude, as the threat to prevent such deviations by the winner of phase 1 has to be credible. Therefore, condition (2) assures that a split deviation that tries to win the remaining share in phase 2 for a higher split price than p_e^σ yields a lower expected payoff than a bid in equilibrium. Because of the pooling prices, there is no additional information about the cost type of the winner in phase 1, which is why updating of beliefs is not critical for the derivation of the equilibrium strategy. Next, we show that, unlike the FPSB split-award auction, the efficient σ equilibrium is unique in a Dutch split-award auction.

Proposition 4 *In the Dutch split-award auction model with $n = 2$ risk-neutral bidders with DSE, there is neither a WTA nor a hybrid equilibrium.*

In the FPSB split-award auction, bidders are able to play a WTA strategy, because they can unilateraly exclude the split by submitting high bid-to-lose prices for the 50% share. However, this is not possible in the Dutch auction, because there is always a profitable split deviation for high cost types playing a potential WTA equilibrium. Hence, such a strategy cannot be an equilibrium. All possible types of hybrid equilibria can be excluded as well. Thus, if a σ equilibrium exists in the Dutch split-award auction with $n = 2$ bidders, it is the unique equilibrium.

3.2.3. The Dutch-FPSB Split-Award Auction Next, we analyze equilibrium bidding in the Dutch-FPSB split-award auction. The auction format combines the two first-price mechanisms and the bidding behavior contains elements from the equilibrium strategies of both auction formats. The necessary conditions for a split price in a σ equilibrium are summarized in corollary 3.

Corollary 3. *Consider the Dutch-FPSB split-award auction model with $n = 2$ risk-neutral bidders and DSE. Then, if a σ equilibrium exists, the split price p_e^σ must be constant and $p_e^\sigma \in [\overline{\Theta}C, \underline{\Theta}(1-C)]$ for all bidders with cost types $\Theta \in [\underline{\Theta}, \overline{\Theta}]$.*

Multiple constant split prices in a given range are possible in a σ equilibrium. As in the FPSB split-award auction, a bidder can only observe ex-post, whether or not his opponent played a σ equilibrium or not. Hence, it is easier for the bidders to implement a σ equilibrium strategy. As described above, a threat must be realized, and becomes payoff-relevant, when the opponent deviates in a Dutch split-award auction. When phase 2 is a sealed-bid stage, this problem disappears and it suffices that the threat prevents the opponent from deviating. When this is fulfilled, the threat never becomes effective and the expected payoff of the winner of phase 1 remains the same. Proposition 5 summarizes the results for the existence of pure σ equilibria:

Proposition 5 *In the Dutch-FPSB split-award auction model with $n = 2$ risk-neutral bidders and DSE, there are different efficient σ equilibria with a strategy profile $(p_e^{\sigma 1}(\cdot), p_e^{\sigma 2}(\cdot))$ and beliefs $\mu(\cdot \mid p_e^{\sigma 1}(\cdot)) = \{\mu_w(\cdot \mid p_e^{\sigma 1}(\cdot)), \mu_l(\cdot \mid p_e^{\sigma 1}(\cdot))\}$ after the loser of phase 1 observes $p_e^{\sigma 1}(\cdot)$, if*

$$\Theta \leq G(\Theta, p_e^\sigma) = p_e^\sigma + \frac{p_e^\sigma - C\overline{\Theta}F(\Theta)}{1 - F(\Theta)} \text{ for all } \Theta \in [\underline{\Theta}, \overline{\Theta}] \tag{3}$$

applies.

Both bidders accept the split award at a pooling price $p_e^{\sigma 1}(\cdot) = p_e^\sigma \in [\overline{\Theta}C, (1-C)\underline{\Theta}]$.

The loser of phase 1 with cost type Θ_l submits the pooling price of phase 1, $p_e^{\sigma 2l}(\Theta_l) = p_e^\sigma$, in the sealed-bid stage in phase 2, whereas the winner of phase 1 with cost type Θ_w, $w \neq l$, submits a credible threat for the remaining share at a price of $p_e^{\sigma 2w}(\Theta_w) = \max\{p_e^\sigma, \Theta_w - p_e^\sigma\}$.

The beliefs about the opponent's cost type do not change after phase 1, i.e. $\mu_w(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = F(\Theta)$ for $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ and $\mu_l(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = F(\Theta)$ for $\Theta \in [\underline{\Theta}, \overline{\Theta}]$

The reasoning here is similar to that in proposition 3. As the auctioneer offers the shares according to his indifference condition in phase 1, sole source deviations are not possible in equilibrium. Condition (3) is important to assure that the credible threat of the winner of phase 1 is possible without violating the assumption of individual rationality. Note that the function $G(\cdot,\cdot)$ is the same as in Anton and Yao (1992). This means that exactly the same σ equilibria as in the FPSB split-award auction can emerge. Furthermore, if condition (3) applies, all split deviations in phase 2 can be excluded in a σ equilibrium with split price p_e^σ. As in the Dutch split-award auction, only efficient σ equilibria can emerge.

Proposition 6 *In the Dutch-FPSB split-award auction model with $n = 2$ risk-neutral bidders with DSE, there is neither a WTA nor a hybrid equilibrium.*

We omit the proof for the proposition 6 as it follows that of proposition 4. The same efficient equilibria as in the FPSB auction emerge without additional restrictions. Furthermore, it can be shown that WTA and hybrid equilibria are excluded as equilibrium bidding strategies, which reduces the coordination problem to efficient equilibria. As we will see in the welfare analysis below, such a coordination problem can be solved via payoff dominance.

The characteristics of both first-price mechanisms influence the equilibrium bidding behavior in the Dutch-FPSB split-award auction. The combinatorial Dutch auction in phase 1 is sufficient to exclude inefficient equilibria. The sealed-bid mechanism in phase 2 also allows for σ equilibria with various split prices in the same range as in the FPSB auction, as the winner in phase 1 can credibly threaten to punish deviations from a σ equilibrium.

3.3. The n-Bidder Model

Next, we analyze the bidding behavior with more than two suppliers. The n-bidder case leads to differences in how the equilibrium strategies are derived and in the outcome compared to the 2-bidder case. In particular, a pooling equilibrium at high prices that exists in all first-price auction formats for both bidders in the 2-bidder case cannot be maintained anymore.

3.3.1. The FPSB Split-Award Auction We start with analyzing bidding behavior in the FPSB split-award auction. First, it is interesting to see that there is a WTA equilibrium in DSE with $n > 2$ suppliers, even though bidders have less power to veto a split award for their opponents:

Proposition 7 *In the FPSB split-award auction with $n > 2$ risk-neutral bidders and DSE, a WTA equilibrium $(p_e^s(\Theta), p_e^\sigma(\Theta))$ is given by*

$$p_e^s(\Theta) = \Theta + \frac{\int_\Theta^{\overline{\Theta}} (1 - F(t))^{n-1} dt}{(1 - F(\Theta))^{n-1}}$$
$$p_e^\sigma(\Theta) = p_e^s(\Theta) - C\underline{\Theta}.$$

By following such an equilibrium strategy, the split-award auction is reduced to a single-object auction for 100% of the business, because the split is excluded for all bidders (and the auctioneer) due to sufficiently high split prices. The expected payoffs of all possible (unilateral) split deviations are zero with probability 1. Therefore, the sole source price must be equal to the equilibrium strategy in a single-object auction in order to maximize the expected profit for winning the whole business.

In addition to the WTA equilibrium, a σ equilibrium exists with DSE as well. This equilibrium always results in the efficient allocation, the split award.

Proposition 8 *In the FPSB split-award auction with $n > 2$ risk-neutral bidders and DSE, a σ equilibrium $(p_e^s(\Theta), p_e^\sigma(\Theta))$ is given by*

$$p_e^s(\Theta) = \max\{\overline{\Theta}C + p_e^\sigma(\Theta), \Theta\}$$
$$p_e^\sigma(\Theta) = \Theta C + C\frac{\int_\Theta^{\overline{\Theta}}(1-F(t))^{n-1} + (n-1)F(t)(1-F(t))^{n-2}dt}{(1-F(\Theta))^{n-1} + (n-1)F(\Theta)(1-F(\Theta))^{n-2}}.$$

if either $C < \frac{\underline{\Theta}}{2\overline{\Theta}}$ or

$$\begin{aligned}E[\Pi_e^\sigma(\Theta)] >& (p_e^\sigma(x_1) + p_e^\sigma(x_2) - \Theta)P(p_e^\sigma(x_1) + p_e^\sigma(x_2) < \min\{p_e^\sigma(\Theta_{1:n-1}) + \min\{p_e^\sigma(\Theta_{2:n-1}), p_e^\sigma(x_2))\}, \\ & \max\{\Theta_{1:n-1}, p_e^\sigma(\Theta_{1:n-1}) + \overline{\Theta}C\}\}) \\ & + (p_e^\sigma(x_2) - C\Theta)P(p_e^\sigma(x_2) < p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) \geq p_e^\sigma(\Theta_{1:n-1}))\end{aligned} \tag{4}$$

applies for all $\Theta \in [\underline{\Theta}, 2C\overline{\Theta})$ and $x_1 < x_2$ with $x_1 \in [\underline{\Theta}, \overline{\Theta}], x_2 \in (x_1, \overline{\Theta}]$.

The split price is derived by maximizing the expected payoff of being amongst the two suppliers winning the split award in order to rule out split deviations. In contrast to the setting with two bidders, in which the suppliers' bids for the split award are constant, split prices are increasing with costs and the highest cost type $\overline{\Theta}$ makes a payoff of zero in equilibrium. Bidders who concentrate on winning the split award, submit sole source prices at least as high as the buyer's maximal purchasing costs in the σ equilibrium, $2\overline{\Theta}C$. However, sole source or even hybrid deviations are nevertheless possible, if the efficiency parameter is not too small, i.e. $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$.

In contrast to a split deviation, a deviating bidder is not dependent on another competitive bid for the same share. Thus, there are sole source and hybrid deviations with positive expected payoff, which is why the proof of excluding both types of deviations is the most challenging part of the proof. While sole source deviations can be excluded in general for all possible settings within the model assumptions, the additional condition 4 as stated in the proposition is needed to assure the exclusion of all possible hybrid deviations. When condition 4 is verified, proposition 8 provides a closed-form solution for an efficient Bayesian Nash equilibrium strategy in a sealed-bid combinatorial first-price auction and a general number of bidders n. Condition 4 can be approximated by a stricter condition that is simple to evaluate and numerical experiments yield that it holds for a wide range of distributions.

Proposition 9 *In the FPSB split-award auction with $n > 2$ risk-neutral bidders and DSE, there is no hybrid equilibrium.*

With a hybrid equilibrium, there must be at least one definite cost type τ, for which the winning allocation stays the same provided that all cost draws of the $n > 2$ bidders are higher than τ. It can be shown that there is no such potential cost type that fulfills all the required conditions with DSE, which has as a consequence that such equilibria do not exist. Therefore, a bidder faces a coordination problem in a FPSB split-award auction, for which he has to decide whether to play a WTA or σ equilibrium. We discuss this coordination problem in section 4.

3.3.2. The Dutch Split-Award Auction As presented in section 2.2, the Dutch split-award auction comprises two phases, if a single bidder has accepted a counteroffer for the 50% share in phase 1. Therefore, an efficient equilibrium strategy has to maximize the bidders' expected payoff in both phases and has to consider the asymmetric cost structure of the suppliers in phase 2. After the result of phase 1 is observed, there are two different types of suppliers, one winner and $n-1$ losers of phase 1. Whereas all suppliers have the chance to win the whole business in phase 1, this only applies for one supplier in phase 2. When the winner of phase 1 accepts a counteroffer for the split in phase 2, he is the winner of the whole business.[8] Hence, we have to define the equilibrium strategies for both of these different types of bidders, because suppliers are not symmetric anymore in phase 2.

Proposition 10 *In the Dutch split-award auction with $n > 2$ risk-neutral bidders and DSE, the unique σ equilibrium bidding strategy $(p_e^{\sigma 1}(\cdot), p_e^{\sigma 2}(\cdot))$ for phase 1 and 2 together with the beliefs $\mu(\cdot \mid p_e^{\sigma 1}(\cdot)) = \{\mu_w(\cdot \mid p_e^{\sigma 1}(\cdot)), \mu_l(\cdot \mid p_e^{\sigma 1}(\cdot))\}$ of the suppliers after observing the outcome in phase 1, can be defined as follows:*

Phase 1:
A supplier with cost type Θ accepts the counteroffer for the 50% share at a price of

$$p_e^{\sigma 1}(\Theta) = \frac{\int_{\Theta}^{\overline{\Theta}} p_e^{\sigma 2l}(t)(n-1)(1-F(t))^{n-2} f(t)dt}{(1-F(\Theta))^{n-1}}.$$

[8] The buyer can also exclude the winning supplier from the auction in phase 2. All the results apply for both auction variants with DSE. The format without the possibility of a requote is easier tractable with other efficiency settings.

Phase 2:
The winner in phase 1 with cost type Θ_w accepts the counteroffer for the 50% share at a price of

$$p_e^{\sigma 2w}(\Theta_w) = (1-C)\Theta_w.$$

The $n-1$ losers in phase 1 with cost types Θ_l accept the counteroffer for the 50% share at a price of

$$p_e^{\sigma 2l}(\Theta_l, \Theta_w) = C\Theta_l + C\frac{\int_{\Theta_l}^{\overline{\Theta}}(1-F(t))^{n-2}dt}{(1-F(\Theta_l))^{n-2}}.$$

The cost type of the winner of phase 1 is revealed, as the bidders can infer it from the equilibrium strategy in phase 1, i.e. the beliefs of the losers about the winners cost types, $\mu_w(\Theta \mid p_e^{\sigma 1}(\cdot))$, is a distribution function over $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ with

$$\mu_w(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = \begin{cases} 0 & \textit{if } \Theta < (p_e^{\sigma 1}(\Theta_w))^{-1} \\ 1 & \textit{if } \Theta \geq (p_e^{\sigma 1}(\Theta_w))^{-1} \end{cases}.$$

The cost types of the $n-1$ losers remain private and every bidder knows that all losers have cost types higher than Θ_w. Hence, the beliefs about the losers of phase 1, $\mu_l(\Theta \mid p_e^{\sigma 1}(\cdot))$, can be defined as a distribution function over $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ with

$$\mu_l(\Theta \mid p_e^{\sigma 1}(\Theta_w)) = \begin{cases} 0 & \textit{if } \Theta < (p_e^{\sigma 1}(\Theta_w))^{-1} \\ \frac{F(\Theta)-F(\Theta_w)}{(1-F(\Theta_w))} & \textit{if } \Theta \geq (p_e^{\sigma 1}(\Theta_w))^{-1} \end{cases}.$$

In order to exclude split deviations in phase 1 and 2, we take the equilibrium strategy of an ex-ante split-award auction, in which the 50% share is awarded sequentially to two different suppliers. This strategy maximizes the expected payoff for the split award in both phases. Hence, it only remains to be shown that there is no sole source deviation, which is more profitable than the σ equilibrium. An assessment of the expected payoff of such deviations yields the desired result for phase 1. Additionally, it can be shown that the winner of phase 1 has no chance to win the remaining 50% share in phase 2. One of his opponents secures himself the remaining 50% share before this award becomes attractive for him. Because this would be the only possible sole source deviation in phase 2, the proof is complete.

In the FPSB split-award auction, bidders are able to play a WTA strategy, as they can exclude the split by submitting high bid-to-lose prices in equilibrium. However, this is not possible in the

Dutch auction, as there is always a profitable split deviation for high cost types playing a potential WTA equilibrium. Hence, such a strategy cannot be a Bayesian Nash equilibrium. Furthermore, similar deliberations as in the proof of proposition 9 show that no hybrid equilibrium exists with DSE in the Dutch split-award auction.

3.3.3. The Dutch-FPSB Split-Award Auction The equilibrium analysis for the Dutch-FPSB split-award auction is identical to the Dutch auction with more than 2 bidders. All the equilibrium strategies of section 3.3.2 are equivalent.

4. Welfare Analysis

In this section we first study the efficiency of all three auction formats and then discuss differences in the procurement costs for the auctioneer.

4.1. The 2-Bidder Model

As in the equilibrium analysis, the setting with only two suppliers needs to be analyzed separately from the setting with $n > 2$ suppliers.

4.2. Efficiency Analysis

Bidding behavior in the Dutch split-award auction is straightforward for suppliers, as there is a unique and efficient σ equilibrium strategy. In addition, the Dutch-FPSB split-award auction, in which multiple σ equilibria exist, always results in efficient allocation. The σ equilibrium with the highest possible split price $p_e^\sigma = \underline{\Theta}(1-C)$ is payoff-dominant over all other efficient equilibria; obviously, a σ equilibrium with a lower split price yields less payoff, as the probability to win does not increase as the split price decreases.

The coordination problem in the FPSB split-award auction is more challenging for bidders, because a WTA, multiple σ equilibria with split prices $p_e^\sigma \in [\overline{\Theta}C, \underline{\Theta}(1-C)]$, and multiple hybrid equilibria with different strategic parameters $\tau \in (\underline{\Theta}, \overline{\Theta})$ can exist in this auction format. Proposition 5 in Anton and Yao (1992) describes a setting in which the σ equilibrium is payoff-dominant over the WTA equilibrium for the bidders. We extend these results by considering hybrid equilibria as well; corollary 4 gives conditions for which an efficient σ equilibrium is payoff-dominant over all other types of equilibria.

Corollary 4. *In the FPSB split-award auction model with $n = 2$ risk-neutral bidders, a constant parameter $\tau \in (\underline{\Theta}, \overline{\Theta})$ and DSE, a σ equilibrium is payoff-dominant for all bidders if the following conditions apply:*

(i) $p_e^\sigma = \underline{\Theta}(1-C)$
(ii) $E[\Pi_e^\tau(\Theta,\tau)] \leq \underline{\Theta}(1-C) - C\Theta \ \forall\ \Theta \in [\underline{\Theta},\overline{\Theta}]$
(iii) $E(\Theta) < 2\underline{\Theta}(1-C)$

The expected payoff of a type Θ *in a hybrid equilibrium with strategic parameter* τ *is*

$$E[\Pi_e^\tau(\Theta,\tau)] = \begin{cases} \tau(1-2C)(1-F(\tau)) + \int_\Theta^\tau (1-F(x))dx & \text{if } \Theta < \tau \\ (\tau(1-C) - C\Theta)(1-F(\tau)) & \text{if } \Theta \geq \tau \end{cases}.$$

As mentioned above, only the σ equilibrium with the highest pooling price $p_e^\sigma = \underline{\Theta}(1-C)$ can be payoff-dominant over all other equilibria. When condition (ii) applies, there is no cost type with a higher equilibrium payoff in a hybrid than in a σ equilibrium. Condition (iii) assures that the σ equilibrium is payoff-dominant over its WTA counterpart. Corollary 4 describes the strategic complexity of bidding in a FPSB split-award auction. The conditions are very restrictive and it is hard to find a setting, for which these conditions are fulfilled simultaneously. Furthermore, the strategic parameter τ, for which bidders change the strategy must be known to all bidders and to the buyer when hybrid equilibria are possible. This additional assumption is hard to motivate in procurement practice.

4.3. Comparison of Purchasing Costs

We use payoff dominance to overcome the bidders' coordination problem. A prediction on the expected procurement costs of a buyer can only be done in settings with a payoff-dominant equilibrium. This applies for the Dutch as well as the Dutch-FPSB split-award auction, although not always for the FPSB auction.

Corollary 5. *In the split-award auction model with* $n = 2$ *risk-neutral bidders and DSE, there is cost equivalence between the Dutch and the Dutch-FPSB auction formats. The buyer's expected procurement costs are* $E[p^b] = 2\underline{\Theta}(1-C)$ *in these auctions. This applies for the FPSB auction if the conditions of corollary 4 are valid.*

We omit the proof, as it is trivial and follows directly from the equilibrium analysis. Because the split is awarded with probability 1 in a σ equilibrium, the purchasing costs in the Dutch and Dutch-FPSB split-award auctions equal twice the split price $p_e^\sigma = \underline{\Theta}(1-C)$. This only applies for the FPSB split-award auction when all conditions of corollary 4 are fulfilled and there is a payoff-dominant σ equilibrium. Otherwise, cost equivalence between the (partly) ascending auction formats and the sealed-bid variant fails. According to the equilibrium analysis, bidders are able to coordinate on very high split prices in a σ equilibrium, although they face lower average costs for 50% than for 100% of the business. In settings in which the expected costs of a σ equilibrium are lower than in a single-unit auction, a buyer should prefer one of the two (partly) ascending split-award auctions (Dutch and Dutch-FPSB) in order to achieve higher efficiency at lower costs in equilibrium.

Furthermore, it is interesting to observe that for all possible cost draws of the two suppliers, costs of the auctioneer in an efficient sigma equilibrium of the analyzed first-price auctions are always lower than the VCG costs. This doesn't hold for $n > 2$ as we will see, and is due to the fact that the VCG payments depend on the second bidder's costs for the package, which is high in DSE.

COROLLARY 6. *The purchasing costs in a first-price split-award auction, for which a payoff-dominant σ equilibrium exists, are lower than the VCG costs with $n = 2$ risk-neutral bidders and DSE independent of the cost draws of the suppliers.*

In other words, cost equivalence not only fails between the different first-price split-award auctions, but also does not hold between first- and second-price split-award auctions in the 2-Bidder-Model. By applying a Dutch or Dutch-FPSB split-award auction instead of a VCG mechanism, the auctioneer achieves full efficiency and procurement costs are expected to be lower.

4.4. The n-Bidder Model

In what follows, the efficiency and procurement costs with more than two bidders are analyzed.

4.4.1. Efficiency Analysis Bidding behavior in the Dutch or Dutch-FPSB split-award auction is straightforward for suppliers, as there is a unique and efficient equilibrium strategy. However, in the FPSB split-award auction, bidders face a coordination problem between the WTA and the σ equilibrium. Payoff dominance can be a remedy in equilibrium selection problems, but it does not help in the FPSB auction.

Proposition 11 *In the FPSB split-award auction, the WTA equilibrium cannot be payoff-dominant over the σ equilibrium for all cost types $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ with $n > 2$ risk-neutral bidders and DSE.*

In the proof, we show that bidders with a high cost draw always prefer a σ equilibrium due to higher expected profits regardless of the parameters $n, C, F(\cdot)$ or the support $[\underline{\Theta}, \overline{\Theta}]$. Therefore, only the σ equilibrium can be payoff-dominant with DSE. However, if there is at least a single cost type $\Theta \in [\underline{\Theta}, \overline{\Theta}]$, whose expected profits in a WTA are higher than in a σ equilibrium, the equilibrium coordination problem cannot be solved by payoff dominance. The proof of this proposition is omitted, because it follows directly from the comparison of equilibrium payoffs of a σ and WTA equilibrium.

Proposition 12 *Neither the σ nor the WTA equilibrium are payoff-dominant in a FPSB split-award auction with $n>2$ risk-neutral bidders and DSE, if for at least one cost type $\Theta \in [\underline{\Theta}, \overline{\Theta}]$*

$$C < \frac{\int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1} dx}{\int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2} dx}$$

applies.

The condition in proposition 12 applies in many environments, e.g. whenever the cost parameters are uniformly distributed over any support $[\underline{\Theta}, \overline{\Theta}]$.

Corollary 7. *There is no setting, for which either the WTA equilibrium or the σ equilibrium is payoff-dominant for all cost types in a FPSB split-award auction with $n>2$ risk-neutral bidders and DSE, if $\Theta \sim U[\underline{\Theta}, \overline{\Theta}]$.*

Thus, the coordination problem makes it very hard to predict the bidding behavior and the outcome in a FPSB split-award auction. An inefficient WTA or an efficient σ equilibrium are possible as equilibrium outcomes. These problems do not arise in the Dutch or Dutch-FPSB split-award auction, because there is a unique and efficient σ equilibrium.

4.5. Comparison of Purchasing Costs

For the FPSB split-award auction we can only define the expected costs for the buyer on the condition that all bidders follow the same equilibrium strategy. Hence, we get expected costs for the WTA and for the σ equilibrium in the FPSB split-award auction. When bidders chose a WTA equilibrium, the price for the auctioneer equals the purchasing costs in a single-unit auction. If the bidders choose the σ equilibrium, then bidders in all auction formats, the descending, the Dutch, the FPSB, and the VCG auction, aim for a single share in equilibrium.

With this symmetric σ equilibrium, $n>2$ and identical 50% shares, the auctions are strategically equivalent to traditional multi-unit auctions with single-unit demand such that we can draw on the well-known revenue equivalence theorem for this environment (Myerson 1981, Engelbrecht-Wiggans 1988).

Bidders typically do not have single-unit demand in combinatorial auctions. However, by playing a σ equilibrium and submitting non competitive bid-to-lose prices for 100% of the business, the sole source award is off-equilibrium and the results are outcome equivalent to an ex-ante split-award auction, in which bidders cannot win more than 50% of the business. This is the reason, why the assumption of single-unit demand can be applied to bidders playing a σ equilibrium and the purchasing costs in the Dutch auction equal the costs in the VCG or descending auction with DSE. This is only true for the FPSB auction provided that bidders are able to coordinate on the split.

Although the split is efficient, purchasing costs in a σ equilibrium are not necessarily lower than in a WTA equilibrium. However, this applies for most of the settings with DSE.

COROLLARY 8. *In the FPSB split-award auction with $n > 2$ risk-neutral bidders and DSE, the price for the buyer in the σ equilibrium is always lower than in the WTA equilibrium,*

- *if either $C < \frac{\underline{\Theta}}{2\overline{\Theta}}$ applies or*
- *if $\Theta \sim U[\underline{\Theta}, \overline{\Theta}]$ applies.*

The expected price for the auctioneer in a σ equilibrium raises with a higher efficiency parameter C, whereas prices in the WTA equilibrium are independent of C. Hence, if the efficiency parameter C is sufficiently low, the costs for the auctioneer are always lower in the σ equilibrium.

Additionally, we show that for all possible $C < \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$ and uniformly distributed cost types, the σ equilibrium yields lower purchasing costs than the WTA equilibrium. Corollary 7 states that there is always a coordination problem in such a setting and no equilibrium is payoff-dominant. Thus, when costs are assumed to be uniformly distributed over any support $[\underline{\Theta}, \overline{\Theta}]$, the auctioneer should prefer the Dutch or Dutch-FPSB over the FPSB split-award auction with DSE not only because of its efficiency properties and lower strategical complexity for the bidders but also because of lower expected purchasing costs.

Note that the effect of adding a third bidder on procurement costs is substantial. In a split-award auction, with uniformly distributed cost types $\Theta \in [100, 140]$ and an efficiency parameter $C = 0.3$ procurement costs are reduced by 44.3% in expectation[9], if a third bidder is added. An additional fourth bidder only has and impact of minus 2.6%, a fifth supplier only an impact of minus 1.7%.

5. Experimental Evaluation

Before discussing efficiency and procurement costs and the bidder behavior in our experiments in detail, we will first describe the experimental design.

[9] Procurement prices are expected to drop from a σ equilibrium with the highest pooling price, $2\underline{\Theta}(1-C)$, to $2E[\Theta_{3:3}]$ in the σ equilibrium with three bidders.

5.1. Experimental Design

In our human subject experiments, we tested our proposed theory for a two-bidder and a three-bidder environment of the three first-price split-award auction mechanism FPSB, Dutch and Dutch-FPSB. Thus, our treatment variables were:

- Auction Format: FPSB, Dutch, and Dutch-FPSB split-award auctions
- Number of Bidders: 2 and 3 bidders

At the beginning of every period in all treatments the bidders are informed about their own cost draws for the supply of 50% or 100% of a fictitious order. Each bidder's cost parameter Θ is uniformly and independently distributed on the interval $[100.00, 140.00]$. The efficiency parameter is set to $C = 0.3$. Thus, a bidder's costs for the 100% share, Θ, lie within the range $[100.00, 140.00]$ and his costs for the 50% share, $C * \Theta$, lie within $[30.00, 42.00]$. Although every bidder knows his own costs only and not those of his competitors, common knowledge of the cost parameter distribution and the efficiency parameter is given. Consecutively, the respective auctions proceeded as described in section 2.2.

		Sample Size				
		group 1	group 2	group 3	group 4	Σ
2x2 Setting	FPSB	12	12	12	10	46
	Dutch-Dutch	12	12	12	12	48
	Dutch-FPSB	12	12	12	12	48
2x3 Setting	FPSB	12	12	12	0	36
	Dutch-Dutch	12	12	12	12	48
	Dutch-FPSB	12	12	12	12	48
						274

Table 1 Matching Group Sample Sizes

Upper bounds are implemented in each auction format. In the FPSB auction, each bidder is allowed to submit one bid of up to 150.00 for the 50% share and one bid of up to 300.00 for the 100% share at the start of every period. Both values can be entered in step sizes of 0.5.

In the Dutch and in the first phase of the Dutch-FPSB auction, the price for the 50% share starts at a price of 30 and increases at step sizes of 0.5 every half second. The price for the 100% share is twice the price of the smaller share and rises accordingly. Both prices cannot exceed upper bounds of 150.00 and 300.00, respectively. We conducted two sessions for each of the treatment variables. Each session consisted of two matching groups in each of which we let 12 subjects participate in 15 consecutive first-price split-award auctions. The subjects were matched randomly to the auctions in each of the 15 periods and no interaction between subjects across matching groups occurred. Each subject participated in one session only. However, for the FPSB auction with 2 bidders, one out of the four matching groups contained only 10 subjects, and with three bidders we were able to carry out the experiment for only three matching groups. In total, 274 subjects participated in the experiments. The sample sizes of the different treatments are summarized in Table 1.

In the two-bidder treatments, in each period the 12 subjects were randomly divided into six auctions consisting of two bidders each. In total, we conducted 360 auctions for the Dutch and Dutch-FPSB formats and 345 auctions for the FPSB format. In the three-bidder treatments, four auctions took place in every period, which resulted in 240 auctions for the Dutch and Dutch-FPSB auctions and 180 for the FPSB format. The number of auctions for the different matching groups are depicted below in Table 2.

		Number of Auctions				
		group 1	group 2	group 3	group 4	Σ
2x2 Setting	FPSB	90	90	90	75	345
	Dutch-Dutch	90	90	90	90	360
	Dutch-FPSB	90	90	90	90	360
2x3 Setting	FPSB	60	60	60	0	180
	Dutch-Dutch	60	60	60	60	240
	Dutch-FPSB	60	60	60	60	240
						1725

Table 2 Number of Auctions per Matching Group

At the beginning of each session the instructions were read aloud to all subjects. The subjects then had time to go through the instructions on their own and answer the comprehension questions. The interaction in the experiment was computerized and entirely anonymous. Communication or personal interaction between the subjects did not take place.

The experiments were conducted at the experimenTUM, the laboratory for experimental economic studies of the Technical University of Munich in 2016. Subjects were undergraduate and graduate students from the Technical University of Munich from a wide range of different study programs. Our experiments were computerized using the experimental software z-Tree (Fischbacher 2007). The sessions lasted on average one and a half to two hours and subjects were paid their cumulative earnings from all periods including a show-up fee of 6 € (US$6.56). On average subjects earned 20.85 € (US$22.78).

5.2. Theoretical Predictions

In this section, we determine the equilibrium strategies for the chosen experimental setting from Section 5.1 for two and three bidders. Note that the parameters in our experimental setting are chosen such that a σ equilibrium exists in the FPSB, Dutch, and Dutch-FPSB split-award auctions.

5.2.1. FPSB Split-Award Auction In the chosen experimental setting the σ equilibrium of the FPSB auction is characterized by the following range of pooling prices for the 50% share: $p_e^\sigma \in [54.06, 70.00]$. Each of these prices is supported by a bid on the 100% share according to $p_e^s(\Theta) \le G(p_e^\sigma, \Theta)$ for all $\Theta \in [\underline{\Theta}, \overline{\Theta}]$, where $G(p_e^\sigma, \Theta) = p_e^\sigma + \frac{40p_e^\sigma - 42\Theta + 4200}{140 - \Theta}$. The range of equilibrium pooling prices is restricted by the off-equilibrium non-negative profit condition $\Theta \le G(p_e^\sigma, \Theta)$ for all $\Theta \in [\underline{\Theta}, \overline{\Theta}]$. In the WTA equilibrium, the optimal bid on 100% of the business $p_e^s(\Theta) = 0.5\Theta + 70$ is supported by any bid on 50% share of at least $p_e^\sigma \ge p_e^s - 30$.

In our experimental setting, the σ equilibrium with the highest pooling price of $p_e^\sigma = 70.00$ is payoff-dominant over the WTA equilibrium as well as over any hybrid equilibrium with $\tau \ge 117.78$.[10] We predict split-award prices as high as 70.00.

In the setting with three bidders, there is an inefficient WTA equilibrium with competitive prices for the package of two units, $p_e^s(\Theta) = \frac{2\Theta}{3} + \frac{140}{3}$, and high bid-to-lose prices for the single unit, $p_e^\sigma(\Theta) = p_e^s(\Theta) - 30$. A sole source price as high as the bidder's cost type Θ is sufficient to support competitive split prices of $p_e^\sigma = \frac{0.1(-(300-2\Theta)\Theta + 280\Theta - 2800)}{\Theta - 60}$ in the efficient σ equilibrium.

Neither the σ nor the WTA equilibrium is payoff-dominant in this setting as shown in corollary 7 and it is not possible to predict by this criterion, which equilibrium the bidders should select.

5.2.2. Dutch Split-Award Auction In our experimental setting the σ equilibrium of the Dutch auction is defined as a unique pooling price of $p_e^{\sigma 1} = p_e^{\sigma 2} = p_e^\sigma = 70.00$ on the 50% share for phases 1 and 2. The winner of phase 1 threatens to accept the remaining 50% of the business at a price of $p_e^{\sigma 2}(\Theta) = 0.7\Theta$. The conditions for a σ equilibrium hold, and we expect the same split prices of 70.00 as in the FPSB format.

With more than two bidders there still is only a unique and efficient σ equilibrium in the Dutch auction, which always results in split allocation with two bidders winning a single unit sequentially in each phase. The bidder with the lowest cost draw should win the first unit for a price of $p_e^{\sigma 1}(\Theta) = 0.15(\Theta + 140) + 0.05(140 - \Theta)$ and play a threat of $\Theta(1 - C)$ in phase 2. The second-lowest cost draw is supposed to accept a counteroffer of $p_e^{\sigma 1}(\Theta) = 0.15(\Theta + 140)$ for the remaining share in phase 2.

[10] It is difficult to find realistic settings for which payoff dominance of the σ over the hybrid equilibrium is satisfied for all τ.

5.2.3. Dutch-FPSB Split-Award Auction The range of pooling prices for the 50% share in phases 1 and 2 in the σ equilibrium of the Dutch-FPSB auction is analogous to the FPSB format with $p_e^{\sigma 1} = p_e^{\sigma 2} = p_e^{\sigma} \in [54.06, 70.00]$. Each of the equilibrium pooling prices is supported by the winner of 50% of the business from phase 1 threatening to submit a price of $p_e^{\sigma 2}(\Theta_A) = \max\{p_e^{\sigma}, \Theta - p_e^{\sigma}\}$ for the remaining 50% of the business in phase 2. The σ equilibrium exists, and based on payoff dominance we predict the Dutch-FPSB auction to produce the same split prices as its Dutch counterpart. As the Dutch-FPSB is strategically equivalent to the Dutch auction with $n > 2$ bidders, the same σ equilibrium as in section 5.2.2 emerges.

5.2.4. Efficiency and Purchasing Costs As the two ascending auctions are characterized solely by efficient σ equilibria we expect the latter two formats to yield the efficient split award more often than the FPSB format in the experiments. The expected procurement costs for the buyer in the Dutch and the Dutch-FPSB split-award auctions are $E[p^b] = 140.00$ in the two-bidder environment. This applies as well for FPSB auctions in which all σ equilibria are payoff-dominant over the respective hybrid equilibria. If we consider non-payoff-dominant σ equilibria clear predictions about an expected purchasing price cannot be made.

The predictions concerning efficiency are independent of the number of bidders. However, for $n = 3$ bidders the coordination problem in the FPSB auction involves solely the σ and the WTA equilibrium as hybrid equilibria do not exist anymore. The expected VCG price for buying the split award is 78.00 in the three-bidder setting, which equals the costs for the auctioneer in each format, when bidders coordinate on the efficient σ equilibrium. Whereas this can be expected in the ascending formats, also a WTA equilibrium with expected costs for the auctioneer of 126.67 can be supported in the FPSB auction.

5.3. Welfare Results

We first discuss our aggregate results on efficiency and procurement costs.

5.3.1. Efficiency

Result 1 *With three bidders and two shares, the three auction formats almost always implement the efficient split award. With only two bidders, the Dutch-FPSB auction leads to higher proportion of split awards (82%) than the Dutch auction (64%), for which again the split is more often awarded than in the FPSB split-award auction (45%).*

First, we analyze the proportion of auctions that result in the efficient split out of all non-deleted auctions.[11] The allocations of the different treatments are summarized in Table 3 below. We omitted auctions with bids below costs. Contrary to theory, many sole source awards are observed in all three auction formats in the two-bidder setting. The Dutch-FPSB auction results most often in the split allocation, followed by the Dutch, and then the FPSB auction ($p = 0.00$).[12] The split was awarded in nearly all of the auctions independent of the auction design. Only the Dutch auction ended in a profitable sole source award.

Furthermore, we compare the share of efficient allocations, i.e. the proportion of allocations, which resulted in an efficient split award for the two lowest cost types. 71.8 % and 70.1 % of all auctions ended up in such an efficient split allocation in the FPSB and Dutch-FPSB auction with three bidders. With only 64.4 % the Dutch was the least efficient auction with respect to this metric. We conjecture that this results from the overall higher prices in this format, which can lead to inefficient allocations especially in settings, for which the two highest cost types lie very close together. Obviously, the two different shares are identical in the 2x2 setting.

		Efficiency				
		Total Auctions	Omitted Auctions	Split Awards	Efficient Allocations	Allocative Efficiency
2 x 2 Setting	FPSB	345	2	44.9 %	44.9 %	79.4 %
	Dutch	360	12	64.4 %	64.4 %	86.6 %
	Dutch - FPSB	360	15	81.4 %	81.4 %	93.2 %
2 x 3 Setting	FPSB	180	10	100 %	71.8 %	98.8 %
	Dutch	240	7	100 %	64.4 %	98.4 %
	Dutch-FPSB	240	16	100 %	70.1 %	99.1 %

Table 3 Efficiency

Often a relative measure of allocative efficiency is used to characterize the result of combinatorial auctions. The last column of Table 3 provides the mean allocative efficiency based on the definition

[11] The omitted auctions are the result of a small number of participants repeatedly bidding below their costs. In the two-bidder setting two individuals bid below their costs in the FPSB auction, eight do so in the Dutch auction, and nine do so in the Dutch-FPSB format. In the three-bidder setting, seven participants violate individual rationality in the FPSB split-award auction, seven do so in the Dutch format, and six do so in the Dutch-FPSB counterpart.

[12] A "test of equal proportions" is used for all significance tests between two samples.

of Kwasnica et al. (2005). [13] The mean allocative efficiency of the Dutch-FPSB is the highest in both settings with 93.2% and 99.1 %. Whereas there are again significant differences in the 2x2 setting, the Allocative efficiency is close to 100% for all different auction formats with three bidders.

To explain the surprisingly high number of sole source awards in each split-award auction format and the discrepancy in efficiency between the two ascending auctions, we analyze the bidding behavior in more detail in Section 5.4. In the three-bidder setting, the split is almost always awarded and allocations are highly efficient independent of the auction format.

5.3.2. Procurement Costs

Result 2 *With only two bidders, the FPSB and Dutch-FPSB auction formats yield substantially lower procurement costs of 129.98 and 130.12 , respectively, than the Dutch split-award auction with a value of 155.11. Ranging from 75.5 to 79.16, the procurement costs of all auction formats with three bidders are considerably lower than in the two-bidder setting.*

The overall procurement costs are defined as the price the auctioneer has to pay in each treatment. The procurement costs of the different treatments are summarized in Table 4 below, with the standard deviations (sd) given in brackets. The Dutch auction results in higher average costs than the other two auction formats in the two-bidder environment. There is no significant difference in costs between the FPSB and the Dutch-FPSB split-award auctions (p =0.42), but there is a significant difference between the FPSB and the Dutch and the Dutch-FPSB and the Dutch auctions (p =0.00).[14] The procurement costs for the FPSB and Dutch-FPSB auctions are significantly below the theoretical prediction of 140 with 95% confidence intervals of $[128.36, 131.60]$ and $[128.18, 132.05]$, whereas the purchasing price for the Dutch format is significantly above the theoretical prediction with a 95% confidence interval of $[152.77, 157.44]$.[15]

There are slight but significant differences in procurement costs among the auction formats in the three-bidder markets at a 5% significance level. The overall procurement costs in the three-bidder setting are substantially lower compared to the two-bidder setting and they are close to but still significantly different from (p =0.00) the predicted VCG prices.

[13] Assume $N^{\sigma}_{winner}, N^{s}_{winner}$ are the sets of bidders, who won the split, respectively the sole source award, and $N^{\sigma}_{optimal}$ is the set of the two bidders with the lowest cost type per auction. Then, the allocative efficiency of a split-award auction with DSE is defined as

$$Allocative\ efficiency = \frac{\Sigma_{i \in N^{\sigma}_{optimal}} C\Theta_i}{\Sigma_{i \in N^{\sigma}_{winner}} C\Theta_i + \Sigma_{i \in N^{s}_{winner}} \Theta_i}.$$

[14] A two-sample Mann-Whitney test is used for all significance tests between two samples.

[15] A student t-test is used for all single sample significance tests.

		Average Procurement Costs		
		Overall	Split Award	Sole Source Award
2 x 2 Setting	FPSB	129.98	$2 \times 62.30 = 124.60$ (sd = 22.60)	134.37 (sd = 12.29)
	Dutch	155.11	$2 \times 76.01 = 152.02$ (sd = 24.94)	160.67 (sd = 20.07)
	Dutch-FPSB	130.12	$2 \times 64.19 = 128.38$ (sd = 21.41)	137.78 (sd = 16.60)
2 x 3 Setting	FPSB	75.50	$2 \times 37.75 = 75.50$ (sd = 5.46)	-
	Dutch	79.16	$2 \times 39.50 = 79.00$ (sd = 4.46)	115
	Dutch-FPSB	76.40	$2 \times 38.20 = 76.40$ (sd = 4.99)	-

Table 4 Procurement Costs

In the two-bidder setting, there is a tradeoff between the higher efficiency of the Dutch auction and the lower procurement costs of the FPSB auction. We find higher prices for both the sole source and the split award in the Dutch auction than in the FPSB auction. The Dutch-FPSB auction achieves low prices and high efficiency and thus has advantages for the procurement manager in this respect, even resulting in higher efficiency than the Dutch auction with procurement costs that are not significantly different from those of the FPSB auction. Conversely, in the three-bidder setting all three auction formats are equally efficient. Again, the Dutch auction is the most expensive format tested at a 5% level, followed by the Dutch-FPSB, and the FPSB format. However, the differences are much smaller than in the two-bidder case.

5.4. Bidding Behavior

We next discuss the individual bidder behavior in the two-bidder and three-bidder environments for each of the three different auction formats. We estimated fixed-effects regressions for bids and prices of bidders in the split and sole-source award in all treatments and attached the outcomes in appendix B subsection B.1. We also included univariate regressions in which the cost draw is the single independent variable. These regressions allow us to interpret all plots of bids and prices on cost draws in subsection B.2. These plots and the corresponding univariate regressions provide intuitive insights on the subjects' bidding behavior. Finally, appendix B subsection B.3 contains plots of bids/prices in split allocations for all treatments across periods. These plots visualize any adaptation in bidding behavior with repeated interaction of the bidders.

Especially in the two-bidder setting, the derived σ equilibria have a collusive flavour. In order to describe the different forms of tacit collusion we apply the following distinction. We define that *pooling behavior* includes high split prices, which lie (1) above the highest possible cost draw for the split, $C\overline{\Theta}$, and (2) within the range of equilibrium predictions. Furthermore, the regression analysis should show that (3) these pooling prices are not significantly influenced by the own cost draw of the bidder. When this is not the case and only (1) and (2) apply, we talk about *tacit collusion*. Prices, which are even higher than the equilibrium predictions, are defined as *strong pooling* or *strong tacit collusion*.

5.4.1. Two-Bidder FPSB Split-Award Auction

Result 3 *As theory predicts, split-award winners show pooling behavior as nearly all split prices are above 42 ($C\overline{\Theta}$) and the average split price is 62.3. Furthermore, the own cost draw does not significantly influence the split prices in the fixed-effects regression analysis. In the sole-source allocations bidders tried to exclude the split award with high bids on one unit and submitted competitive bids on two units.*

Remember, there is an inherent equilibrium coordination problem in the FPSB auction format, because the inefficient WTA equilibrium can be simply implemented by the unilateral use of veto bids. Solely the existence of these veto bids might make the σ equilibrium less attractive in practice as its implementation requires both bidders not to use veto bids. Moreover, it is strategically complex for two bidders to coordinate on a split. To support any single-unit pooling price bidders have to bid at least twice the amount on two units such that the revenue maximizing auctioneer chooses the split award. Note, however, that such bids only lead to an equilibrium if both bidders actually pool at the same bid for one unit which is very unlikely.

We observe that bidders who win the split-award bid low on the single unit with an average bid of 62.3 and submit a high average bid of 145.7 on the double-unit package. Such bids facilitate coordination on the split award independent of slight deviations by the opponent, especially regarding the choice of an alternative pooling price. However, this strategy is vulnerable against a combination of high single-unit prices and low double-unit bids. In sole source award allocations that do not involve veto bids (9% of all bidders make use of their veto power), winners submit average bids of 68.8 and 135.5, and losers bid on average 78.3 and 157.1 on one and two units, respectively. As a result, the sum of both parties average single-unit bids (147.1) exceeds the winner's average bid on two units. In other words, the winners do not bid high enough on two units and the losers bid too high on one unit, thus, preventing coordination on the split award.

Figure 1 shows the single- and double-unit bids of split award winners plotted against the cost draws. The figure already indicates that winners of the split award pool their single-unit bids and submit high double-unit bids as theory predicts. Also, the bids of split-award winners on the

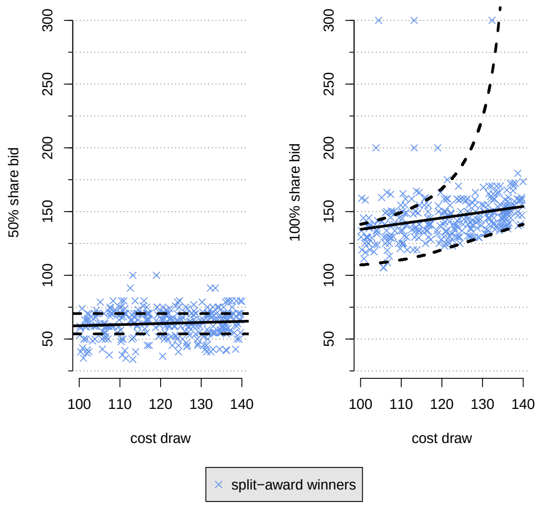

Figure 1 **Bids of Split-Award Winners in FPSB (n=2)**

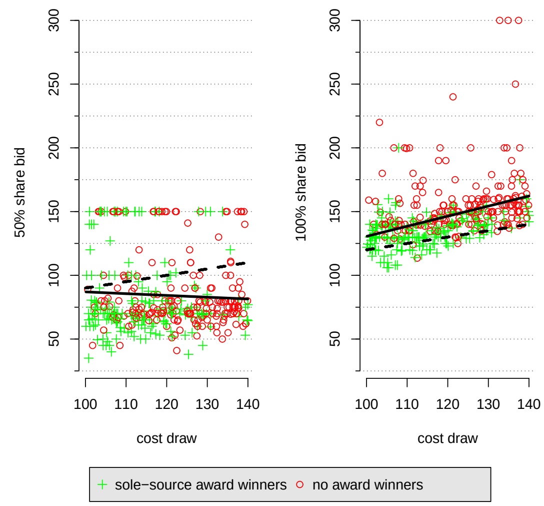

Figure 2 **Bids of Sole-Source-Award Winners in FPSB (n=2)**

package are increasing with the cost draw. The regression line for the single-unit bids lies within the predicted pooling boundaries and the regression line for the double-unit bids is within the predicted support, too. The winners of the sole-source award submit bid-to-lose split prices as can be seen in the left plot of figure 2 in which the univariate regression even has a negative slope. The bid for two units is increasing in the cost draw as predicted by theory and illustrated in the right plot of the figure. We observe overbidding which is mainly caused by the very high double-unit bids of the sole-source award losers. We conjecture that many of them aim for the split award.

In the appendix B we summarize the results of a number of fixed-effects regressions, which support the graphical analysis. We also analyze the bidder behavior across the periods in a session. For split-award winners the cost draw is not a significant explanatory variable for the height of the single-unit bid. Manual inspection yields that many bidders stick to their strategy of either bidding on the sole source or the split over time. We observe a significant decrease in the height of the single-unit bid for ten out of 46 sole-source award bidders across all 15 periods. This might suggest that some sole-source bidders adapt and try to win the split award but do not alter their double-unit bids. The latter bids of split-award winners and the distribution of allocations appears to remain constant over periods. Details can be found in the appendix B in figures 17 and 18, respectively.

5.4.2. Two-Bidder Dutch Split-Award Auction

Result 4 *Split-award winners achieve above-equilibrium prices with an average of 74.80 in the first phase and an average of 77.23 in the second phase. This can be seen as a form of strong tacit collusion, where bidders agree on higher payoffs.*

In the Dutch auction there is a unique perfect Bayesian split equilibrium. Therefore, strategic complexity is lower compared to the FPSB format as bidders observe increasing single- and double-unit prices publicly and can constantly compute their respective profits. Furthermore, a payoff-maximizing bidder will not accept two units at a double-unit price below 140 (which corresponds to a single-unit price of 70) as the split is more profitable. In the experiments, fewer than 10% of all direct sole-source winners accept a double-unit price below 140. With this common knowledge there is a low risk in letting the single-unit price rise to 70 as the equilibrium suggests. Even if the opponent accepts at a lower price in the first phase, there is always a chance to win the second unit.

Interestingly, we find average split prices of 74.80 in phase 1 and 77.23 in phase 2, which strictly exceed the equilibrium prediction of 70. It appears that bidders implicitly agree upon letting the price rise above the equilibrium prediction to make higher profits in the split allocation. The publicly increasing prices allow both bidders to constantly reinforce this agreement on strong tacit collusion until one bidder accepts, which apparently leads both bidders to exceed the equilibrium price. The first bidder to accept the single-unit price, signals at which price the opponent should accept the second unit. Assume that one unit was sold in phase 1 for a price of 74. The loser of phase 1 knows that the price for the remaining share is likely to rise again at least to 74, as otherwise the opponent would have accepted both units at a price of 148 in phase 1. In fact, 78% of the bidders only accepted the counteroffer for one unit in phase 1, when their payoff for the split award was strictly higher than for the sole source award. Therefore, the loser of the first unit might not want to accept the second unit at a lower price than his opponent in phase 1, because it is not credible for the winner of phase 1 to accept the second unit below the price of the first unit. The price of the first unit is a natural lower bound for the price of the second unit and typically bidders try to go even a bid higher at the risk that the opponent takes both shares.

Of course, if the loser of the first phase lets the price in phase 2 rise too high it will become more profitable for the winner to accept the second consecutive unit. Similarly, if the price in phase 1 rises too high it might become more profitable for one of the two bidders to directly accept two units. For example, if the first unit is sold at a price of 74, a bidder with cost draw 110 makes a payoff of 74-33=41. Now, if the second unit reaches a price of 78, the winner of the first unit should accept this price as well, as it provides a higher payoff of 152-110=42. Thus, if bidders let the split prices rise too high the sole-source award becomes more profitable at some point. This is also what we see in the data. There is a substantial proportion of sole-source awards in the Dutch auction due to such behavior, although the proportion of efficient split awards has increased significantly compared to the FPSB format.

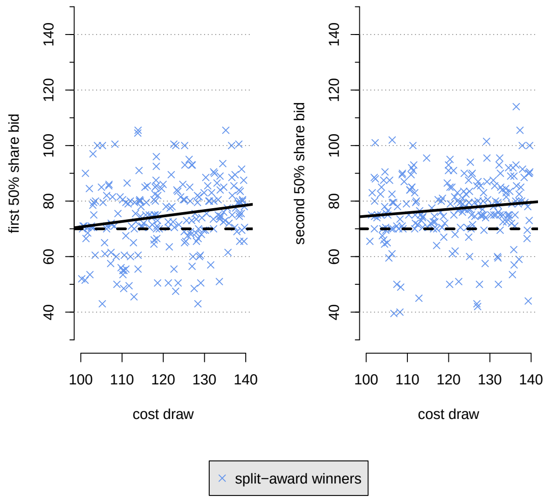

Figure 3 **Bids of Split-Award Winners in Dutch-Dutch (n=2)**

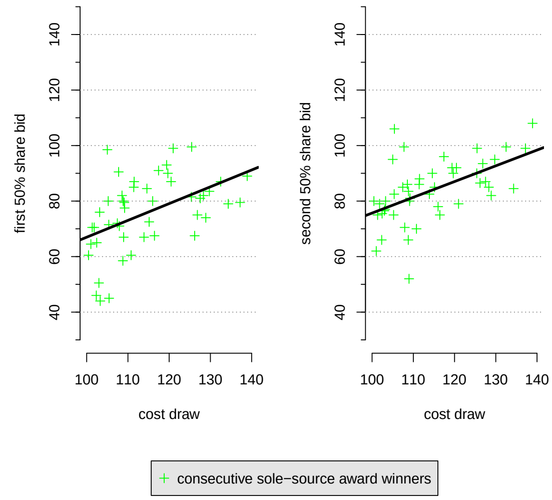

Figure 4 **Bids of Consecutive Sole-Source-Award Winners in Dutch-Dutch (n=2)**

The first-unit price was taken as a signal for the second-unit price, which was higher in 58% of all split allocations. The relation between the two split prices is depicted in figure 3 in which both univariate regression lines lie entirely above the predicted pooling price of 70. The prices in case of bidders winning two consecutive units are depicted in figure 4 and are strongly increasing in costs, but otherwise show a similar pattern than the split prices. Figure 9 in appendix B contains the prices at which two units were directly accepted which are on average higher than in the split award. Similar to the FPSB auction the distribution of allocations does not change across the periods as is shown in figure 20 in the appendix. The fixed-effects regressions for split-award winners in appendix B yield that the cost draws have a significant but small effect on the bid price of the first phase, but not in the second phase. In the second phase, the bid price of the first phase is a significant covariate.

5.4.3. Two-Bidder Dutch-FPSB Split-Award Auction

Result 5 *Split allocations involve tacit collusion of the bidders with average single-unit prices of 61.80 and 66.57 in phases 1 and 2, respectively. The lower prices avoid sole-source bids as they happen in the Dutch auction. Repeated auctions lead to significantly higher prices in this auction format, as bidders learn to coordinate on higher split prices.*

In the Dutch-FPSB auction we observe average single-unit prices of 61.80 and 66.57 in phases 1 and 2, respectively. Efficient equilibria, in which bidders tacitly collude on split prices below 70, are in line with the theoretical predictions for this format. In contrast to the Dutch auction, bidders have a credible threat with low sealed bids on the second unit that would result in a lower overall profit for a bidder, but forces his opponent not to bid too high on the second unit and win. Interestingly, we observe that 62% of the winners of the first unit submit bids for the second unit to make as much profit when winning both units as when winning one unit. Such bids are no credible threats in the Dutch-FPSB auction. The strategic option of using threats to implement split prices below 70 is rarely used in the experiments, probably because it cannot be directly observed by the opponent.

The credible threat is the reason for the different equilibrium bidding strategies between both ascending auction formats. The fact that this credible threat is not used might suggest that one would see prices similar to the Dutch auction. However, prices are lower in the Dutch-FPSB auction. It appears less certain for a bidder to win the second unit, and bidders tend to accept the first unit already at a lower price in order to secure one unit. Furthermore, we conjecture that the possibility of possible threats in itself leads to higher insecurity and lower bidding already in phase 1. Similar to the Dutch auction, the price of the first unit is a signal for the second unit, and we observe slightly higher prices for the second unit. In summary, bidders are faced with relatively low split prices at which sole-source deviations are less likely to occur. Therefore, we observe even fewer sole-source allocations in the Dutch-FPSB auction than in the Dutch format. Overall, this increases efficiency.

Plots of the bids in the Dutch-FPSB auction can be found in appendix B. In the fixed-effects regressions we find a small but significantly positive effect of the cost draws on both units, and again the price of the first phase was a significant covariate for the price in the second phase (see tables 7 in appendix B). As in the Dutch auction the impact of the cost draws on the bid is very small (e.g., 0.13 for the first bid), which is close to the constant bid price predicted by the theory. In contrast to the other auction formats, we found a significant positive impact of the number of periods on the bid price for the first price. This indicates that the subjects adapted and learned to tacitly collude on higher split prices over time.

5.4.4. Three-Bidder FPSB, Dutch, and Dutch-FPSB Split-Award Auction

Result 6 *With three bidders almost always the efficient split award is implemented. In the FPSB auction format it is difficult for bidders to realize a sole source allocation, and they need to rely on another bidder. In the two ascending auction formats sole source awards are no equilibrium and they do not happen in the lab.*

Contrary to the ascending auctions, there is an inefficient WTA equilibrium in the FPSB auction. To implement a sole-source allocation in this format at least two bidders have to exclude the split award with high bid-to-lose prices for one unit. Note, however, that in this case only the bidder with the lowest cost type wins and both losers know that they would have won one unit with certainty by coordinating on the split award. This implies that both losers may regret their decision to chose the WTA strategy. In contrast, in the split allocation the only loser could not have won the sole-source award by unilaterally playing a bid-to lose strategy for the split. He could only regret not to bid aggressive enough for the split award after the winning bids are disclosed. The anticipation of this form of loser's regret is a possible explanation for too aggressive bidding in first-price sealed-bid auctions (Filiz-Ozbay and Ozbay 2007). In our experiments, not only the winning bid(s) but also the winning allocation is disclosed to all bidders after each auction. Hence, we conjecture that anticipated regret of the losers in a WTA equilibrium as described above prevent bidders from trying to win the sole-source allocation in an environment with three bidders, whereas these equilibria can be observed in the experiments with two bidders.

Furthermore, a simple and effective strategy in such an environment is to bid competitive for the split as well as for the sole source award. Such a bidding behavior excludes the sole source and is not vulnerable to deviations even from both opponents. Such a strategy is a form of a σ equilibrium with a moderate bid-to-lose price for 2 units.[16] Assume a bidder with costs of 120 for two units and 36 for one unit submits bids of 124 and 38, respectively. If all bidders follow a σ strategy with bid-to-lose prices on two units, he has good chances to win the split award. In the case where both opponents try to coordinate on the sole source award with high bid-to-lose prices for one unit and competitive bids for two units, his chances (with regard to his cost draw) are still good to win both units.

We conjecture that the combination of both phenomena leads to the high efficiency and non-appearance of sole source awards in the FPSB auction with three bidders. As predicted, in the ascending auction formats competition and the inherent DSE cost structure lead to low prices for the first and the second unit consecutively, so that the sole-source award is always unprofitable.

As expected in case of competitive bidding for the split award with three bidders in all auction formats the fixed-effects regressions in tables 8 to 10 in appendix B contain a cost parameter with significant explanatory power that is positively correlated with the corresponding dependent variable. We used the Wald test to test for the correspondence between fixed-effects within regression model and the derived equilibrium strategy.

Although the single-unit bids of split-award winners in the three-bidder setting of the FPSB auction are significantly different from the equilibrium strategy (Wald test with $p < 2.2e - 16$), the

[16] Remember, that in our setting bidding the cost draw for two units is enough to exclude the split award.

univariate regression line in the left plot of figure 13 in appendix B still indicates correspondence. Similar to the experimental evaluation of bidding in single-unit auctions we observe underbidding in our setting. Moreover, in the right plot the double-unit bids weakly exceed the cost type as predicted by theory.

As can be seen from the regression line in the left plot of figure 14, the equilibrium strategy is a good predictor for the bidding behavior of split-award winners for the first unit in the Dutch auction (Wald test with $p = 0.3443$). Also, the bidding behavior for the second unit corresponds to the theoretical prediction (Wald test with $p = 0.07675$). The left plot of figure 15 in appendix B shows that split-award winners of the first unit in the Dutch-FPSB auction submit bids weakly below the equilibrium strategy and the fixed-effects regression differs significantly (Wald test with $p < 2.2e-16$). Although split-award winners of the second unit appear to bid closer to the theoretical prediction in figure 16, their bidding behavior does not correspond to the equilibrium strategy (Wald test with $p = 0.001158$).

Similar to the experimental literature on first-price single-item auctions, we observe underbidding for the split-award winners in the FPSB and the Dutch-FPSB auctions with three bidders. This causes the fixed-effects regression lines to differ significantly from the equilibrium predictions. Moreover, the first- and second-unit bids in the Dutch-FPSB format neither differ from each other (Wald test with $p = 0.9912$), nor from the single-unit bid of split-award winners in the FPSB auction (Wald test with $p = 0.2343$ and $p = 0.2552$ respectively). We conjecture that the strategical differences between both auction formats do not influence average bidding behavior.

Similar to our two-bidder setting, average single-unit prices in the Dutch auction with three bidders are significantly higher than in the other two formats. Interestingly, the bids for the first- and second units do not differ significantly from the predictions although they do not differ from each other (Wald test with $p = 0.07219$). These higher prices in the Dutch format may explain the lower allocative efficiency as well as the lower share of efficient allocations compared to the other two auctions.

6. Conclusions

Ex-post split-award auctions are a widely used form of combinatorial procurement auctions. In particular, first-price auctions are often chosen for their simplicity. There is little competition in procurement particularly if there are only a few qualified suppliers for specific products to be procured. Unfortunately, bidding strategies of such auctions are not well understood. After the complete-information analysis by Bernheim and Whinston (1986), the analysis by Anton and Yao

(1992) showed that, for the wide-spread FPSB split-award auction with two bidders, there is an efficient split equilibrium and an inefficient WTA equilibrium, leaving the bidders with a veritable coordination problem. The 2-bidder model is specific, because bidders can veto the split award unilateraly, and it is unclear if the results carry over to markets with more than two bidders, and if it is predictive in the lab.

We extend the analysis to $n > 2$ bidders, and also analyze the Dutch split-award auction and the Dutch-FPSB split-award auction, which are wide-spread in procurement practice, but have not yet been studied. For markets with two bidders, we show that the Dutch split-award auction has a unique split equilibrium with a constant pooling price. The Dutch-FPSB also exhibits only efficient split equilibria, but it allows for multiple equilibrium prices. The strategic differences for the bidders arise because of differences in the revealed information in the three auction formats. There is cost equivalence between the Dutch and the Dutch-FPSB auction, while this only applies for the FPSB format when suppliers coordinate in a split equilibrium. In markets with more than two bidders, the FPSB auction still exhibits a WTA equilibrium even though the veto power of bidders ceases. The theoretical results organize important patterns in the experimental results such as pooling prices in the two-bidder auction and the equilibrium selection problem.

In our experimental assessments, we found that bidders in the two-bidder FPSB auction indeed selected both types of awards. It is interesting to see that many more split allocations emerged in the Dutch auctions at prices even beyond the equilibrium pooling price. We conjecture that bidders interpret non-acceptance of counteroffers at low prices as an implicit agreement on high prices. These high prices sometimes lead bidders in the first phase to bid on the package of two units or accept also a high price in the second phase, such that there are inefficient sole-source awards. The introduction of a sealed-bid stage in phase 2 of the Dutch-FPSB auction led to lower prices in the first phase. As a consequence, winning the package became less attractive, which led to even higher efficiency. The Dutch-FPSB auction appears as a robust and simple alternative that yields high efficiency and low procurement cost.

Interestingly, with a single additional bidder the inefficiency of the 2-bidder environment is largely gone, even in the FPSB split-award auction, where there is still an equilibrium selection problem. The veto power of bidders vanishes and instead they want to win the 50% share and coordinate with others. Furthermore, we found that prices drop substantially in all three auction formats as theory predicts. Competition was very effective in the laboratory even though bidders in the FPSB split-award auction could also choose a sole-source award in theory. In summary, first-price combinatorial auctions are highly efficient in our setting. The lower strategic complexity and high efficiency of the Dutch auction formats can be seen as an advantage.

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Appendix A: Proofs

A.1. Proof of Corollary 2

Using the same logic as in proposition 1 of Anton and Yao (1992), it is possible to show that split prices in a σ equilibrium have to be constant. Otherwise, the bidder with the lower split price always has an incentive to deviate from the equilibrium strategy. We will show that the only possible split price in a σ equilibrium is $p_e^{\Sigma} = \underline{\Theta}(1-C)$. First, we assume a σ equilibrium with a constant split price $p_e^{\sigma} > (1-C)\underline{\Theta}$ exists. In this case, there is always a profitable sole source deviation for a bidder $\hat{\Theta} = \underline{\Theta}$ by accepting the counteroffer for 100% of the business at a price of $\hat{p}^s(\hat{\Theta}) = 2p_e^{\sigma} - \varepsilon$ with $\varepsilon \searrow 0$, as $E[\hat{\Pi}^s(\underline{\Theta})] > E[\hat{\Pi}^{\sigma}(\underline{\Theta})]$ implies $p_e^{\sigma} > (1-C)\underline{\Theta} + \varepsilon$.

Second, we assume a σ equilibrium such that $p_e^{\sigma} < (1-C)\underline{\Theta}$ exists and bidder A with $\Theta_A \neq \overline{\Theta}$ sticks to the equilibrium strategy and accepts the 50% share in the first phase; then, he makes a profit of $p_e^{\sigma} - \Theta_A > 0$ in equilibrium. However, in this case there is a split deviation for the loser of phase 1, which generates a higher payoff than in equilibrium. This player knows that the additional costs for providing 100% of the business are $\Theta_A(1-C)$ for the winner of phase 1. Hence, the deviating bidder can accept the remaining share at a price of $\underline{\Theta}(1-C) - \varepsilon$ knowing that the other bidder cannot accept any previous offer without reducing his already achieved payoff. Even when the winner threatens to accept the 50% share for a lower price than $\underline{\Theta}(1-C)$, this is not credible because both bidders are assumed to be payoff-maximizing. Following such a strategy, a deviating bidder wins 50% of the business with probability 1 and achieves a higher payoff than by playing a σ equilibrium with a split price $p_e^{\sigma} < (1-C)\underline{\Theta}$. Q.E.D.

A.2. Proof of Proposition 3

The Dutch split-award auction is modeled as a two-stage game, where the action of phase 1 is observed by both players before phase 2 starts. Because the σ equilibrium includes pooling prices, which are independent on the respective cost type, the loser of phase 1 does not get any information about the cost type of his opponent. Hence, his beliefs μ are not updated after phase 1 and remain the same as ex-ante.

Furthermore, we have to prove that the strategy profile $(p_e^{\sigma 1}(\cdot), p_e^{\sigma 2}(\cdot))$ is sequentially rational given the system of beliefs. Therefore, we have to prove that there is neither a sole source nor a split deviation for any bidder $i \in \{A, B\}$ that yields a higher expected payoff than the equilibrium payoff of $\Pi_e^{\sigma}(\Theta_i) = (1-C)\underline{\Theta} - C\Theta_i$.

Sole source deviations: In each round r, the counteroffers c_r^{σ} and c_r^s are presented according to the pricing rule, $2c_r^{\sigma} = c_r^s$, which satisfies the buyer's indifference condition. Thus, only sole source deviations with prices $\hat{p}^s(\hat{\Theta}) > 2\hat{\Theta}(1-C)$ have to be considered. For all other sole source deviations, a bidder makes a higher profit by accepting the split in the same round. However, sole source deviations with prices greater than $2(1-C)\hat{\Theta}$ can never be realized, as the opponent who sticks to the equilibrium strategy accepts the split at a price of $(1-C)\underline{\Theta}$ in equilibrium, and counteroffers for 100% of the business greater than $2(1-C)\underline{\Theta}$ are not presented. This excludes sole source deviations in phase 1. As the remaining 50% share is offered in phase 2, only the winner of phase 1 can follow a sole source deviation in this stage. Again, if the other bidder sticks to the equilibrium strategy, the sum of the split price in phase 1 and a counteroffer for the remaining share cannot be greater than $2(1-C)\underline{\Theta}$, which excludes profitable sole source deviations in phase 2 as well.

Split deviations: The expected payoff of all possible split deviations with split prices lower than $\underline{\Theta}(1-C)$ is obviously less than the equilibrium payoff, as the probability of winning 50% does not increase with a lower split price.

In phase 1, a split deviation with a split price higher than $(1-C)\underline{\Theta}$ is not possible, when the other bidder follows the equilibrium strategy. However, this does not apply in phase 2 because it would not be a credible threat for bidder A, the winner of phase 1 with a cost of type $\Theta_A \in (\underline{\Theta}, \overline{\Theta}]$, to accept the remaining 50% share in phase 2 for a price $(1-C)\underline{\Theta}$. Bidder A can only accept a counteroffer for the remaining 50% share when his additional costs $(1-C)\Theta_A$ in the case of winning the remaining 50% share are covered and he makes at least the same payoff as in phase 1. Although such split deviations with split prices $\hat{p}^{\sigma 2}(\hat{\Theta}) > \underline{\Theta}(1-C)$ are possible, we show in the following that they yield lower than expected payoff than the equilibrium strategy.

Assume a bidder tries to deviate with a split price $(1-C)\underline{\Theta} < \hat{p}^{\sigma}(\hat{\Theta}) \leq (1-C)\overline{\Theta}$ in phase 2. The upper boundary arises because the winner of phase 1 accepts the counteroffer for the remaining 50% share at a price of $(1-C)\overline{\Theta}$. Such a strategy can be expressed by $\hat{p}^{\sigma}(x) = x(1-C)$ with the variable $x \in (\underline{\Theta}, \overline{\Theta}]$. A deviating bidder B risks losing the whole business because he knows that supplier A will fulfill his threat at $p_e^{\sigma 2w}(\Theta_A) = \Theta_A(1-C)$. The deviating bidder faces a trade-off because he does not know the cost type of the other bidder; recall that the split price in phase 1 is constant and independent of the cost type Θ_A, which is why the loser does not get any information about the cost type of the winner in phase 1.

Split deviations with $\hat{p}^{\sigma}(x)$ can only be excluded, when for all cost types Θ and for all x, the expected payoff of such a deviation is less than the σ equilibrium payoff, i.e., $\Delta^{\Pi}(x,\Theta) = E[\hat{\Pi}^{\sigma}(\Theta)] - E[\Pi_e^{\sigma}(\Theta)] < 0 \;\forall\; \Theta \in [\underline{\Theta}, \overline{\Theta}] \;\forall\; x \in (\underline{\Theta}, \overline{\Theta}]$. As $\Delta^{\Pi}(x,\Theta)$ is strictly decreasing in Θ, it suffices to show that $\Delta^{\Pi}(x,\overline{\Theta}) < 0$ applies for all x. This is fulfilled by the necessary condition (2). The strategy in phase 2 of the winner of the 50% share in phase 1 is credible, as the payoff of this threat equals the equilibrium payoff when it must be carried out.

Since bid prices are constant in equilibrium and $(1-C)\underline{\Theta} > C\overline{\Theta}$, the probability of winning the split by deviating does not depend on your own cost type, but the equilibrium payoff decreases with Θ. Thus, if we show that a deviation is not profitable for the highest cost type $\overline{\Theta}$, it is not profitable for any cost type. The expected payoff of a split deviation with split price $(1-C)\underline{\Theta} < \hat{p}^{\sigma}(\hat{\Theta}) < (1-C)\overline{\Theta}$ is

$$
\begin{aligned}
E[\hat{\Pi}^{\sigma}(\overline{\Theta})] =& (\hat{p}^{\sigma}(\overline{\Theta}) - C\overline{\Theta})P(\hat{p}^{\sigma}(\overline{\Theta}) \leq \Theta_i(1-C)) \\
=& (x(1-C) - C\overline{\Theta})P(x(1-C) \leq \Theta_i(1-C)) \\
=& (x(1-C) - C\overline{\Theta})(1-F(x))
\end{aligned}
$$

For $x = \underline{\Theta}$, the expected payoff of such a deviation equals the equilibrium payoff of the highest cost type $\overline{\Theta}$. Hence, it must apply for all $x \in [\underline{\Theta}, \overline{\Theta}]$ that

$$
\begin{aligned}
E[\hat{\Pi}^{\sigma}(\overline{\Theta})] \leq& E[\Pi^{\sigma}(\overline{\Theta}] \\
(x(1-C) - C\overline{\Theta})(1-F(x)) \leq& \underline{\Theta}(1-C) - C\overline{\Theta}
\end{aligned}
$$

Q.E.D.

A.3. Proof of Proposition 4

If there is a WTA equilibrium strategy, such a strategy would have to be payoff-dominant over all possible split and sole source deviations.

Sole source deviations: In the WTA equilibrium in proposition 7, a bidder must accept a counteroffer for the sole source award, c_r^s, in round r for a price of $p_e^s(\Theta) = \Theta + \frac{\int_{\Theta}^{\overline{\Theta}}(1-F(t))dt}{(1-F(\Theta))}$ in order to ensure that there is no sole source deviation that yields a higher payoff than the equilibrium strategy.

Split deviations: Next, we show that if all bidders play such a WTA strategy, there is at least one bidder who has an incentive to deviate. Consider a bidder with the highest cost type $\overline{\Theta}$ who makes a payoff of zero in equilibrium by accepting the counteroffer $c_r^s = p_e^s(\overline{\Theta})$ in round r. With DSE, there is always a round q preceding round r with a counteroffer $c_q^s = 2\overline{\Theta}C + \varepsilon < c_r^s$, in which this bidder makes a higher payoff than in equilibrium. Hence, there is no WTA equilibrium in a Dutch split-award auction with DSE. Q.E.D.

A hybrid equilibrium consists of disjunct cost intervals $I_1^s, I_2^s, ..., I_t^s$ and $I_1^\sigma, I_2^\sigma, ..., I_u^\sigma$. Bidders with cost types belonging to the first intervals focus on winning the sole source award, while bidders with other cost types try to coordinate on the split award. Furthermore, $t, u \in \mathbb{N}$ and $I_1^s \dot\cup I_2^s \dot\cup ... \dot\cup I_t^s \dot\cup I_1^\sigma \dot\cup I_2^\sigma \dot\cup ... \dot\cup I_u^\sigma = [\underline{\Theta}, \overline{\Theta}]$ applies. We divide the different hybrid equilibria in two types: hybrid equilibria in which the highest cost types focus on winning the sole source award, i.e., $I_t^s = (\tau, \overline{\Theta}]$; and hybrid equilibria in which the bidder with costs of $\overline{\Theta}$ tries to win the split, i.e., $I_u^\sigma = [\tau, \overline{\Theta}]$. The strategic parameter τ indicates the cost type for which bidders change their strategy in a hybrid equilibrium. Using the same reasoning as above for the WTA equilibrium, the bidder with the highest cost type has an incentive for a split deviation in the hybrid equilibrium with $I_t^s = (\tau, \overline{\Theta}]$. Hence, such hybrid equilibria do not exist.

Next, assume a hybrid equilibrium with an interval $I_u^\sigma = [\tau, \overline{\Theta}]$ in which the highest cost type tries to win the split award. As in the analysis of (Anton et al. 2010), a bidder with cost type τ must be indifferent between winning the split for $p_e^\sigma(\tau)$ or the sole source award for $2p_e^\sigma(\tau)$. Otherwise, this bidder would not change his strategy, i.e., $E[\Pi_e^s(\tau)] = E[\Pi_e^\sigma(\tau)]$ implies $p_e^\sigma(\tau) = \tau(1-C)$. This directly implies that $p_e^\sigma(\tau) = \tau(1-C)$ is the equilibrium split price for bidder τ and all other bidders with $\Theta \in I_1^\sigma \dot\cup I_2^\sigma \dot\cup ... \dot\cup I_u^\sigma$ as well, as split prices must be constant based on the same reasoning as in corollary 2. However, all cost types for which the strategy changes in equilibrium must be indifferent between both awards, such as τ for $I_t^\sigma = [\tau, \overline{\Theta}]$. Because this is never true with a constant split price and multiple disjunct intervals in which bidders compete for the split award, a hybrid equilibrium with $u > 1$ can be excluded. Hence, it suffices to show that no hybrid equilibrium with $I_1^s = [\underline{\Theta}, \tau)$ and $I_1^\sigma = [\tau, \overline{\Theta}]$ exists.

In such an equilibrium, the highest cost type makes an expected payoff of $(1-C)\tau - \overline{\Theta}C > 0$. Furthermore, the equilibrium price for the sole source award of the lowest cost type, must be greater than $2\underline{\Theta}(1-C)$ in order to exclude a more profitable split deviation. Therefore, there is always a profitable split deviation for the highest cost type by accepting the offer for the 50% share for a price of $\underline{\Theta}(1-C)$, as

$$E[\hat{\Pi}(\overline{\Theta})] > E[\Pi_e(\overline{\Theta})]$$

$$\underline{\Theta}(1-C) - \overline{\Theta}C > ((1-C)\tau - \overline{\Theta}C)(1-F(\tau)).$$

This is the necessary condition for a σ equilibrium.

Hence, we proved that when a σ equilibrium exists, it is unique.

Assume there is such an equilibrium with an arbitrary parameter $\tau \in (\underline{\Theta}, \overline{\Theta})$. The sole source price $p_e^s(\Theta)$ for $\Theta \in [\underline{\Theta}, \tau)$ must assure that there is no sole source deviation in equilibrium, i.e., that $E[\Pi_e^s(\Theta^*, \Theta)] = (p_e^s(\Theta^*) - \Theta)(1 - F(\Theta^*))$ is maximized for $\Theta^* = \Theta$. As in the analysis of (Anton et al. 2010), this applies with the following sole source prices

$$p_e^s(\Theta) = \Theta + \tau(1-2C)\frac{(1-F(\tau))}{(1-F(\Theta))} + \frac{\int_{\Theta}^{\tau} 1 - F(t)dt}{(1-F(\Theta))}$$

for types $\Theta \in [\underline{\Theta}, \tau)$. Furthermore, for a type τ there must be no incentive to deviate from equilibrium by accepting the offer for 50% for a price lower than $0.5p_e^s(\tau)$. Such a deviation would yield the following expected payoff dependent on variable x, which defines the deviating split price: $E[\hat{\Pi}^\sigma(x, \tau)] = (0.5p_e^s(x) - \tau C)(1 - F(x))$. Solving the first-order condition yields: $-f(x)(x - 2C\tau) = 0$.

$$\begin{aligned} \frac{d}{dx} E[\hat{\Pi}^\sigma(x,\tau)] &= 0 \\ 0.5(-f(x)(x-2C\tau) + (1-F(x)) - (1-F(x))) &= 0 \\ -f(x)(x-2C\tau) &= 0 \end{aligned}$$

The solution of the first-order condition is $x = 2C\tau < \tau$, and it can be shown that it is the unique maximum of the expected payoff function, as f is strictly positive and the derivative of $E[\hat{\Pi}^\sigma(x, \tau)]$ is positive (negative) for all values for x that are lower (higher) than $2C\tau$. Hence, there is always a profitable split deviation with $x = \tau - \varepsilon$ for a bidder with cost type τ in a hybrid equilibrium, as $E[\hat{\Pi}^\sigma(\tau - \varepsilon, \tau)] > E[\hat{\Pi}^\sigma(\tau, \tau)] = E[\Pi_e^\sigma(\tau)]$ and no such hybrid equilibrium can emerge.   Q.E.D.

A.4. Proof of Corollary 3

The results for (i) and (ii) of proposition 1 in Anton and Yao (1992) can be easily transferred to the Dutch-FPSB split-award auction case. In the Dutch split-award auction, the bidding strategy of a bidder A, the winner of phase 1, has to assure that it yields at least the same payoff as in phase 1, as the payoff is realized by carrying out the threat. However, when phase 2 is an FPSB mechanism, it suffices that the offer for the remaining share impedes split deviations. Then, as long as the auctioneer bids at least p_e^σ for the remaining share, the probability that the threat must be carried out and the payoff of bidder A is changed to zero, as the opponent has no incentive to deviate from equilibrium. This makes it easier for the winner to exclude split deviations in phase 2 and various split prices can emerge in equilibrium.

A.5. Proof of Proposition 5

As in the proof of proposition 3, the pooling price in phase 1 does not allow for any updating of the beliefs about the opponent's cost type. In what follows, we show that deviations from the pooling equilibrium are unprofitable for all different cost types in every stage of the game. Hence, the sequential rationality assumption for perfect Bayesian equilibria applies.

Assume a σ equilibrium in which both bidders win 50% of the business for a constant split price $p_e^\sigma \in [\overline{\Theta}C, \underline{\Theta}(1-C)]$ in equilibrium.

Sole source deviations: With the same logic as in the proof of proposition 3 payoff-dominant bidders have no incentive to deviate for the sole source award in equilibrium .

Split deviations: Split deviations in phase 1 can be easily excluded, as the expected payoff is either strictly lower than the equilibrium payoff (for deviations with a lower split price than p_e^σ) or zero (for deviations with higher split prices). Bidder A, the winner of the first 50% share, submits a quote of $\max\{p_e^\sigma, \Theta_A - p_e^\sigma\}$ in phase 2 in order to implement the σ equilibrium strategy. This threat is credible as his equilibrium payoff of phase 1 does not change in expectation. The probability that his opponent deviates is zero, as the expected payoff of such a split deviation is lower than the equilibrium payoff. This is assured by condition (3). We skip this line of reasoning because it is similar to the proof of proposition 1 (Case 1 deviation) in Anton and Yao (1992) for the FPSB split-award auction. Q.E.D.

A.6. Proof of Proposition 7

As in a WTA equilibrium for two bidders (Anton and Yao 1992), the existence of such an inefficient equilibrium can be shown with $n > 2$ bidders:

Sole source deviations: In order to avoid any sole source deviation, the sole source price $p_e^s(\cdot)$ has to maximize the expected payoff of winning 100% of the business

$$E[\Pi_e^s(\Theta)] = (p_e^s(\Theta) - \Theta)P(p_e^s(\Theta) \leq p_e^s(\Theta_{1:n-1})) \text{ for every } \Theta \in [\underline{\Theta}, \overline{\Theta}]$$

Split deviations: Additionally, split deviations can be excluded by sufficiently high equilibrium prices for the split, which satisfy

$$p_e^\sigma(\Theta) > \overline{\Theta} - \underline{\Theta}C.$$

These prices assure that the auctioneer never choses the split, as the sum of any possible split deviation $\hat{p}^\sigma(\hat{\Theta}) \geq \underline{\Theta}C$ and the lowest split price of another supplier is strictly greater than $\overline{\Theta}$, the highest possible price the auctioneer has to pay for 100% of the business in equilibrium. Hence, the payoff of such a deviating strategy is always zero. Q.E.D.

A.7. Proof of Proposition 8

We will show that in equilibrium neither sole source nor split deviations are profitable for bidders.

Sole source deviations: With a sole source deviation $(\hat{p}^s(\hat{\Theta}), \hat{p}^\sigma(\hat{\Theta}))$, a bidder aims to win the sole source award and excludes the split (for himself). By differentiating the following cases, it is shown that there is no sole source deviation, which yields a higher expected payoff than a σ equilibrium.

i) $2\overline{\Theta}C \leq \underline{\Theta}$

All possible deviating sole source prices $\hat{p}^s(\hat{\Theta})$ have to be greater than or equal to $\underline{\Theta}$ to fulfill the assumption of individual rationality. As all other bidders play the σ equilibrium, the highest possible price for the auctioneer is $2\overline{\Theta}C$ by awarding the split. Therefore, the auctioneer never allocates the sole source award to any bidder with $\hat{p}^s(\hat{\Theta}) \geq \underline{\Theta} > 2\overline{\Theta}C$ and the expected payoff of such deviations is zero.

ii) $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\overline{\Theta}+\underline{\Theta}}$:

We show that there is no sole source deviation with prices $\hat{p}^s(\hat{\Theta})$. A sole source deviation with price $\hat{p}^s(\hat{\Theta}) = 2p_e^\sigma(\underline{\Theta})$ is payoff-dominant over all possible deviations with lower sole source prices, because the probability to win the sole source award is 1 for all deviations of this type. Hence, it suffices to show that deviations with prices greater than or equal to $2p_e^\sigma(\underline{\Theta})$ are not attractive for the bidders. The upper bound for deviating sole source prices is $2\overline{\Theta}C$, because the probability that a sole source award is chosen is zero, when a bidder submits a higher price for the sole source award.

As first step, the optimal deviating sole source price $\hat{p}^s(\hat{\Theta}) \in [2p_e^\sigma(\underline{\Theta}), 2\overline{\Theta}C]$ for all possible cost types $\hat{\Theta} \in [\underline{\Theta}, 2\overline{\Theta}C]$ is derived. Because $p_e^\sigma(\Theta)$ is continuous, we can express all possible $\hat{p}^s(\hat{\Theta})$ by $2p_e^\sigma(\Theta)$ with variable $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ and the expected profit of a sole source deviation as

$$\begin{aligned} E[\hat{\Pi}^s(\hat{\Theta})] &= (\hat{p}^s(\hat{\Theta}) - \hat{\Theta})\ P(\hat{p}^s(\hat{\Theta}) \leq p_e^\sigma(\Theta_{2:n-1}) + p_e^\sigma(\Theta_{1:n-1})) \\ &= (2p_e^\sigma(\Theta) - \hat{\Theta})\ P(2p_e^\sigma(\Theta) \leq p_e^\sigma(\Theta_{2:n-1}) + p_e^\sigma(\Theta_{1:n-1})) \end{aligned}$$

Obviously, the probability that the deviating sole source price is lower than the split prices of the second lowest and lowest order statistic is always less than the probability that it is lower than two times the split price of the second lowest order statistic. Hence, the payoff

$$\begin{aligned} E[\hat{\Pi}^{s'}(\hat{\Theta})] &= (2p_e^\sigma(\Theta) - \hat{\Theta})\ P(2p_e^\sigma(\Theta) \leq 2p_e^\sigma(\Theta_{2:n-1})) \\ &= (2p_e^\sigma(\Theta) - \hat{\Theta})\ P(p_e^\sigma(\Theta) \leq p_e^\sigma(\Theta_{2:n-1})) \\ &= (2p_e^\sigma(\Theta) - \hat{\Theta})\ ((1-F(\Theta))^{n-1} + (n-1)F(\Theta)(1-F(\Theta))^{n-2}) \\ &< (p_e^\sigma(\Theta) - C\hat{\Theta})\ ((1-F(\Theta))^{n-1} + (n-1)F(\Theta)(1-F(\Theta))^{n-2}) \\ &\leq E[\Pi_e^\sigma(\hat{\Theta})] \end{aligned}$$

as $\Theta = \hat{\Theta}$ maximizes the payoff function and

$$\begin{aligned} C &< \frac{\underline{\Theta}}{\underline{\Theta} + \overline{\Theta}} \\ \overline{\Theta}C &< \underline{\Theta}(1-C) \\ p_e^\sigma(\Theta) &< \hat{\Theta}(1-C) \text{ because } p_e^\sigma(\Theta) \leq \overline{\Theta}C \text{ and } \underline{\Theta} \leq \hat{\Theta} \quad | + p_e^\sigma(\Theta) \\ 2p_e^\sigma(\Theta) - \hat{\Theta} &< p_e^\sigma(\Theta) - C\hat{\Theta} \end{aligned}$$

Split deviations: Split prices $p_e^\sigma(\Theta)$ have to maximize the expected payoff of winning 50% of the business for a bidder with cost type Θ

$$E[\Pi_e^\sigma(\Theta)] = (p_e^\sigma(\Theta) - \Theta C)P(p_e^\sigma(\Theta) \le p_e^\sigma(\Theta_{2:n-1})) \text{ for every } \Theta \in [\underline{\Theta}, \overline{\Theta}]$$

in equilibrium. The first-order condition can be simplified to:

$$\frac{d}{d\Theta}\{((1-F(\Theta))^{n-1} + (n{-}1)F(\Theta)(1-F(\Theta))^{n-2})p_e^\sigma(\Theta)\} = \\ \Theta C((n-1)(n-2)(1-F(\Theta))^{n-3}F(\Theta)(-f(\Theta))$$

By applying boundary condition $p_e^\sigma(\overline{\Theta}) = \overline{\Theta}C$ and integration on both sides, we get

$$((1-F(\Theta))^{n-1} + (n-1)F(\Theta)(1-F(\Theta))^{n-2})p_e^\sigma(\Theta) \mid_{\Theta}^{\overline{\Theta}} = \\ \int_{\Theta}^{\overline{\Theta}} xC((n-1)(n-2)(1-F(x))^{n-3}F(x)(-f(x))dx.$$

Solving for $p_e^\sigma(\Theta)$ results in the equilibrium split price for a bidder with cost type Θ:

$$p_e^\sigma(\Theta) = C\Theta + C\frac{\int_{\Theta}^{\overline{\Theta}}(1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2}dx}{(1-F(\Theta))^{n-1} + (n-1)F(\Theta)(1-F(\Theta))^{n-2}}$$

Hybrid deviations: Least, one have to exclude hybrid deviations, i.e. deviating strategies, for which the sole source as well as the split award can emerge with (strictly) positive probability.

i) $2\overline{\Theta}C \le \underline{\Theta}$

When this condition applies, hybrid deviations are excluded due to the individual rationality assumption following the same logic as as discussed above for sole source deviations.

ii) $\frac{\underline{\Theta}}{2\overline{\Theta}} < C < \frac{\underline{\Theta}}{\overline{\Theta}+\underline{\Theta}}$:

In contrast to pure deviations, the bids of the deviating bidder are influencing each other, as a low bid for the sole source award can lower the probability of winning the split award and vice versa. Because all other bidders follow the equilibrium strategy, only bidders with cost types $\hat{\Theta} \le \overline{\Theta}C + \hat{p}^\sigma(\hat{\Theta})$ have positive probability of winning the sole source award. The lower bound $2p_e^\sigma(\underline{\Theta}) \le \hat{p}^s(\hat{\Theta})$ emerges, as all lower deviating sole source prices are dominated. The same logic applies for deviating split prices, which are bounded below by $p_e^\sigma(\underline{\Theta})$ and above by $\overline{\Theta}C$. The expected payoff of such deviations $(\hat{p}^s(\hat{\Theta}), \hat{p}^\sigma(\hat{\Theta}))$ is

$$E[\hat{\Pi}] = (\hat{p}^s(\hat{\Theta}) - \hat{\Theta})P(\hat{p}^s(\hat{\Theta}) < \min\{p_e^\sigma(\Theta_{1:n-1}) + \min\{p_e^\sigma(\Theta_{2:n-1}), \hat{p}^\sigma(\hat{\Theta})\}, \max\{\Theta_{1:n-1}, p_e^\sigma(\Theta_{1:n-1}) + \overline{\Theta}C\}\}) \\ + (\hat{p}^\sigma(\hat{\Theta}) - C\hat{\Theta})P(\hat{p}^\sigma(\hat{\Theta}) < p_e^\sigma(\Theta_{2:n-1}) \wedge \hat{p}^s(\hat{\Theta}) \ge \hat{p}^\sigma(\hat{\Theta}) + p_e^\sigma(\Theta_{1:n-1}))$$

We define $\hat{p}^s(\hat{\Theta}) = p_e^\sigma(x_1) + p_e^\sigma(x_2)$ and $\hat{p}^\sigma(\hat{\Theta}) = p_e^\sigma(x_2)$ with $x_1 \in [\underline{\Theta}, \overline{\Theta}], x_2 \in [\hat{\Theta}, \overline{\Theta}]$ and $p_e^\sigma(x_1) + p_e^\sigma(x_2) > 0$. It is known that

$$\begin{aligned}
C &< \frac{\underline{\Theta}}{\underline{\Theta} + \overline{\Theta}} \\
\overline{\Theta}C &< \underline{\Theta}(1-C) \\
p_e^\sigma(x_1) &< \Theta(1-C) \text{ as } p_e^\sigma(x_1) \qquad \leq \overline{\Theta}C \text{ and } \underline{\Theta} \leq \Theta \\
p_e^\sigma(x_2) + p_e^\sigma(x_1) - \Theta &< p_e^\sigma(x_2) - \Theta \ \ | + p_e^\sigma(x_2)
\end{aligned}$$

We want to find x_1 and x_2 such that the following expected payoff function is maximized:

$$\begin{aligned}
E[\hat{\Pi}(\hat{\Theta})] = &(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta})P(p_e^\sigma(x_1) + p_e^\sigma(x_2) < \min\{p_e^\sigma(\Theta_{1:n-1}) + \min\{p_e^\sigma(\Theta_{2:n-1}), p_e^\sigma(x_2))\}, \\
&\max\{\Theta_{1:n-1}, p_e^\sigma(\Theta_{1:n-1}) + \overline{\Theta}C\}\}) \\
&+ (p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) < p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) + p_e^\sigma(x_2) \geq p_e^\sigma(x_2) + p_e^\sigma(\Theta_{1:n-1}))
\end{aligned}$$

For deviations with $x_1 \geq x_2$ we can show that

$$\begin{aligned}
E[\hat{\Pi}(\hat{\Theta})] <&(p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})) \\
&+ (p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) \geq p_e^\sigma(\Theta_{1:n-1})) \\
\leq&(p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1})) \\
\leq&E[\Pi(\hat{\Theta})]
\end{aligned}$$

If assumption 4 applies, deviations with $x_1 < x_2$ can be excluded. Q.E.D.

If one wants to test assumption 4 for a specific setting, it is better to use the stricter condition

$$\begin{aligned}
E[\hat{\Pi}(\hat{\Theta})] <&(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})) \\
&+ (p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_1) \geq p_e^\sigma(\Theta_{1:n-1})) \\
<&(p_e^\sigma(x_1) + p_e^\sigma(x_2) - \hat{\Theta})P(p_e^\sigma(x_1) < p_e^\sigma(\Theta_{1:n-1})) \\
&+ (p_e^\sigma(x_2) - C\hat{\Theta})P(p_e^\sigma(x_2) \leq p_e^\sigma(\Theta_{2:n-1}) \wedge p_e^\sigma(x_2) \geq p_e^\sigma(\Theta_{1:n-1})) \ | \ \text{because } x_1 < x_2
\end{aligned}$$

and check, whether it is lower than the equilibrium payoff for all possible cost types and all possible combinations of x_1 and x_2.

A.8. Proof of Proposition 9

Assume there is a hybrid equilibrium with cost intervals $I_1^s, I_2^s, ..., I_t^s$ and $I_1^\sigma, I_2^\sigma, ..., I_u^\sigma$, for which the sole source award, respectively the split award, is the equilibrium outcome when the interval includes the two lowest cost draws of the n competitors. Furthermore, $t, u \in \mathbb{N}$ and

$$I_1^s \cup I_2^s \cup ... \cup I_t^s \cup I_1^\sigma \cup I_2^\sigma \cup ... \cup I_u^\sigma = [\underline{\Theta}, \overline{\Theta}]$$

applies. The functions $p_i^s(\Theta)$ for $i \in 1,...,t$ and $p_j^\sigma(\Theta)$ for $j \in 1,...,u$ are the relevant equilibrium prices for the sole source and split awards in the intervals I_i^s and I_i^σ, respectively. Then, every possible hybrid equilibrium must include a strategic parameter $\tau_1 \in (\underline{\Theta}, \overline{\Theta})$, for which the equilibrium results in the same award, when all bidders have cost draws higher than τ. Without loss of generalization, we assume for the proof that $I_1^s = [\underline{\Theta}, \tau_1]$ and $I_1^\sigma = (\tau_1, \overline{\Theta}]$. Then, the following conditions have to apply for a hybrid equilibrium:

(i) $p_1^s(\tau_1) = 2p_1^\sigma(\tau_1)$

(ii) $p_1^s(\tau_1) - \tau_1 = p_1^\sigma(\tau_1) - C\tau_1$

(iii) $p_1^\sigma(\tau_1) \leq \overline{\Theta}C$

The first two conditions for hybrid equilibria have been established by Anton et al. (2010) for the FPSB auction and two bidders. When all cost types belong to I_1^s, the auctioneer choses the sole source award in equilibrium; the split is selected, when all cost types are in I_1^σ. Then, the auctioneer must be indifferent between both awards in the case that all bidders have the same cost type τ_1, as the price functions $p_e^s(\Theta)$ and $p_e^\sigma(\Theta)$ are increasing and continuous. Furthermore, a bidder must be indifferent of winning the sole source award or the split award, when his cost parameter is τ_1. Otherwise, he would not change his strategy for this cost type in equilibrium. The third condition is a standard requirement for split prices in equilibrium with more than two bidders. [17]

In what follows, we show that these three assumptions can never be met simultaneously with DSE:

Combining (i) and (ii) results in $p_1^\sigma(\tau_1) = \tau_1(1-C)$. Hence, with (iii)

$$p_1^\sigma(\tau_1) \leq \overline{\Theta}C$$
$$C \geq \frac{\tau_1}{\tau_1 + \overline{\Theta}}$$

must apply. This is never true for $\tau_1 \in (\underline{\Theta}, \overline{\Theta})$ with DSE. Q.E.D.

A.9. Proof of Proposition 10

In order to prove sequential rationality, it has to be shown that there are no payoff-dominant sole source or split deviations in phase 1 as well as in phase 2. In contrast to the setting with two bidders, the equilibrium strategies for both phases are increasing. Hence, the losers of phase 1 have full information about the cost type of the winner of phase 1, while their cost types remain private. This updating process has to be considered for the derivation of the equilibrium strategies.

Split deviations: The equilibrium bidding strategy in phase 1 and 2 is similar to the equilibrium strategy in an ex-ante split-award auction, in which two times 50% are auctioned off sequentially in two FPSB auctions. The winner of the first auction cannot participate in the auction for the remaining 50% share,[18] which is the reason why only deviations for the split award are possible. Hence, a strategy which maximizes

[17] This is different to the case with two bidders, in which split prices are constant and can be greater than or equal to $\overline{\Theta}C$. In such a setting, hybrid equilibria cannot be excluded.

[18] Otherwise it would not be an ex-ante split-award auction.

the expected payoff in a sequential ex-ante split-award auction, also excludes all split deviations in the Dutch ex-post split-award auction. In proposition 15.1 in (Krishna 2009, p. 217), an equilibrium in a sequential multi-unit (forward) auction is characterized, which can be easily transformed to an ex-post split-award auction in our setting. Both settings are comparable, as we will see below that the winner of the first 50% share cannot win the remaining 50% share due to DSE and the assumption of individual rationality. Hence, the following equilibrium emerges:

Bidders accept the counteroffer for the split award in phase 1 at a price of

$$p_e^{\sigma 1}(\Theta) = \frac{\int_{\Theta}^{\overline{\Theta}} p_e^{\sigma 2l}(t)(n-1)(1-F(t))^{n-2} f(t)dt}{(1-F(\Theta))^{n-1}}.$$

For phase 2, bidders are asymmetric, because $n-1$ losers and one winner of phase 1 compete for the remaining 50% share. The losers of phase 1 approve the counteroffer for the 50% share at a price of

$$p_e^{\sigma 2l}(\Theta, \Theta_w) = C\Theta + C\frac{\int_{\Theta}^{\overline{\Theta}} (1-F(t))^{n-2} dt}{(1-F(\Theta))^{n-2}}.$$

in equilibrium.

Additionally, we have to define a strategy for the winner of phase 1, which is off-equilibrium. When, for example, supplier w wins 50% of the business in phase 1 and is also the winner of the remaining 50% share in phase 2, the auctioneer pays him the sum of both split prices. Due to individual rationality this sum must be at least as high as the costs for 100% of the business, Θ_w. Furthermore, the overall payoff of bidder w has to be at least as high as the payoff in phase 1, as bidders are assumed to be payoff-maximizing and otherwise the strategy would not be a credible threat. Therefore, bidder w can only accept counteroffers in phase 2, which are higher than his additional costs $(1-C)\Theta_w$. This is the case at a price of

$$p_e^{\sigma 2w}(\Theta_w) = (1-C)\Theta_w.$$

A loser of phase 1 with cost type Θ_l faces the following maximization problem in phase 2:

$$\begin{aligned}
\max_z E[\Pi(z)] &= (p_e^{2\sigma l}(z, \Theta_w) - \Theta_l) P(p_e^{2\sigma l}(z, \Theta_w) < \min\{p_e^{2\sigma l}(\Theta_{1:n-2}, \Theta_w), p_e^{\sigma 2w}(\Theta_w)\} \mid \Theta_{1:n-2} \geq \Theta_w) \\
&= (p_e^{2\sigma l}(z, \Theta_w) - \Theta_l) \frac{(1-F(z))^{n-2}}{(1-F(\Theta_w))^{n-2}}
\end{aligned}$$

As $(1-C)\Theta_w > p_e^{\sigma 2l}(\Theta, \Theta_w)$ applies for all $\Theta, \Theta_w \in [\underline{\Theta}, \overline{\Theta}]$ with DSE, the $n-1$ losers know that Θ_w never wins the remaining share in equilibrium. Otherwise it would not be a a σ equilibrium and this is also the reason, why $\overline{\Theta}C$ and not $p_e^{\sigma 2w}(\Theta_w)$ is the upper limit for $p_e^{\sigma 2l}(\Theta, \Theta_w)$. Nevertheless, the beliefs about the cost types of the $n-1$ losers are updated after phase 1, as every supplier knows that the costs of every loser of phase 1 cannot be lower than Θ_w, which is identical to the ex-ante format discussed by Krishna (2009). One can easily check that inserting the equilibrium strategy $p_e^{2\sigma l}(\Theta_l, \Theta_w)$ with $z = \Theta_l$ from above maximizes the expected payoff.

The expected equilibrium payoffs in phase 2 has to be considered for the derivation of the strategy in phase 1 as well. Because the winner of phase 1 never wins the remaining share in phase 2, the same logic of the proof of ex-ante split-award auctions yields $p_e^{\sigma 1}(\Theta)$.

However, there could be profitable sole source deviations, which are only possible in the ex-post format. In what follows, we show that such deviations do not exist with DSE.

Sole source deviations: In phase 1, a bidder can deviate from equilibrium by accepting the 100% share before the first 50% is awarded. Such a sole source deviation is only possible for types $\hat{\Theta} \in [\underline{\Theta}, 2\overline{\Theta}C]$ and has to satisfy

$$2p_e^{\sigma 1}(\underline{\Theta}) \leq \hat{p}^s(\Theta) \leq 2\overline{\Theta}C.$$

The probability of winning the auction by accepting a counteroffer for 100% at a price of $2p_e^{\sigma 1}(\underline{\Theta})$ is 1. Therefore, no price which is lower than this bound can yield a higher expected payoff and such deviations can be neglected. There is an upper bound for sole source deviations, because the split is accepted by a bidder at the latest in round r with counteroffers $c_r^s = 2\overline{\Theta}C$ and $c_r^\sigma = \overline{\Theta}C$.

Because all price functions are continuous, the deviating sole source price $\hat{p}^s(\hat{\Theta})$ can be expressed by $\hat{p}^s(\hat{\Theta}) = 2p_e^{\sigma 1}(\Theta)$ with $\Theta \in [\underline{\Theta}, \overline{\Theta}]$. In what follows we show that the expected payoff for a sole source deviation is strictly lower than the expected payoff in a σ equilibrium:

$$\begin{aligned}
E[\hat{\Pi}^s(\hat{\Theta})] &= (\hat{p}^s(\hat{\Theta}) - \hat{\Theta})P(\hat{p}^s(\hat{\Theta}) < 2p_e^{\sigma 1}(\Theta_{1:n-1})) \\
&= (2p_e^{\sigma 1}(\Theta) - \hat{\Theta})P(2p_e^{\sigma 1}(\Theta) < 2p_e^{\sigma 1}(\Theta_{1:n-1})) \\
&= (2p_e^{\sigma 1}(\Theta) - \hat{\Theta})P(p_e^{\sigma 1}(\Theta) < p_e^{\sigma 1}(\Theta_{1:n-1})) \\
&= (p_e^{\sigma 1}(\Theta) - \hat{\Theta}C + p_e^{\sigma 1}(\Theta) - (1-C)\hat{\Theta})(1-F(\Theta))^{n-1} \\
&< (p_e^{\sigma 1}(\Theta) - \hat{\Theta}C)(1-F(\Theta))^{n-1} + \underbrace{(\overline{\Theta}C - (1-C)\hat{\Theta})}_{<0, \text{ as } C < \frac{\hat{\Theta}}{\hat{\Theta}+\overline{\Theta}}}(1-F(\Theta))^{n-1} \\
&< (p_e^{\sigma 1}(\Theta) - \hat{\Theta}C)(1-F(\Theta))^{n-1} \\
&\leq \max_{\Theta}(p_e^{\sigma 1}(\Theta) - \hat{\Theta}C)(1-F(\Theta))^{n-1} \\
&= E[\Pi_e^\sigma(\hat{\Theta})].
\end{aligned}$$

The winner w of phase 1 has the chance to deviate for the sole source award in phase 2. We know that in equilibrium the highest possible price for the auctioneer is $C\overline{\Theta}$, which determines the upper bound for any sole source deviation. However, accepting the split for this price is unprofitable regardless of the cost type of bidder w, because the additional costs for producing 100% of the business are not covered at this price.

$$C\overline{\Theta} < (1-C)\Theta_w$$
$$C < \frac{\Theta_w}{\Theta_w + \overline{\Theta}}$$

This applies for all Θ_w with DSE. Q.E.D.

A.10. Proof of Corollary 4

The expected payoff in a σ equilibrium, $E[\Pi_e^\sigma(\Theta)] = \underline{\Theta}(1-C) - C\Theta$, must be for all cost types $\Theta \in [\underline{\Theta}, \overline{\Theta}]$ greater than or equal to the expected payoff of any other possible equilibrium. Condition (i) is necessary in order to achieve payoff dominance over all σ equilibria. The derivation of the expected payoff function used in condition (ii), which assures payoff dominance of the split over hybrid equilibria with parameter τ, can be found in (Anton et al. 2010). Condition (iii) is adapted from proposition 5 of Anton and Yao (1992).

A.11. Proof of Corollary 6

It is known that truthful bidding, i.e. $C\Theta_i$ for 50% as well as Θ_i for 100% of the business is a dominant strategy for bidders $i \in \{A, B\}$ in a VCG mechanism. Hence, the price of the auctioneer can be calculated as

$$\begin{aligned}
p^b_{VCG}(\Theta_A, \Theta_B) &= \Theta_A C + (\Theta_B - (\Theta_A C + \Theta_B C)) + \Theta_B C + (\Theta_A - (\Theta_A C + \Theta_B C)) \\
&= (\Theta_A + \Theta_B)(1-C) \\
&\geq 2\underline{\Theta}(1-C) \\
&= p^b_\sigma(\Theta_A, \Theta_B) \text{ for all } \Theta_A, \Theta_B \in [\underline{\Theta}, \overline{\Theta}]
\end{aligned}$$

For every possible combination of cost types the sum of the VCG prices is lower than or equal to the purchasing costs in a payoff-dominant σ equilibrium in one of the three first-price split-award auctions analyzed above. Q.E.D.

A.12. Proof of Proposition 11

For the following proofs, we need the ex-interim expected payoffs for a cost type Θ

$$E[\Pi_e^s(\Theta)] = \int_\Theta^{\overline{\Theta}} (1-F(x))^{n-1} dx \text{ in a WTA equilibrium and}$$
$$E[\Pi_e^\sigma(\Theta)] = C \int_\Theta^{\overline{\Theta}} (1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2} dx \text{ in a } \sigma \text{ equilibrium.}$$

In what follows, it is shown that for all possible settings with DSE, regarding the parameters C, F, $\underline{\Theta}$, $\overline{\Theta}$ and n, there is always an interval $(\Theta^*, \overline{\Theta}]$ for which the σ equilibrium yields a higher payoff than the WTA equilibrium, i.e., for which

$$E[\Pi_e^s(\Theta)] < E[\Pi_e^\sigma(\Theta)] \text{ for all } \Theta \in (\Theta^*, \overline{\Theta}]$$
$$E[\Pi_e^s(\Theta)] - E[\Pi_e^\sigma(\Theta)] < 0 \text{ for all } \Theta \in (\Theta^*, \overline{\Theta}].$$

We show that this is true for high cost types:

$$
\begin{aligned}
\lim_{\Theta\to\overline{\Theta}} E[\Pi_e^s(\Theta)] - E[\Pi_e^\sigma(\Theta)] &< 0 \\
\lim_{\Theta\to\overline{\Theta}} \int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1}dx - C\int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2}dx &< 0 \\
\lim_{\Theta\to\overline{\Theta}} \frac{\int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1}dx}{\int_{\Theta}^{\overline{\Theta}} (1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2}dx} &< C \quad \| \frac{0}{0} \text{ L'Hospital} \\
\lim_{\Theta\to\overline{\Theta}} \frac{-(1-F(t))^{n-1}}{-(1-F(t))^{n-1} - (n-1)F(t)(1-F(t))^{n-2}} &< C \\
\lim_{\Theta\to\overline{\Theta}} \frac{1}{1+(n-1)\frac{F(t)}{1-F(t)}} &< C \\
0 &< C
\end{aligned}
$$

Because this inequality applies for all possible C, the σ equilibrium always yields more payoff than the WTA equilibrium for cost types in an interval $(\Theta^*, \overline{\Theta}]$.
As a result, a WTA equilibrium cannot be payoff-dominant for all cost types with DSE. Q.E.D.

A.13. Proof of Corollary 7

When the split equilibrium is payoff-dominant for an arbitrary setting with DSE, more than two bidders and uniformly distributed cost parameters with support $[\underline{\Theta}, \overline{\Theta}]$, it must apply that

$$E[\Pi_e^s(\Theta)] - E[\Pi_e^\sigma(\Theta)] < 0 \ \forall \Theta \in [\underline{\Theta}, \overline{\Theta}].$$

We know from proposition 11 that the σ equilibrium yields higher expected payoff than the WTA equilibrium for bidders with high cost types. Therefore, it suffices to show that for a bidder with cost type $\Theta = \underline{\Theta}$, the WTA equilibrium yields a higher expected profit than the σ equilibrium.

$$
\begin{aligned}
E[\Pi_e^s(\underline{\Theta})] - E[\Pi_e^\sigma(\underline{\Theta})] &> 0 \\
\int_{\underline{\Theta}}^{\overline{\Theta}} (1-F(x))^{n-1}dx - C\int_{\underline{\Theta}}^{\overline{\Theta}} (1-F(x))^{n-1} + (n-1)F(x)(1-F(x))^{n-2}dx &> 0 \\
(1-C)\int_{\underline{\Theta}}^{\overline{\Theta}} \frac{(\overline{\Theta}-x)^{n-1}}{(\overline{\Theta}-\underline{\Theta})^{n-1}}dx - C\int_{\underline{\Theta}}^{\overline{\Theta}} (n-1)\frac{(x-\underline{\Theta})(\overline{\Theta}-x)^{n-2}}{(\overline{\Theta}-\underline{\Theta})^{n-1}}dx &> 0 \quad \| *(\overline{\Theta}-\underline{\Theta})^{n-1} \\
(1-C)\left[-\frac{(\overline{\Theta}-x)^n}{n}\right]_{\underline{\Theta}}^{\overline{\Theta}} - C(n-1)\left[-\frac{(\overline{\Theta}-x)^n}{n(n-1)}\right]_{\underline{\Theta}}^{\overline{\Theta}} &> 0 \\
\underbrace{(1-2C)}_{>0}\underbrace{\frac{(\overline{\Theta}-\underline{\Theta})^n}{n}}_{>0} &> 0
\end{aligned}
$$

Hence, for small cost types, a WTA equilibrium yields higher payoff than a σ equilibrium. Q.E.D.

Integration by parts:

$$\int_{\underline{\Theta}}^{\overline{\Theta}} (x-\underline{\Theta})(\overline{\Theta}-x)^{n-2}dx = \left[-\frac{(\overline{\Theta}-x)^{n-1}}{n-1}(x-\underline{\Theta})\right]_{\underline{\Theta}}^{\overline{\Theta}} - \int_{\underline{\Theta}}^{\overline{\Theta}} -\frac{(\overline{\Theta}-x)^{n-1}}{n-1}dx$$
$$= 0 + \left[-\frac{(\overline{\Theta}-x)^{n}}{n(n-1)}\right]_{\underline{\Theta}}^{\overline{\Theta}}$$

A.14. Proof of Corollary 8

We show that the expected price for the auctioneer in the σ equilibrium is lower than in the σ equilibrium, when either C is sufficiently low or the cost types are uniformly distributed with DSE.

First, assume $C < \frac{\underline{\Theta}}{2\overline{\Theta}}$. Then

$$E[p_b^\sigma] = 2CE[\Theta_{3:n}] < E[\Theta_{2:n}] = E[p_b^{WTA}]$$
$$C < \frac{E[\Theta_{3:n}]}{2E[\Theta_{2:n}]}$$

The function $f(x,y) = \frac{x}{2y}$ is decreasing in y and increasing in x. Because $E[\Theta_{2:n}] < \overline{\Theta}$ and $E[\Theta_{3:n}] > \underline{\Theta}$, the inequality applies. Q.E.D.

Second, with a uniform distribution the expectation value of the k-th lowest order statistic $\Theta_{k:n}$ can be expressed by

$$E[\Theta_{k:n}] = \underline{\Theta} + \frac{k}{n+1}(\overline{\Theta}-\underline{\Theta}).$$

Hence, we can show that

$$E[p_b^\sigma] = 2CE[\Theta_{3:n}] < E[\Theta_{2:n}] = E[p_b^{WTA}]$$
$$2C\left(\underline{\Theta} + \frac{3}{n+1}(\overline{\Theta}-\underline{\Theta})\right) < \underline{\Theta} + \frac{2}{n+1}(\overline{\Theta}-\underline{\Theta})$$
$$C < \frac{\underline{\Theta} + \frac{2}{n+1}(\overline{\Theta}-\underline{\Theta})}{2\underline{\Theta} + \frac{6}{n+1}(\overline{\Theta}-\underline{\Theta})} \text{ RHS } \nearrow 0.5 \text{ with } n \nearrow \text{inf}$$

Therefore, it suffices to show that this applies for $n = 3$ and $C = \frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}}$:

$$\frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}} < \frac{\underline{\Theta} + \frac{1}{2}(\overline{\Theta}-\underline{\Theta})}{2\underline{\Theta} + \frac{3}{2}(\overline{\Theta}-\underline{\Theta})}$$
$$\frac{\underline{\Theta}}{\underline{\Theta}+\overline{\Theta}} < \frac{\underline{\Theta}+\overline{\Theta}}{\underline{\Theta}+3\overline{\Theta}}$$
$$3\overline{\Theta}\underline{\Theta} + \underline{\Theta}^2 < \underline{\Theta}^2 + 2\overline{\Theta}\underline{\Theta} + \overline{\Theta}^2$$
$$0 < \overline{\Theta}(\overline{\Theta}-\underline{\Theta})$$

Q.E.D.

Appendix B: Supporting Documentation for the Reviewing: Statistical Analysis and Lab Environment

This appendix contains fixed-effects regressions of bids and prices for bidders in split and sole-source awards for all treatments in subsection B.1. Moreover, fixed-effects regressions of the allocation (split vs. sole-source award) are included. We also added the univariate regressions of bids and prices on the cost draws and plotted them in subsection B.2. Finally, plots of split-award winner bids and prices over periods of all treatments are attached to provide insight into the development of bidding behavior with repeated interaction in auctions. We also added plots to visualize the distribution of allocations over periods for the two-bidder treatments.

B.1. Appendix: Regression Tables

In tables 5 to 10 the dependent variable is depicted in the left column and the intercept as well as all independent variables in the columns to the right. For each dependent variable we provide estimates for the coefficients of all explanatory variables for the univariate regression on cost draws _X and the fixed-effects regression _X(P). The corresponding p-values are included in brackets below the coefficients. The columns "subjects" and "subjects per period" contain the number of subjects with significant (at least at the 5% level) fixed-effects and fixed-effects over the periods of the experiment, respectively. The column "period" is the fixed-effect over periods for the reference subject and included for completeness. NA-values for the latter column occur if a subject ends up in the corresponding allocation only once. The right outermost column contains the R^2 as a measurement for the explanatory power of the linear regression model. Note that we used a logistic regression for the fixed-effects model of the binary outcome allocation with ajusted measurement of explanatory power McFadden R^2.

allocation	**intercept**	**#subjects**	**period**	**#subjects per period**	**McFadden R^2**		
FPSB_2_A	0.82151	2 (all negative)/47	-0.15915	3 (all positive)/48	0.15533		
	(0.4737)	*	(0.2383)	*			
Split Bidders							
single-unit bid	**intercept**	**teta**	**double-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_S_S	51.88973	0.08566					
	(1.04e-13)***	(0.117)					
FPSB_2_S_S(P)	34.49385	0.0571	0.07806	8 (all positive)/46	0.35459	2 (one positive and negative)/46	0.5286
	(8.66e-05)***	(0.242545)	(0.012520)*	*	(0.633163)	*	
double-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_S_D	91.11576	0.44934					
	(7.21e-13)***	(9.22e-06)***					
FPSB_2_S_D(P)	62.887305	0.481243	0.368754	3 (all positive)/46	0.094282	2 (all negative)/46	0.3685
	(0.001058)**	(3.50e-06)***	(0.012520)*	*	(0.953452)	*	
Sole-Source Bidders							
single-unit bid	**intercept**	**teta**	**double-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_SS_S	100.8732	-0.1387					
	(8.7e-10)***	(0.303)					
FPSB_2_SS_S(P)	81.1726	-0.3662	0.444	5 (4 positive, 1 negative)/46	3.4386	11 (1 positive, 10 negative)/46	
	(6.98e-05)***	(0.001212)**	(2.85e-12)***	*	(0.033868)*	*	
double-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
FPSB_2_SS_D	51.50754	0.7907					
	(1.18e-05)***	(6.31e-15)***					
FPSB_2_SS_D(P)	1.70465	0.83653	0.35606	7 (all positive)/46	-1.25169	1 (positive)/46	
	(0.926719)	(<2e-16)***	(2.85e-12)***	*	(0.390034)	*	

Table 5 FPSB (n=2) Regressions

In the fixed-effects regression FPSB_2_S_S(P) in Table 5 the cost draw is not a statistically significant explanatory variable for the single-unit bid of split award winners whereas each bidder's double-unit bid possesses relevant explanatory power. In regression FPSB_2_S_D(P) the double-unit bid strictly increases in costs and the single-unit bid has explanatory power. In the fixed-effects regressions FPSB_2_SS_S(P) and FPSB_2_SS_D(P) the single- and double unit bids of sole-source award bidders are statistically significant increasing in the bid for the alternative number of units. The latter possesses more explanatory power than for the split award winners which corresponds to our conjectures about the different relations of single- and double-unit bids for split award winners and sole-source bidders. The negative influence of the cost parameter in FPSB_2_SS_S(P) might be explained by a large number of sole-source bidders, who submit high bids on one unit to exclude the split award.

allocation	**intercept**	**#subjects**	**period**	**#subjects per period**	**McFadden R^2**		
DU_2_A	1.90367	0	-0.013362	0/49	0.1415133		
	(0.2575)	*	(0.9398)	*			
Split Bidders							
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_S_1	51.20217	0.19497					
	(3.22e-08)***	(0.00854)**					
DU_2_S_1(P)	52.2339	0.14209		4 (1 positive, 3 negative)/47	4.51003	2 (1 positive, 1 negative, 2 NA)/47	0.6171
	(1.93e-05)***	(0.01678)*		*	0.10722	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_S_2	62.10713	0.12452					
	(5.98e-12)***	(0.0767)					
DU_2_S_2(P)	46.61149	0.06771	0.50969	25 (24 positive, 1 negative)/47	-0.71126	8 (6 positive and 2 negative, 3 NA)/47	0.8001
	(9.74e-09)***	(0.088325)	(<2e-16)***	*	(0.142994)	*	
Sole-source Winners							
direct double-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_D	95.6622	0.5888					
	(0.00108)**	(0.02207)*					
DU_2_SS_D(P)	122.4552	0.09057		6 (all positive)/33	1.03293	1 (positive, 14 NA)/33	0.6203
	(0.00176)**	(0.76205)		*	(0.46129)	*	
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_1	6.4632	0.6052					
	(0.733072)	(0.000589)***					
DU_2_SS_1(P)	-6.74E+00	5.44E-01		0/30	2.22E+00	0 (16 NA)/30	0.7367
	(0.913)	(0.149)		*	(0.535)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_2_SS_2	18.9572	0.5672					
	(0.212)	(7.56e-05)***					
DU_2_SS_2(P)	54.87064	-0.09605	1.06658	0/30	-3.48774	0 (16 NA)/30	0.7518
	(0.3252)	(0.7897)	(0.0773).	*	(0.3008)	*	

Table 6 Dutch-Dutch (n=2) Regressions

In regression DU_2_S_1(P) in Table 6 the price of the first unit is increasing in the cost parameter. Considering that the average price is above the equilibrium prediction. This suggests that high cost bidders let the price rise higher above 70 as compared to their low cost counterparts. Interestingly, the price for the second unit in the regression model DU_2_S_2(P) does not significantly depend on the cost draw but on the height of the price for the first unit. Especially, the last observation is in line with our explanation of the bidding behavior in the split awards of the Dutch auction with two bidders. Regarding the second unit, bidders appear to be influenced much more by the signal of their counterpart rather than by their own cost type. Eight subjects even increased the price for the second unit statistically across the periods to make even more profit (see figure 19).

In the fixed-effects regressions there are almost no independent variables with explanatory power. This can be explained by our conjecture that sole-source awards occur when bidders let the price of the first and/or the second unit rise too high. Similar to the split regression, the model DU_2_SS_2(P) describes sole-source winners who obtain two single units consecutively. The price for the second unit is influenced by the price of the first unit. If split-award winners hesitate to accept a second-unit price below the first-unit price, sole-source winners are even more likely to do so.

allocation	**intercept**	**#subjects**	**period**	**#subjects per period**	**McFadden R^2**		
DUSB_2_A	-6.82E-03	0/49	2.59E-01	1 (negative)/49	0.1907833		
	(0.9961)	*	(0.2414)	*			
Split Bidders							
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_S_W_1	51.01613	0.09005					
	(3.53e-15)***	(0.0776)					
DUSB_2_S_W_1(P)	51.05587	0.13715		34 (almost all negative)/49	0.26856	26 (almost all positive, 2 NA)/49	0.8192
	(<2e-16)***	(1.83e-07)***		*	(0.512747)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
winner of first unit	49.61836	0.32714					
DUSB_2_S_W_2	(7.18e-07)***	(7.33e-05)***					
DUSB_2_S_W_2(P)	19.79858	0.446	0.124	4 (all positive)	0.07968	6 (4 negative, 2 positive, 2 NA)/49	0.4933
	(0.254066)	(1.11e-08)***	(0.537509)	*	(0.943477)	*	
winner of second	48.96511	0.14544					
DUSB_2_S_L_2	(2.61e-11)***	(0.0127)*					
DUSB_2_S_L_2(P)	4.83E+01	1.20E-01	1.81E-01	23 (almost all positive)/49	-2.73E-01	15 (mix, 1 NA)/49	0.7046
	(1.88e-09)***	(0.002013)**	(0.014269)*	*	(0.628269)	*	
Sole-source Winners							
direct double-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_D	83.8373	0.4807					
	(0.00769)**	(0.07491)					
DUSB_2_SS_D(P)	-10.7642	1.3082		0/13	-0.5096	0 (8 NA)/13	0.415
	(0.8837)	(0.1137)		*	(0.8631)	*	
first single-unit bid	**intercept**	**teta**		**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_1	22.2237	0.3737					
	(0.2648)	(0.0381)*					
DUSB_2_SS_1(P)	6.783	0.4602		0/25	1.517	0 (16 NA)/25	0.8669
	(0.9588)	(0.6678)		*	(0.2148)	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DUSB_2_SS_2	0.5297	0.6516					
	(0.974)	(4.21e-05)***					
DUSB_2_SS_2(P)	216.8512	-2.3889	2.4789	0/25	-2.8574	0 (16 NA)/25	0.9321
	(0.224)	(0.174)	(0.123)	*	(0.201)	*	

Table 7 Dutch-FPSB (n=2) Regressions

In the fixed-effects regressions for the Dutch-FPSB auction in table 7 we observe almost identical relationships between prices and explanatory variables than in the Dutch format. However, in the split-award regression models DUSB_2_S_W_1(P), DUSB_2_S_W_2(P) and DUSB_2_S_L_2(P) the cost draw has statistically significant positive influence on the dependent variable. This is not surprising as with lower prices, that are not supported by tacit collusion to the same extent as in the Dutch auction, bidders are inclined to take into account their single-unit costs. Moreover, the price for the first unit is not a reliable predictor for the bids on the second unit by winners of the first unit. As depicted in the right plot of figure 10 they bid much higher to not risk winning the respective unit and sustain the split. In the left and right plot of this figure the univariate regression lines lie within the predicted boundaries. The prices and bids of sole-source winners are illustrated in figures 11 and 12.

Note, however, that contrary to the Dutch format, 26 winners of the first unit in split-award allocations significantly increase the price for the respective unit over the number of periods as is shown in figure 21. Regarding the second unit 15 winners substantially adapt the price at which they accept although not all of them let the price rise higher. The distribution of allocations does not change over the periods as illustrated in plot 22. These observations might indicate that subjects manage to overcome the initial uncertainty with respect to the second period and eventually end up in a similar strong tacit collusion as described for the Dutch auction and even support prices above equilibrium.

single-unit bid	**intercept**	**teta**	**1/(teta-60)**	**double-unit bid**	**#subjects**	**period**	**#subjects over period**	**adjusted R^2**
FPSB_3_S	2.09512	0.32766						
	(0.753)	(5.53e-09)***						
FPSB_3_S(P)	-7.92E+01	5.75E-01	1.38E+03	2.08E-01	0/34	-8.71E-02	3 (2 positive, 1 negative)/34	0.3942
	(0.117803)	(0.044607)*	(0.142947)	(8.70e-10)***	*	(0.903771)	*	
double-unit bid	**intercept**	**teta**		**single-unit bid**	**#subjects**	**period**	**#subjects over period**	**adjusted R^2**
FPSB_3_D	17.3928	0.9504						
	(0.243)	(8.1e-14)***						
FPSB_3_D(P)	17.35231	0.7494		0.39953	2 (all positive)/34	0.05135	3 (all positive)/34	0.7784
	(0.1358)	(<2e-16)***		(7.05e-10)***	*	(0.95894)	*	

Table 8 FPSB (n=3) Regressions

For the split award with three bidders the fixed-effects regressions in tables 8 to 10 contain a cost parameter with significant explanatory power that is positively correlated with the corresponding dependent variable.

first single-unit bid	intercept	teta	single-unit bid	#subjects	period	#subjects per period	adjusted R^2
DU_3_1	26.9637	0.10899					
	(<2e-16)***	(5.24e-14)***					
DU_3_1(P)	26.12313	0.092455		23 (all positive)/47	0.229195	12 (all negative, 6 NA)/47	0.6826
	(<2e-16)***	(1.09e-14)***		*	(0.028266)*	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects per period**	**adjusted R^2**
DU_3_2	25.18748	0.12198					
	(<2e-16)***	(<2e-16)***					
DU_3_2(P)	15.03993	0.120899	0.253872	3 (3 positive)/48	0.023405	2 (all negative, 3 NA)/48	0.6084
	(9.50e-08)***	(<2e-16)***	(2.09e-05)***	*	(0.84849)	*	

Table 9 Dutch-Dutch (n=3) Regressions

first single-unit bid	intercept	teta		#subjects	period	#subjects over period	adjusted R^2
DUSB_3_W_1	18.61513	0.16846					
	(<2e-16)***	(<2e-16)***					
DUSB_3_W_1(P)	17.888482	0.185402		10 (all positive)/49	-0.20185	8 (all positive, 2 NA)/49	0.7758
	(<2e-16)***	(<2e-16)***		*	(0.03235)*	*	
second single-unit bid	**intercept**	**teta**	**single-unit bid**	**#subjects**	**period**	**#subjects over period**	**adjusted R^2**
winner of first unit	45.426	0.4004					
DUSB_3_W_2	(0.0126)*	(0.0123)*					
DUSB_3_W_2(P)	8.6789	0.32287	1.08283	9 (7 positive, 2 negative)/49	-0.39624	7 (4 positive, 3 negative, 2 NA)/49	0.8237
	(0.651303)	(0.052737).	(0.148866)	*	(0.623098)	*	
winner and loser of second	7.2542	0.28409					
DUSB_3_L_2	(0.238)	(2.28e-08)***					
DUSB_3_L_2(P)	-5.794843	0.26957	0.282631	7 (all positive)/46	-0.042096	6 (1 positive, 5 negative)/46	0.5344
	(0.584099)	(6.51e-10)***	(0.161048)	*	(0.956202)	*	

Table 10 Dutch-FPSB (n=3) Regressions

Also, in regressions FPSB_3_S(P) and FPSB_3_D(P) the height of the single- and double-unit bid significantly depends on the height of the bid for two and one units, respectively. The lower a bidder's competitive bid for one unit the more willing he is to submit a low bid on two units and vice versa as would be expected in a competitive outcome.

Interestingly, in panel regression DU_3_2(P) for the second unit in the Dutch auction the price of the first unit still contains high explanatory power whereas this is not the case for the respective regression DUSB_3_L_2(P) in the Dutch-FPSB format. To explain this observation note that in the Dutch auction 12 subjects significantly decrease the price at which they accept the first unit over the periods. It appears that if some bidders accepted the first unit at a higher price in the earlier periods then the price of the second unit was also accepted at higher prices. This connection then vanishes with more periods. In contrast, in the Dutch-FPSB format eight bidders let the price at which they accept the first unit rise significantly higher with more periods. These bidders have accepted the first unit at rather low prices in early periods which did not constitute a relevant signal for the price of the second unit. Generally, bidder behavior is rather constant over periods as is depicted in plots 23 to 25.

B.2. Figures

In the plots of bids and prices against cost draws in figures 5 to 16 we include the solid univariate regression lines and the dashed equilibrium strategies. If there are no significant fixed and period effects in the linear regressions of bids and prices the univariate regression on the cost draws helps to visualize the subjects' underlying bidding behavior.

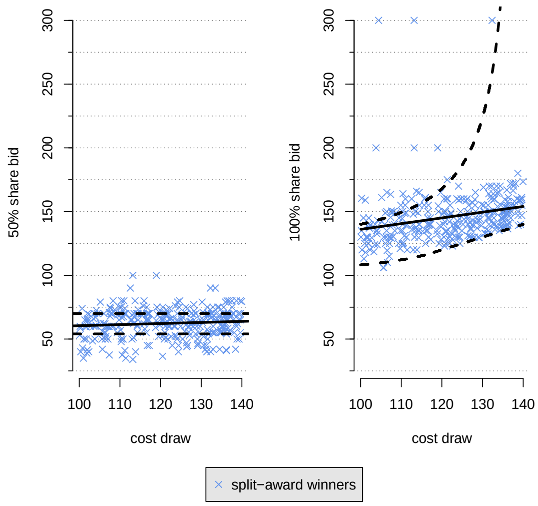

Figure 5 **Bids of Split-Award Winners in FPSB (n=2)**

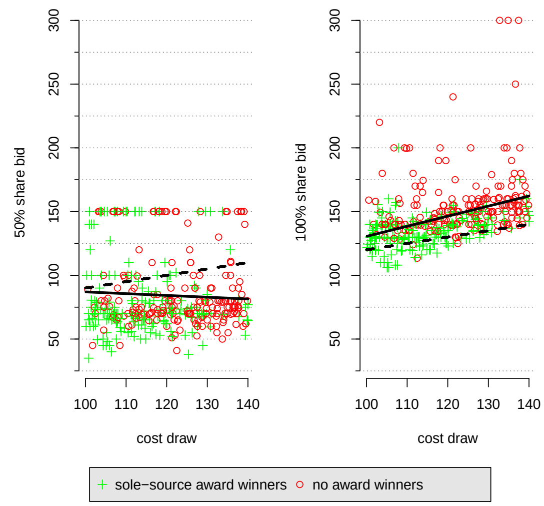

Figure 6 **Bids of Sole-Source-Award Winners in FPSB (n=2)**

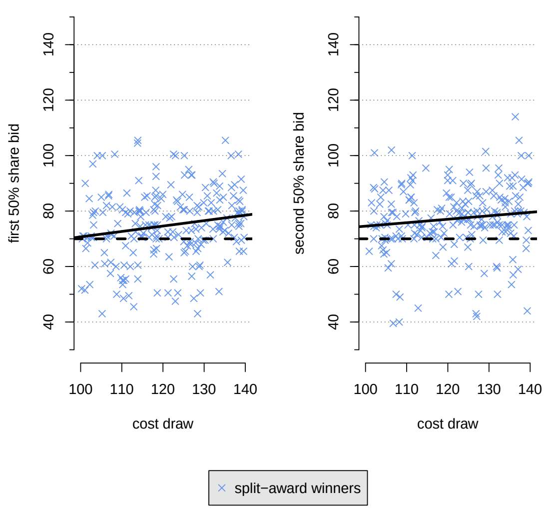

Figure 7 **Bids of Split-Award Winners in Dutch-Dutch (n=2)**

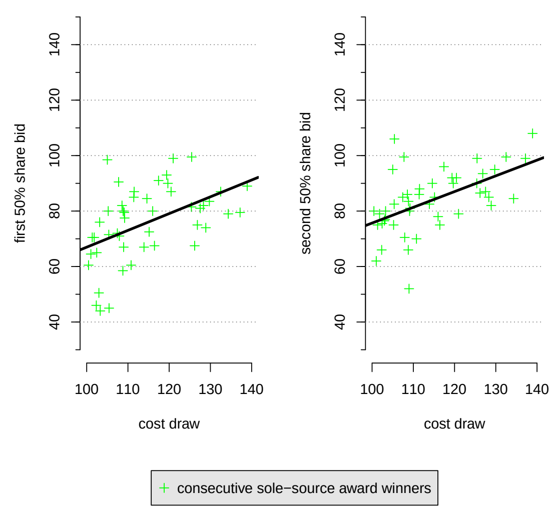

Figure 8 **Bids of Consecutive Sole-Source-Award Winners in Dutch-Dutch (n=2)**

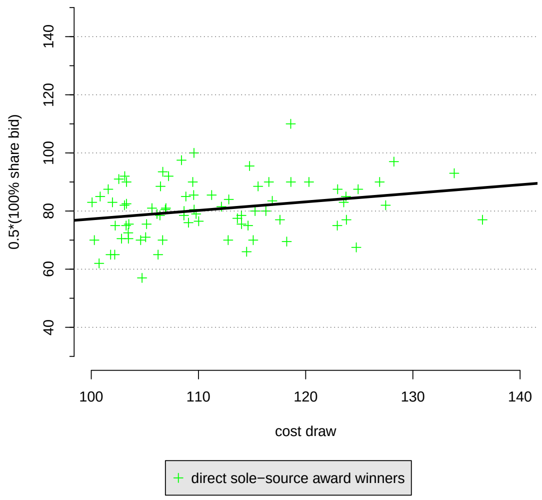

Figure 9 **Bids of Direct Sole-Source-Award Winners in Dutch-Dutch (n=2)**

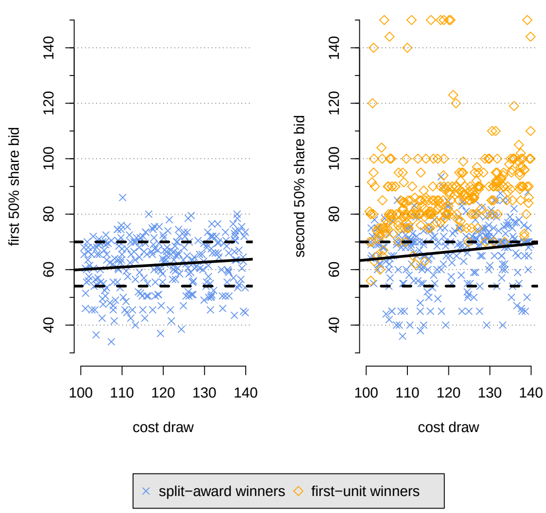

Figure 10 **Bids of Split-Award Winners in Dutch-FPSB (n=2)**

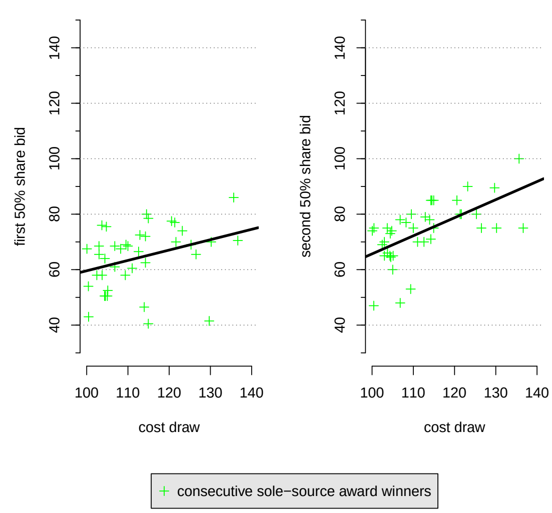

Figure 11 **Bids of Consecutive Sole-Source-Award Winners in Dutch-FPSB (n=2)**

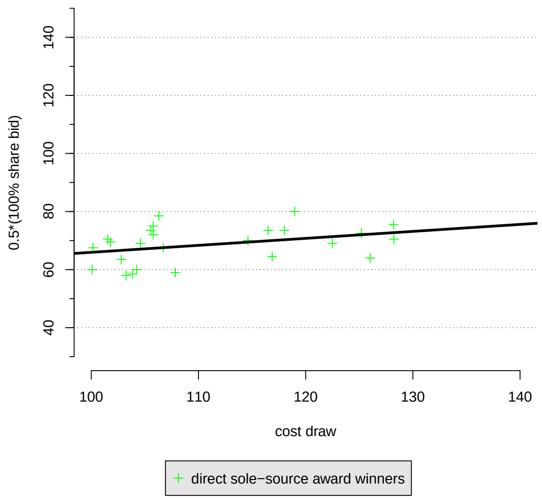

Figure 12 **Bids of Direct Sole-Source-Award Winners in Dutch-FPSB (n=2)**

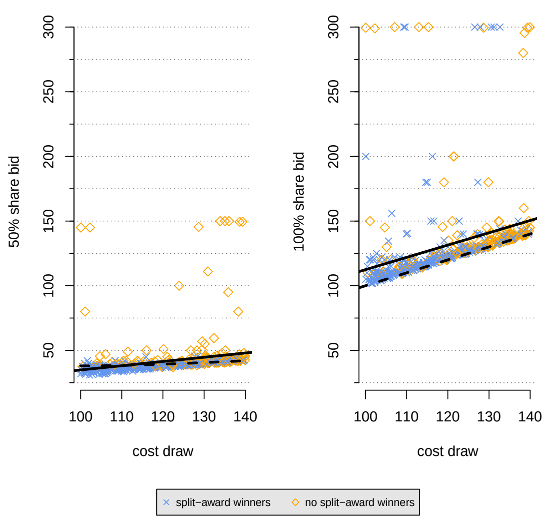

Figure 13 **Bids of Split-Award Bidders in FPSB (n=3)**

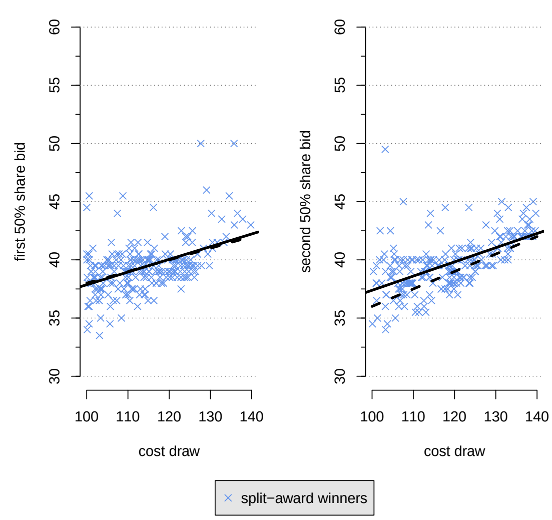

Figure 14 **Bids of Split-Award Bidders in Dutch-Dutch (n=3)**

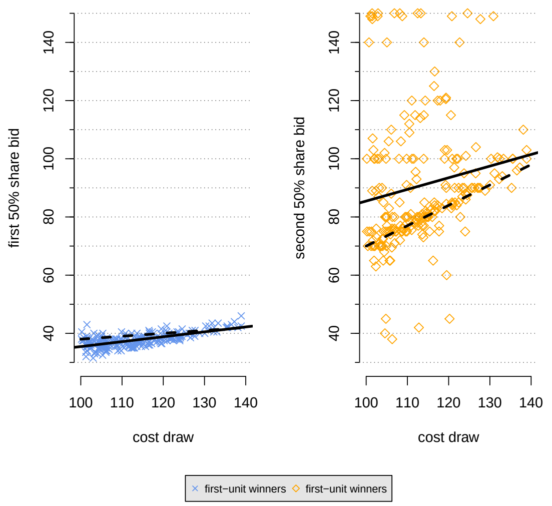

Figure 15 **Bids of First-Unit Split-Award Winners in Dutch-FPSB (n=3)**

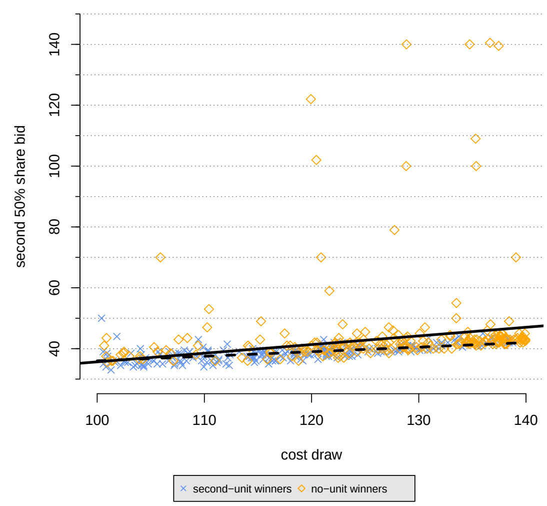

Figure 16 **Bids of Second-Unit Split-Award Winners in Dutch-FPSB (n=3)**

B.3. Period Plots

In the following plots of bids and prices against periods the solid black line represents the average bid/price in period 15 and the solid grey line depicts the average bid/price over all periods. For the two-bidder treatments we included dashed lines for the range of constant pooling prices. Furthermore, for the treatments with two bidders we also added plots of the distribution of allocations against periods.

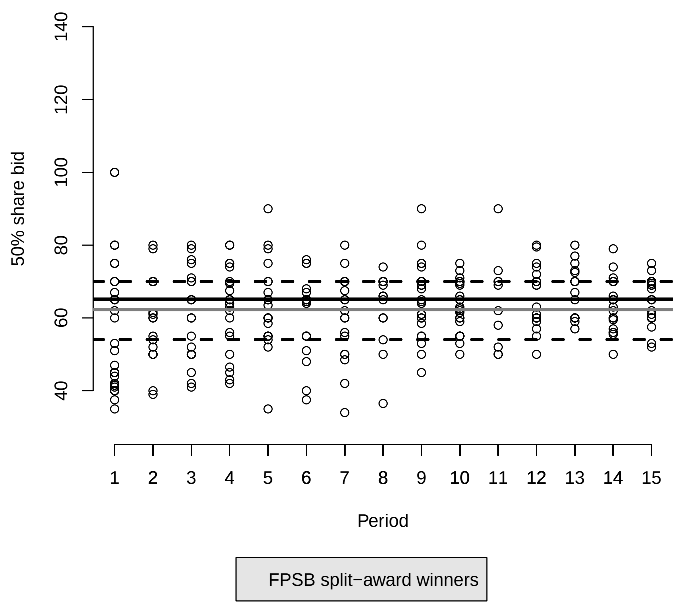

Figure 17 Bids of Split-Award Winners in FPSB (n=2)

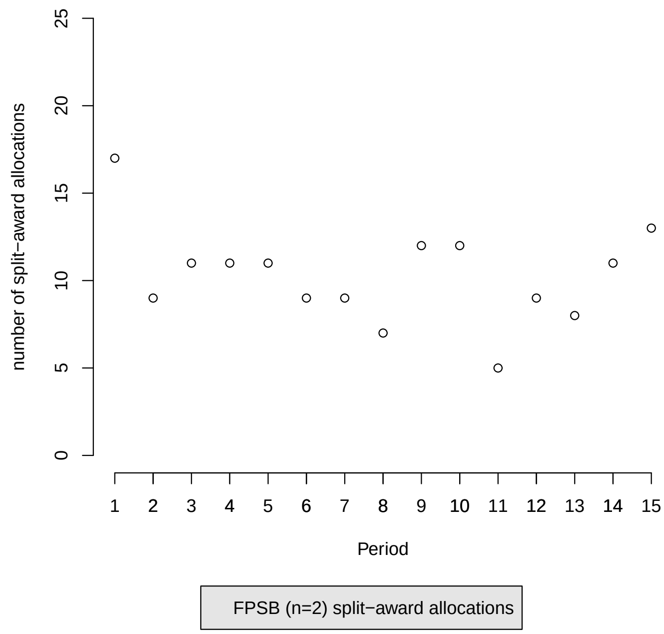

Figure 18 **Distribution of allocations in FPSB (n=2)**

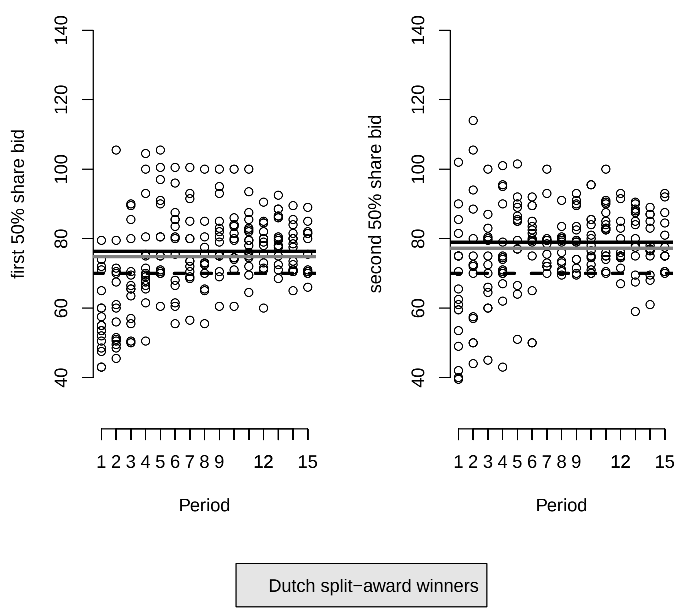

Figure 19 **Prices of Split-Award Winners in Dutch-Dutch (n=2)**

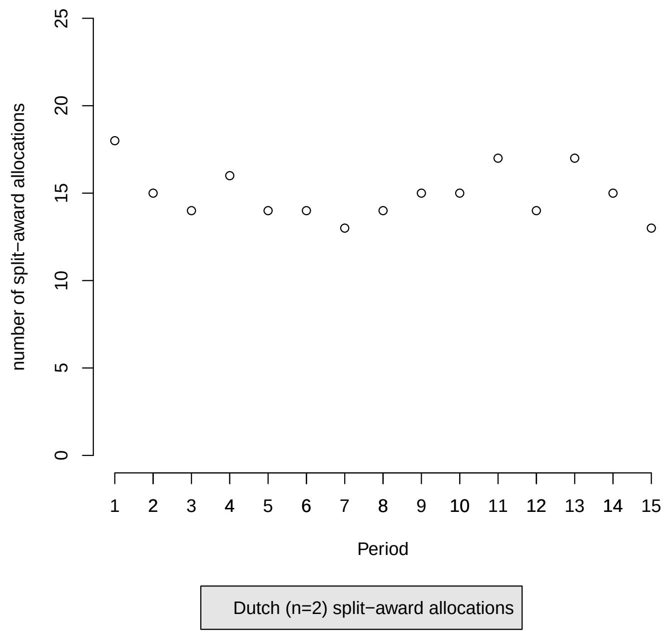

Figure 20 **Distribution of allocations in Dutch (n=2)**

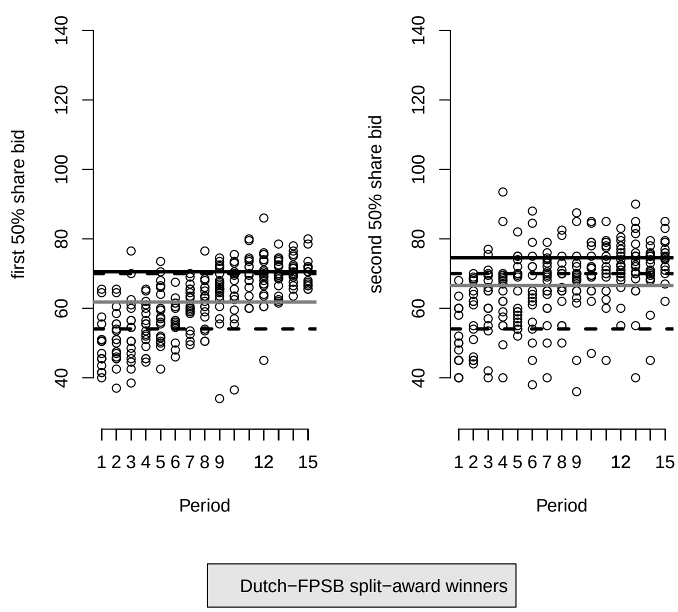

Figure 21 **Prices of Split-Award Winners in Dutch-FPSB (n=2)**

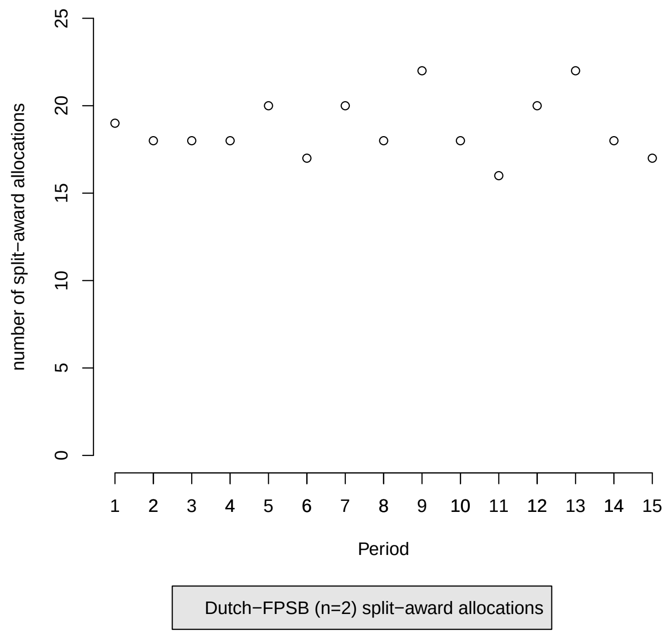

Figure 22 **Distribution of allocations in Dutch-FPSB (n=2)**

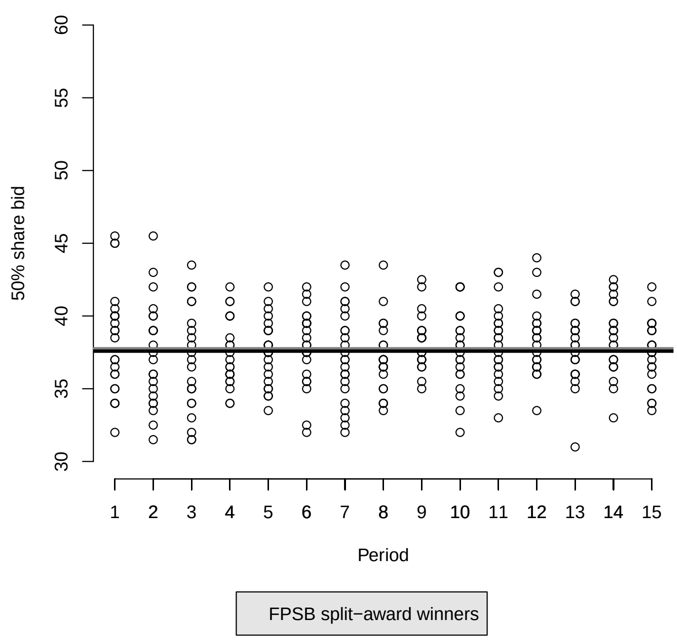

Figure 23 **Bids of Split-Award Winners in FPSB (n=3)**

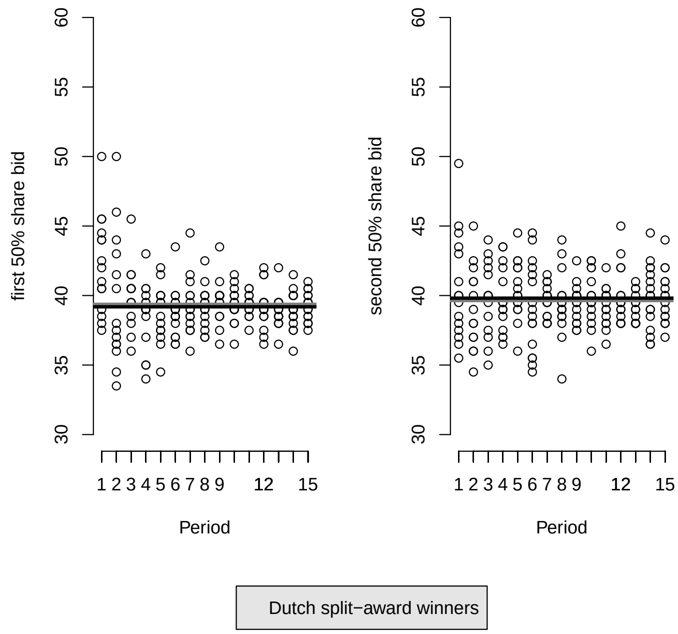

Figure 24 **Prices of Split-Award Winners in Dutch-Dutch (n=3)**

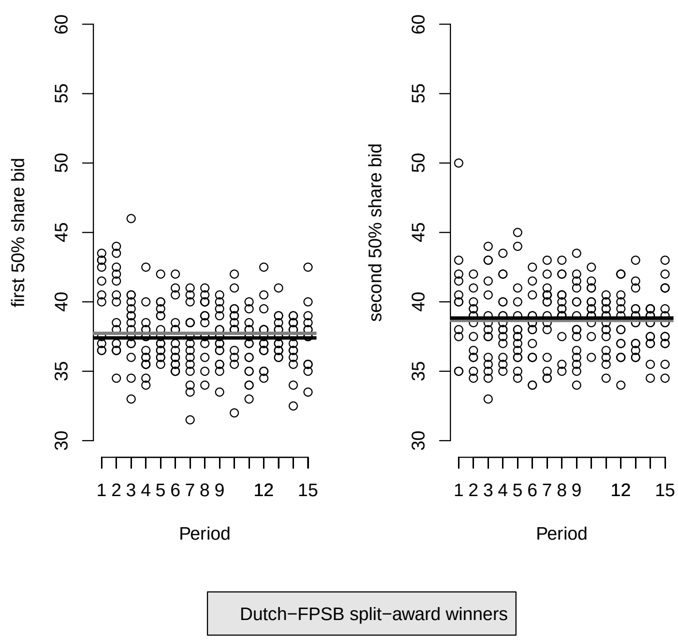

Figure 25 **Prices of Split-Award Winners in Dutch-FPSB (n=3)**

In the following we attach screenshots of the graphical user interface via which the subjects interacted with each other and were presented with the relevant information during the different auction treatments. To avoid redundancies we only provide screenshots for the two-bidder treatments. As our experiments were conucted in German we provide English translations below the respective screens.

B.4. Screenshots in the FPSB Auction

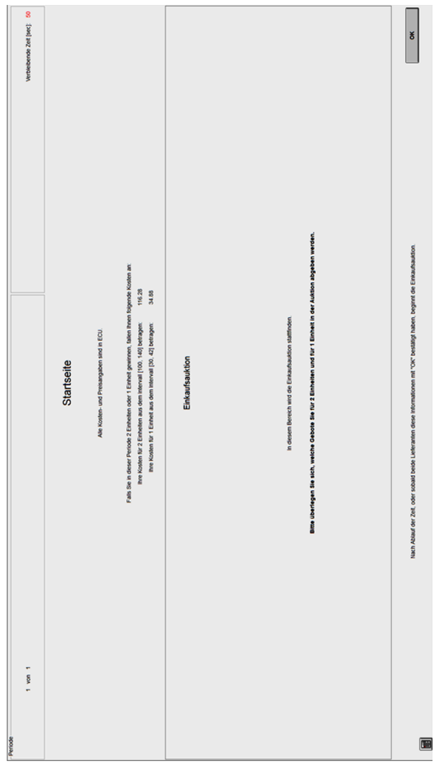

Figure 26 Starting Screen in FPSB Treatments

The bidders are informed about their own costs for the supply of 50% or 100% of a fictitious order. Moreover, a text on the screen indicates where the procurement auction, i.e., the submission of bids is going to take place.

Each bidder is allowed to submit one bid for the 50% share and one bid for the 100% share of the order.

After the termination of the auction each bidder is informed about the winning prices and his own profit.

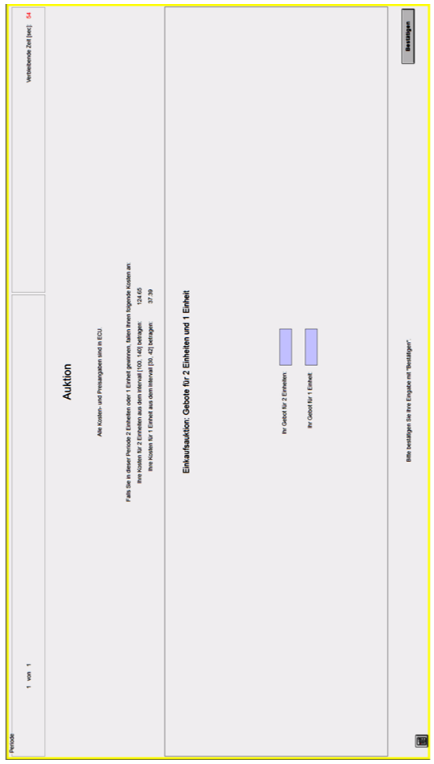

Figure 27 Bid Submission Screen in FPSB Treatments

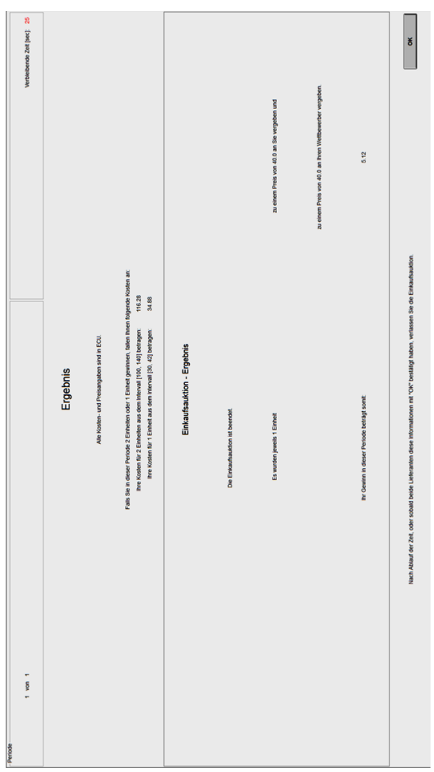

Figure 28 Result Screen in FPSB Treatments

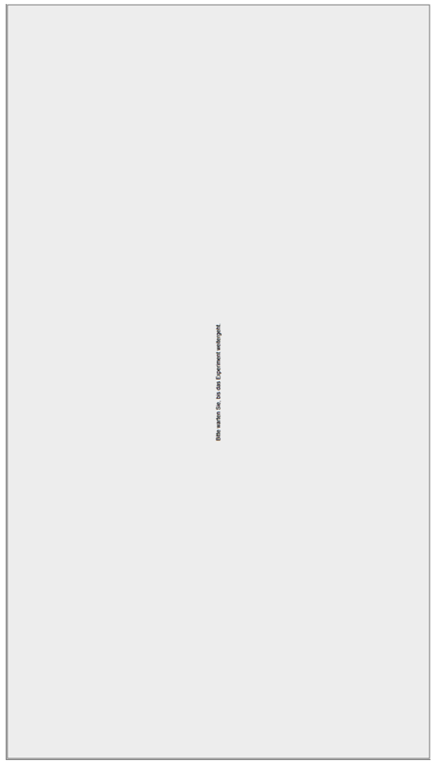

Figure 29 Waiting Screen in all Treatments

B.5. Screenshots in the Dutch Auction

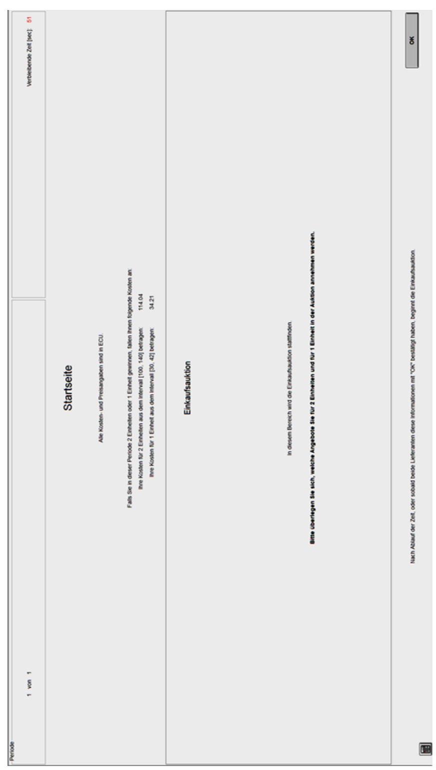

Figure 30 Starting Screen in Dutch Treatments

Each bidder is informed about his costs for the provision of the 50% or the 100% share. A text on the screen indicates where the split-award auction takes place, i.e., where the increasing prices are located.

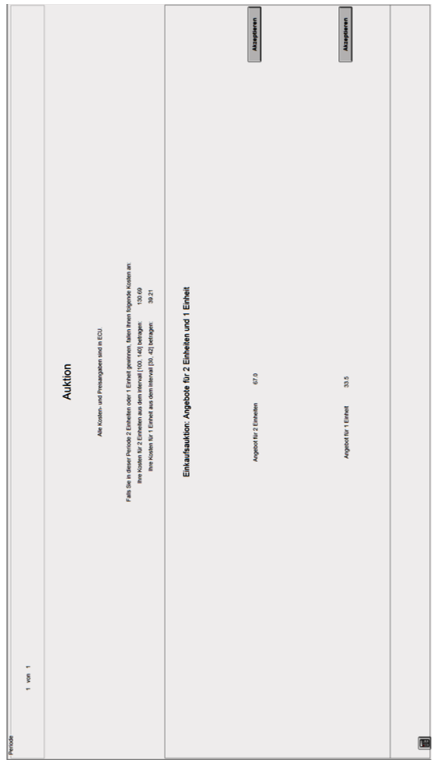

Figure 31 **Screen with Increasing Prices in Dutch Treatments**

The rising prices for the 50% share and the 100% share can be accepted via buttons next to them.

In case the 50% share was accepted the bidders are informed about the winner and the winning price.

The price for the remaining 50% share increases publicly and can be accepted via the button next to it.

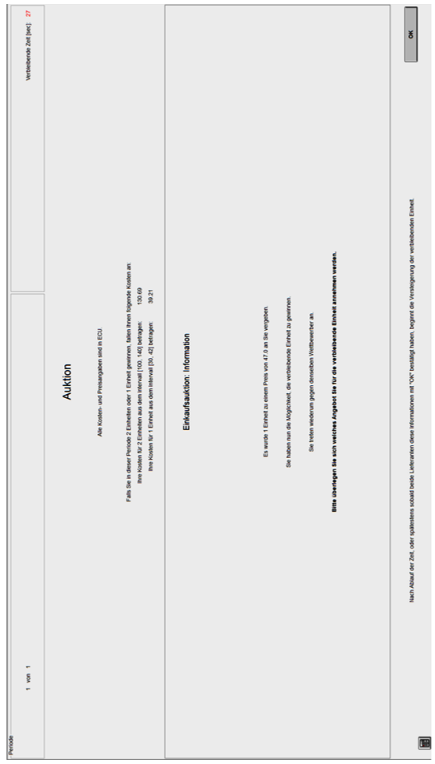

Figure 32 Result Screen for first unit in split of Dutch Treatments

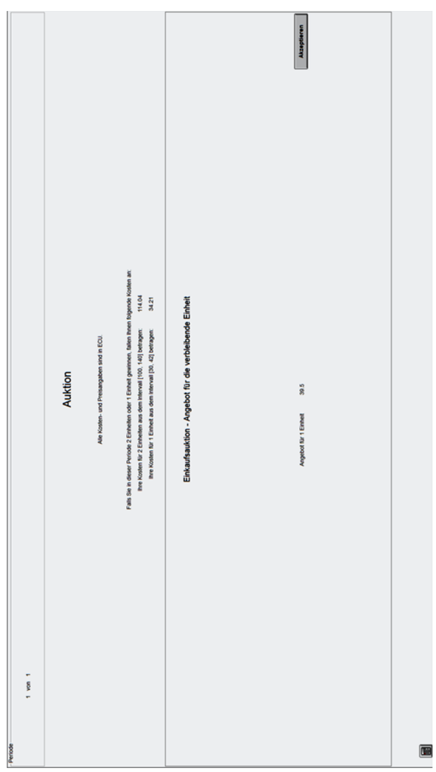

Figure 33 Screen with Increasing Price for remaining unit in split of Dutch Treatments

B.6. Screenshots in the Dutch-FPSB Auction

The Dutch-FPSB auction only differs from the Dutch-Dutch format in case the remaining unit is auctioned off via a sealed-bid stage in the split award.

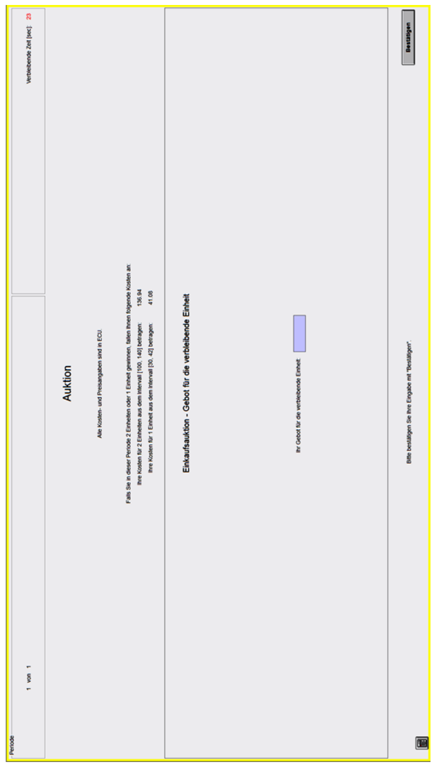

Figure 34 Bid Submission Screen for remaining unit in split of Dutch-FPSB Treatments

All bidders are allowed to submit one bid for the remaining 50% share of the order.