Split-Award Procurement Auctions – Can Bayesian Equilibrium Strategies Predict Human Bidding Behavior in Multi-Object Auctions?

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We analyze if and when symmetric Bayes Nash equilibrium predictions can explain human bidding behavior in multi-object auctions. We focus on two sealed-bid split-award auctions with ex-ante split decisions as they can be regularly found in procurement practice. These auction formats are straightforward multi-object extensions of the first-price sealed-bid auction. We derive the risk-neutral symmetric Bayes Nash equilibrium strategies and find that, although the two auction mechanisms yield the same expected costs to the buyer, other aspects of the two models, including the equilibrium bidding strategies, differ significantly. The strategic considerations in these auction formats are more involved than in single-lot first-price sealed-bid auctions, and it is questionable whether expected utility maximization can explain human bidding behavior in such multi-object auctions. Therefore, we analyzed the predictive accuracy of our equilibrium strategies in the lab. In human subject experiments we found underbidding, which is in line with earlier experiments on single-lot first-price sealed-bid auctions. In order to control for regret we organize experiments against computerized bidders, who play the equilibrium strategy. In computerized experiments where bid functions are only used in a single auction, we found significant underbidding on low-cost draws. In experiments where the bid function is reused in 100 auctions, we could also control effectively for risk aversion, and there is no significant difference of the average bidding behavior and the risk-neutral Bayes Nash equilibrium bid function. The results suggest that strategic complexity does not serve as an explanation for underbidding in split-award procurement auctions, but risk aversion does have a significant impact.

Keywords: multi-object procurement auction, Bayes Nash equilibrium, experiments

1. Introduction

A growing number of papers over recent years have focused on the design of auctions for multiple non-identical objects. The simultaneous multi-round auction, for example, and, more recently, combinatorial auction designs have been used for selling spectrum licenses worldwide (Cramton 2013). Similar auction design problems regularly arise in industrial procurement and logistics where multiple heterogeneous goods or services need to be purchased (Cramton, Shoham et al. 2006).

In this paper, we focus on *split-award auctions*, a simple but widespread type of multi-object auction often used for multi-sourcing, which is a typical requirement in procurement negotiations. Companies such as Sun and HP, for example, procure products worth hundreds of millions of dollars using different types of multiple sourcing auctions (Elmaghraby 2000, Tunca and Wu 2009). These firms usually want to have more than one supplier for risk considerations. In global business often a new supplier with cheaper but possibly unreliable technology enters the marketplace to win orders from firms by beating the price of their reliable but more expensive competitors (Gurnani, Gumus et al. 2012). Split-award auctions can be used to make sure the new entrant does not win the entire quantity and the second source with the required production facilities is available in case the new entrant defaults.

We focus on equilibrium bidding strategies in two forms of sealed-bid auction formats that we frequently encounter in procurement practice for dual sourcing. The lots in such auctions could be the 30% and the 70% share of the demand for a particular raw material, and the buyers choose the split ex-ante and restrict the suppliers to submitting either one single bid or two bids on the two lots. More specifically, we analyze two first-price sealed-bid procurement auctions for auctioning the two lots of a product with different sizes at one go. The first auction model is the so-called *Yankee auction*, in which each bidder just submits one bid for the unit price of the product for which they bid. Then, the bidder with the lowest bid wins the large lot and the bidder with the second lowest bid wins the small lot. The second split-award sealed-bid auction we study is an extension called the *parallel auction*, in which each bidder submits one bid for the unit price of the product for each lot and the bidder with the lowest bid on each lot wins that lot. Each bidder can win at most one lot. Such auction formats with an ex-ante split are easy to implement for procurement managers, and it is important for them to understand the bidding strategies in such auctions.

Corey (1978), Woodside and Vyas (1987), and Seshadri, Chatterjee et al. (1991) have already discussed such split-award contracts with pre-defined splits in a variety of industries, and such auctions are common in e-sourcing nowadays. Interestingly, we are neither aware of a game-theoretical analysis nor of experimental work on these auction formats. Note that the auction formats analyzed in this paper are different from the split-award auctions analyzed in Anton and Yao (1989, 1992) and Anton et al. (2010), where suppliers submit bids on each possible split of a contract and the entire quantity and the split choice is endogenous. Our paper is also different from Perry and Sákovics (2003), who study a setting in which the buyer commits to a split ex-ante, but the items are then sold in a sequential second-price auction. Recently, Gong, Li et al. (2012) assume a single-bid second-price split-award auction with an ex-ante split similar to the Yankee auction in our paper, but their model focuses on incentives of suppliers to invest. We contribute the theoretical analysis and Bayes Nash characterization of these split-award auctions with a predefined split, which is different from the one with endogenous determination of market structure, and analyze to which extent such models have predictive power in lab experiments.

Bayes Nash equilibrium analysis is the standard approach to model sealed-bid auctions and a lot of recent research has tried to extend this type of analysis to multi-object auctions (Krishna 2009). The risk-neutral Bayes Nash equilibrium (RNBNE) of multi-object auctions is technically much more challenging than that of single-lot auctions and, as of yet, there are only a small number of papers deriving RNBNE strategies for specific combinatorial or non-combinatorial multi-object auction formats (Goeree and Lien 2009, Ausubel and Baranov 2010, Sano 2012). Actually, so far there is no Bayes Nash equilibrium characterization for first-price sealed-bid combinatorial auctions. Some very recent research uses numerical methods to derive equilibrium predictions, which also shows the difficulties of Bayesian models for multi-object markets. Given the strategic complexity of these multi-object auctions, it is not clear that RNBNE predictions explain human behavior well. The bidders' decision is single-dimensional only concerning the level of bid-shading for first-price sealed-bid auctions of a single object. Bidders in multi-object first-price sealed-bid auctions also need to decide which objects they want to bid on and how much they want to shade their bids in each of the lots. Bid shading is defined by the difference between the bid and the production costs for a lot. In parallel split-award auctions, for example, not only the number of bidders and the prior distribution, but also the split parameter determines the level of bid shading. In the Yankee auction, the bidders also need to take into account the risk of winning the small lot rather than the large lot with a certain bid price.

The predictive accuracy of RNBNE predictions for multi-object auctions in the lab is largely unexplored. However, there is growing literature on first-price sealed-bid auctions of single objects which shows that bidding behavior in the lab deviates substantially from the RNBNE prediction and overbidding is a common phenomenon in sales auctions. In our procurement auction context this means *underbidding* below the RNBNE equilibrium prediction, and we will refer to underbidding and the underbidding phenomenon throughout unless we want to distinguish between the results of sales and procurement auctions in the lab. The first-price auction controversy and the discussions about underbidding already played out in a controversy among experimental economists in the December 1992 issue of the American Economic Review. It raised the question, how to establish sufficient experimental control in order to establish empirical regularities in the laboratory, and how to modify theory in light of countervailing empirical evidence. Ever since, underbidding has been source of substantial research in the experimental auction literature (see Section 3.1). Risk aversion, regret and uncertainty about the rationality of others have served as explanations for underbidding (Engelbrecht-Wiggans and Katok 2009). A number of authors have challenged the overall approach of models based on rational choice and expected utility maximization (Bourdieu 2005, Nell 2007).

Even if bidders are able to mimic their RNBNE strategy in a single-lot auction, it is far from obvious that RNBNE models would still be a good predictor for multi-object auctions. In single-lot auctions, bidders might estimate the right level of bid shading. As described above, split-award auctions are strategically more complex, and it is interesting to understand if bidders are able to master the strategic complexity and mimic their RNBNE strategy at least in a controlled environment. If this is the case, the model can serve as a baseline starting point from which one can add other known phenomena such as risk aversion or regret. However, if the RNBNE strategy does not explain bidding behavior even in a controlled environment, there is little hope that such models would explain bidding behavior in more complex multi-object auctions such as combinatorial auctions. In summary, we try to understand *if, in spite of the increased strategic complexity of split-award auctions, bidders are able to bid according to the RNBNE bid function in a controlled experiment*. Our contribution is two-fold:

Firstly, we derive closed-form increasing Bayes Nash bidding strategies for the Yankee and the parallel auction in the symmetric independent private values model and also compare and contrast the total cost implications. This Bayes Nash characterization has been missing from the growing literature of multi-object auctions and is particularly relevant for procurement. It is interesting that until now there is no closed-form expression of Bayes Nash equilibrium bidding strategies for first-price combinatorial auctions. We find that, although the parallel and the Yankee auction mechanisms yield the same expected costs to the buyer, other aspects of the two models, including the equilibrium bidding strategies as well as winning bidders' ex post profits, differ significantly. Most of the previous studies in this area (Anton and Yao 1992, Armstrong 2000) focus on the comparison of auction mechanisms in terms of the expected revenue, while here, we also compare the different mechanisms in terms of other measures, such as the equilibrium bidding strategies and winning bidders' ex post profits, which are important considerations in real-world procurement practice.

Secondly, we report on lab experiments with split-award procurement auctions testing RNBNE predictions. We designed lab experiments with different levels of control where human bidders compete either against other human bidders or against computerized bidders. The latter are designed to mitigate the impact of behavioral influences, such as risk aversion, regret and inconsistent expectations. In experiments with human subjects, bidders compete in repeated auctions. These experiments are modeled after procurement auctions as they can be found in the field. We observe underbidding for low-cost draws similar to earlier experiments on single-lot first-price sealed-bid auctions. Although the impact of risk aversion should be reduced with many repeated auctions, residual risk aversion, wrong expectations about other bidders or regret can all serve as explanations for this underbidding. Interestingly, we do not find significant underbidding in our computerized experiments, where bidders submit bids only once, but their bids are reused in 100 computerized auctions. There is no significant difference between the average empirical bid functions and the RNBNE bid function in the single-lot and in both split-award auctions when subjects played against computerized bidders. This provides evidence that strategic complexity or wrong expectations cannot serve as an explanation for underbidding. Computerized experiments where the bid function of a bidder is only used once and not in 100 auctions exhibit significant underbidding, which indicates that risk aversion has considerable impact on the bidding strategies. Andreoni, Che et al. (2007) write that "one element of the theory that cannot be replicated in an experiment is the risk neutrality of bidders, for the risk attitudes of the subjects cannot be controlled." The computerized experiments where the bid function is reused in 100 auctions effectively achieve this goal.

The results provide evidence that bidders in the lab behave as expected utility maximizers in our auctions and they are able to mimic the complicated equilibrium strategies with surprisingly high precision in the computerized experiments, if we control for regret and risk aversion. In other words, the RNBNE model describes the basic strategic considerations of bidders in the lab well in the parallel but also in the more complex Yankee auction, and that this is independent of the split parameter. We also show that risk aversion leads to significant underbidding, once we do not control for risk aversion. In the field wrong expectations about others, risk aversion and regret can all influence bidder behavior. The level of underbidding will depend on the frequency of such auctions and bidder idiosyncrasies. Still, we argue that bidders in split-award auctions are able to understand the strategic situation and that the RNBNE model can serve as a useful baseline model for practitioners and future research.

The paper is organized as follows. The two competing procurement auction models are introduced in Section 2, where we also discuss equilibrium bidding strategies and their implications. In Section 3, the experimental design is introduced and the results are presented in Section 4. Finally, Section 5 concludes with a discussion on further extensions of the models. Technical proofs and the experimental instructions are included in the Appendices.

2. Theoretical Models

2.1 The Auctions

A buyer is to procure a given volume of a product using an auction. For exogenous reasons, the buyer divides the total required volume, normalized to be 1 unit, into two lots with q units in lot 1 and $1-q$ units in lot 2. We further assume that $q > 0.5$, and hence, lot 1 is the large lot. There are N risk-neutral bidders competing for the two lots. Bidder i's private constant marginal production cost c_i is identically and independently distributed according to the distribution $F(\cdot)$ with support $[\underline{c}, \bar{c}]$ and density $f(\cdot)$. In addition to the per-unit marginal production cost c_i, each supplier must also incur a fixed cost K to complete the production. Finally, we assume that there is no reserve price.

2.1.1 The Parallel Auction

The rules of the parallel auction are as follows: After observing his private per-unit constant marginal production cost for the product, c_i, each bidder submits two bids, b_i^1 and b_i^2, as the per-unit price for each lot of the products. The lots are awarded to the lowest bid on each lot and the winning bidder gets the contract with the payment that he bids. If one bidder has the lowest bids on both lots, he will be awarded only the large lot.

We will denote the expected payoff to a generic bidder whose private per-unit production cost is c and whose bids are b_1 and b_2 by $\pi(b_1, b_2, c)$. Then

$$\begin{aligned}
\pi(b_1, b_2, c) &= \Pr(\text{bidder } i \text{ wins both Lots}) * \left[\left(b_1 - c \right) q - K \right] \\
&+ \Pr(\text{bidder } i \text{ wins Lot 1 and loses Lot 2}) * \left[\left(b_1 - c \right) q - K \right] \\
&+ \Pr(\text{bidder } i \text{ loses Lot 1 and wins Lot 2}) * \left[\left(b_2 - c \right) (1 - q) - K \right]
\end{aligned} \tag{1}$$

The three terms in equation (1) reflect the three possible outcomes of the auction game other than losing. The first possibility is that a bidder submits the lowest bid on each lot, and hence, could win both lots. However, according the auction rule, he is only awarded the large lot, that is, lot 1. The second possibility is that he only wins the large lot 1, and the last term is the case where he only wins the small lot 2. Conditional on the other bidders' behavior, bidder *i* chooses the bidding strategies by maximizing the expected payoff (1).

2.1.2 The Yankee auction

The Yankee auction works as follows: After observing his private per-unit constant marginal production cost for the product, c_i, each bidder *i* submits only one bid b_i as the per-unit price for both lots of the products. The large lot is then awarded to the bidder with the lowest bid and the small lot is awarded to the bidder with the second lowest bid. The winning bidders are again compensated according to the bids they make. In this auction model, the expected payoff for a generic bidder whose private per-unit production cost is c and whose bid is b is

$$\pi(b,c) = \Pr(\text{bidder } i \text{ wins Lot 1}) * \left[(b-c)q - K \right]$$
$$+ \Pr(\text{bidder } i \text{ loses Lot 1 and wins Lot 2}) * \left[(b-c)(1-q) - K \right] \tag{2}$$

Since in this auction, bidders only submit one bid, there are only two possible outcomes other than losing. The first case is that bidder *i* submits the lowest bid, and hence, wins the large lot. The other possibility is that his bid is the second lowest and he is awarded the small lot. Conditional on the other bidders' behavior, bidder *i* chooses the bidding strategies by maximizing the expected payoff (2).

2.2 Characteristics of the Equilibria

We will discuss the Bayes Nash equilibrium strategies in what follows. All proofs can be found in Appendix 1.

2.2.1 The Parallel Auction

We restrict our attention to symmetric Bayes Nash equilibrium strategies in which bids are continuous, strictly increasing and, almost everywhere, differentiable functions of costs. Let $(\beta_1(c_i), \beta_2(c_i))$ denote the equilibrium pair of bidding strategies for the two lots.

Proposition 1: *In the parallel sealed-bid first-price procurement auction model with two lots and $N > 2$ risk-neutral bidders, the unique symmetric Bayes Nash equilibrium bidding strategies are given by*

$$\beta_1(c) = c + \frac{K}{q} + \frac{1}{q[1-F(c)]} \int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} \{ q[1-F(x)] + (1-q)(N-1)[F(x) - F(c_i)] \} dx$$

$$\beta_2(c) = c + \frac{K}{1-q} + \int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} dx$$

with the boundary conditions 1) $\beta_1(\bar{c}) = \bar{c} + \dfrac{K}{q}$ *and 2)* $\beta_2(\bar{c}) = \bar{c} + \dfrac{K}{1-q}$.

Furthermore, $\beta_1(c)q > \beta_2(c)(1-q)$ *and* $\beta_2(c) > \beta_1(c)$ *if the entry cost K is greater than a threshold* K_0 *where* $K_0 := -\dfrac{(1-q)}{(2q-1)} \displaystyle\int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} \left[\frac{[Nq-(N-1)][F(x)-F(c)]}{[1-F(c)]} \right] dx$.

Proposition 1 calls for a few comments. Firstly, the equilibrium bidding function for the small lot is the same as the bidding strategy in a standard sealed-bid first-price auction with $N-1$ bidders competing for the small lot. Secondly, the bidders bid less aggressively than the equilibrium bid they submit in a standard sealed-bid first-price auction with N bidders competing for the large lot, because they can still win the small lot. This means that bidders need to take into account the possibility of losing the large lot and winning the small lot when they submit bids for the large lot. The strategy is similar to a sequential auction where the large lot is sold first. Additionally, ex post, the winning bidders make positive profits no matter which lot they win. Lebrun (2006) shows that with standard assumptions on the type distributions such an equilibrium is unique and it is characterized as the solution to the system of differential equations corresponding to the first-order conditions.

Proposition 2: *In the parallel sealed-bid first-price procurement auction model with two lots and N risk-neutral bidders,*

(i) As the number of bidders increases, bidding becomes more aggressive for both lots:

$\frac{\partial \beta_1(c)}{\partial N} < 0$ and $\frac{\partial \beta_2(c)}{\partial N} < 0$.

(ii) As q increases, bidding becomes more (less) aggressive for the large lot 1 (the small lot 2):

$\frac{\partial \beta_1(c)}{\partial q} < 0$ and $\frac{\partial \beta_2(c)}{\partial q} > 0$.

2.2.2 The Yankee Auction

As above, we restrict our attention to strictly increasing differentiable symmetric Bayes Nash equilibrium strategies. At such equilibrium, bidder *i* chooses his bid $b_i = \beta(c_i)$ by maximizing his expected payoff.

Proposition 3: *In the Yankee sealed-bid first-price procurement auction model with two lots and N >2 risk-neutral bidders, the unique symmetric Bayesian Nash equilibrium bidding strategies are given by*

$$\beta(c) = c + \frac{\int_c^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c)\right]^{N-1} q + (N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q) \right\}}$$

$$+K \frac{\left\{1+(N-2)F(c)\right\}}{\left\{\left[1-F(c)\right]q + (N-1)F(c)(1-q)\right\}}$$

with the boundary conditions $\beta(\bar{c}) = \bar{c} + \dfrac{K}{1-q}$.

Corollary 1: *In the Yankee sealed-bid first-price procurement auction model, ex post,*
(i) the winner of the large lot always makes a positive profit; and
(ii) the profit for the winner of the small lot can be either positive or negative.

Corollary 1 reveals an important difference between the two models. In the parallel auction mechanism, bidders bid in such a way that, ex post, they always make positive profits no matter which lot is awarded. By contrast, in the Yankee mechanism, ex post, the bidder who wins the small lot will earn either a positive or negative profit depending on his private production costs.

Proposition 4: *In the Yankee sealed-bid first-price procurement auction model with two lots and* N *risk-neutral bidders,*
(i) As N *increases, bidding becomes more (less) aggressive if the fixed cost* K *is greater than (smaller than) a threshold* K_1*:*

$$\frac{\partial \beta(c)}{\partial N} > 0 (< 0) \text{ if } K > (<) K_1(N,c,F)$$

where

$$K_1(N,c,F) := -\frac{\int_{c_i}^{\bar{c}} \left\{A+B+D\right\} dx}{\left[1-F(c)\right]^N F(c)(2q-1)}$$

$$A = \left[F(c)-F(x)\right]\left[1-F(x)\right]^{N-1}\left[1-F(c)\right]q^2$$

$$B = \left[F(c)-F(x)\right]\left[1-F(x)\right]^{N-2} F(x)F(c)(1-q)^2$$

$$D = \left[F(c)-F(x)\right]\left[NF(c)-2(N-1)F(c)F(x)+NF(x)-1\right]\left[1-F(x)\right]^{N-2} q(1-q)$$

(ii) As q *increases, bidding becomes more (less) aggressive if the fixed cost* K *is greater than (smaller than) a threshold* K_2*:*

$$\frac{\partial \beta(c)}{\partial q} > 0 (< 0) \text{ if } K > (<) K_2(N,c,F)$$

where

$$K_2(N,c,F) := \frac{1}{1+(N-2)F(c)} \frac{N-1}{NF(c_i)-1} \int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)}\right)^{N-2} [F(x)-F(c)]dx$$

Comparative statics analysis of the equilibrium bidding strategy for the Yankee model is more involved. Partial derivatives are not of uniform sign throughout the parameter space, but regions where the partial effects are positive and negative can be characterized in terms of thresholds on the magnitude of the fixed cost K.

2.3 Procurement Cost Comparisons

Having derived the symmetric equilibrium bidding strategies in both the parallel and the Yankee auctions, we can now compare the implications of the two mechanisms on expected cost to the buyer. The proof for Proposition 5 can be found in Appendix 1 and it also follows from Engelbrecht-Wiggans (1988) and Krishna (2009, p. 206).

Proposition 5: *Suppose that the private production costs are independently and identically distributed and all bidders are risk-neutral. Then the sealed-bid first-price parallel procurement auction and the sealed-bid first-price Yankee procurement auction yield the same expected cost to the buyer.*

The equivalence result is a useful piece of information for business decision-makers, because in theory they can now look into other dimension of the differences between the two formats and recommend which format to adopt in practice without worrying about the expected costs by using different auctions.

3. Experimental Setup

Predicting strategies of bidders in sealed-bid auctions turned out to be a challenge, as factors such as the uncertainty about the 'rationality' of other bidders, risk aversion, and regret can all play a role. In fact, the literature on single-lot first-price sealed-bid auctions shows a consistent and significant level of underbidding compared to the Bayes Nash equilibrium prediction (Roth and Kagel 1995). We will firstly summarize lessons learned from the research on underbidding in single-lot first-price sealed-bid auctions before we introduce our experimental design.

3.1 The Underbidding Puzzle in Sealed-bid Auctions

A number of authors have used *risk aversion* to explain bids above the RNBNE (Cox, Smith et al. 1988, Chen and Plott 1998, Kirchkamp and Reiss 2006, Andreoni, Che et al. 2007). However, measuring the risk aversion of lab subjects turned out to be a challenge. Isaac and James (2000) compare estimates of risk preferences from first-price sealed-bid auctions to the Becker-DeGroot-Marshak (BDM) procedure for comparably risky choices. Aggregate measures of risk preferences under the two procedures showed that bidders were risk averse in the first-price auction but risk-neutral, or moderately risk-loving, under the BDM procedure. Overall, risk attitudes not only differed across assessment methods, but also varied within the same method used (Payne, Laughhunn et al. 1980, MacCrimmon and Wehrung 1990, Schoemaker 1993, Krahnen, Rieck et al. 1997). Paired lottery choices, as introduced by Holt and Laury (2005), have become more popular recently, but risk aversion is still recognized as a complex and context specific phenomenon, which is difficult to measure (Dohmen, Falk et al. 2005).

Engelbrecht-Wiggans (1989) introduced post-auction *regret* as another explanation for the bidding behavior in first-price sealed-bid auctions. Engelbrecht-Wiggans and Katok (2009) found support for the regret model in experiments with information feedback about the highest or second-highest bid after the auction. Filiz-Ozbay and Ozbay (2007) also found support for regret. Deviations from the RNBNE bid function could also be due to *wrong expectations* of the bidders about the bids of others (Stahl and Wilson 1995). Goeree, Holt et al. (2002) demonstrate that misperceived probabilities of winning the auction would explain underbidding as well as risk aversion.

Problems in computing a best response are yet another conjecture why bidders might not be able to follow the RNBNE strategy. The mathematical derivation of the RNBNE of split-award auctions in Section 2 is more complex than in single-lot auctions and it is far from obvious that human bidders in the lab would be able to mimic these derivations. Actually, there are contradictory claims as to whether people reason according to Bayesian inference (Gigerenzer and Hoffrage 1995). We will refer to the problem of deriving the RNBNE bid function based on given prior distributions about valuations as the *strategic complexity* of the auction.

The literature on underbidding is extensiv and beyond what we can discuss in this section. Some authors mention spite and joy of winning as potential reasons for deviations from equilibrium predictions in second-price auctions (Cooper and Fang 2008). However, risk aversion, regret, wrong expectations, and strategic complexity are the most natural conjectures for deviations in our experiments.

3.2 Experimental Design and Hypotheses

In our experiments, we wanted to test our theoretical results and used four designs which mirror the decision situation in split-award auctions with different levels of control for the conjectures discussed in the previous section. We organized computerized experiments where subjects competed against computer agents to test hypotheses for underbidding, and human subject experiments to analyze environments which mirror real-world environments.

In all treatments, we used the *strategy method* in a way similar to Selten and Buchta (1999), Güth, Ivanova-Stenzel et al. (2003), and Kirchkamp and Reiss (2011). The strategy method elicits bid functions rather than bids for individual cost draws and it allows us to observe bidding functions in much more detail. Other experiments by Kirchkamp and Reiss (2006) show that bidding behavior that is observed with the strategy method is very similar to the behavior observed with alternative methods. In contrast to this earlier work, in our computerized experiments, we reuse a bid function 100 times in auctions against computer bidders to eliminate risk aversion. We will see that this procedure is very effective. Let us briefly motivate the three groups of treatment combinations in our experiments.

The *first group of treatment combinations (H)* is modeled after real-world procurement practices. These experiments model an environment where four bidders compete in the same type of auction against a pool of unknown bidders, but the competitors change over time. Four bidders received cost draws and competed against each other. Bidders submitted bids in 16 subsequent auctions and they are re-matched randomly after each auction. These experiments allow for learning about the auction format, and the 16 repetitions mitigate risk aversion to some extent, which can be motivated by real-world tenders. After each auction, bidders had a possibility to revise their bid functions and new variable costs are drawn for each bidder independently. After each auction, the bids of all competitors are revealed, as is typically the case, for example, in construction or in public sector auctions. This is often done to combat collusion or bribery (Thomas 1996). In contrast to the computerized experiments, which are designed to understand the potential impact of risk aversion or wrong expectations on underbidding, the results of the treatment combination H should have external validity as they are close to real-world practices. Our main hypothesis for this set of baseline treatment combinations is therefore:

Hypothesis 1: *Bidders in human subject experiments will underbid below the RNBNE bid function.*

As we found underbidding in line with earlier experiments on single-lot auctions, we introduced additional treatments to control for different conjectures why bidders underbid. A s*econd group of treatment combinations* (*C1*) had human subjects compete against computerized agents, which played their RNBNE strategy. Bidders are told that their opponents are rational computer agents, who maximized their expected payoff. Bidders did not learn about other bids in the auction, just whether they won or lost an auction, which should minimize the impact of regret as it was shown in Engelbrecht-Wiggans and Katok (2009). Of course, risk aversion and wrong expectations can still be a driver for deviations from the RNBNE.

Hypothesis 2: *Bidding against computerized agents without information about the bids of others after the auction eliminates underbidding.*

The *third group of treatment combinations (C100)* is identical to C1, but the bid function of a user is reused in 100 auctions, which should mitigate risk aversion. For each new auction, we drew a cost value randomly and determined the bid based on the bid function of a bidder to participate in an auction against computerized bidders. The subject is then paid the average of his winnings in the 100 auctions. The impact of regret should also be minimal, because bidders did not learn about the outcome of individual auctions or the bids of others. The difference between C100 and C1 provides an estimate for the impact of risk aversion in these auctions.

Hypothesis 3: *Bidding against computerized agents without information about the bids of others, where bid functions are reused in 100 auctions, eliminates underbidding.*

The *fourth group of treatment combinations (C100+)* uses the same experimental design as C100, but we also provide explicit information about the RNBNE function of the computerized agents. Since the bidders are all ex-ante symmetric, the information in C100+ tells the subjects implicitly what their equilibrium bidding function would be. Bidders should just replicate the RNBNE strategy of others in order to determine their best response. Here we control for wrong expectations about the computerized bidders, which might be different from wrong expectations that bidders have in human subject experiments. Still, it is valuable to understand which impact explicit information about the bidding strategies of others has on bidders compared to a treatment where this information is not available in C100. We consider deviations from the RNBNE in C100+ as ground noise or irrationality, which provides a baseline for other experiments. Still, bidders might not understand that their best response is to mirror the RNBNE in this treatment and strategic complexity can still be a reason for deviations. However, we cannot expect subjects in experiments with less control to be closer to the RNBNE prediction.

Hypothesis 4: *Bidding against computerized agents without information about the bids of others, where bid functions are reused in 100 auctions, and bidders see the equilibrium bid functions of their computerized opponents, eliminates underbidding.*

Table 1 provides an overview of how we control for different hypotheses for deviations from the RNBNE in the four different treatment combinations. As discussed earlier, in treatment C100+ strategic complexity can still be an explanation for possible deviations from the RNBNE. Bidders in C100 only get the prior cost distributions, and therefore wrong expectations and strategic complexity can both explain deviations. The difference between C100 and C1 is only the number of times in which the bid function is reused. Therefore, this difference can be explained by risk aversion to a large extent. Regret should have little impact, because bidders did not learn about the bids of others in an auction. Treatment combination H allows for all explanations, although the 16 repetitions should mitigate risk aversion to some extent.

	H	C1	C100	C100+
Strategic complexity	+	+	+	+
Wrong expectations	+	+	+	-
Risk aversion	+	+	-	-
Regret	+	-	-	-

Table 1: Control for reasons of deviations from the RNBNE in different treatment combinations. The + sign indicates a possible reason for deviation, while – indicates that this reason is unlikely in this treatment.

The individual treatment combinations are described in Table 2. Overall, 209 subjects were involved in the experiments. In all treatment combinations, variable costs per unit, c_i, were i.i.d. random variables drawn from a uniform distribution with a support of [0, 10.0]. The fixed cost K is 1 for all bidders. The split parameter in the experiments reported in the following with treatment combinations H, C100, and C100+ is q = 0.7.

Treatment	Bid fct. reused	Opponents	Information	Split	Auction format	No. of Subjects
H.S	1	Human	Prior distribution, bids of past auctions	1.0	Single object	16
H.P	1	Human	Prior distribution, bids of past auctions	0.7	Parallel	16
H.Y	1	Human	Prior distribution, bids of past auctions	0.7	Yankee	16
C1.S	1	Computer	Prior distribution	1.0	Single object	12
C1.P	1	Computer	Prior distribution	0.7	Parallel	12
C1.Y	1	Computer	Prior distribution	0.7	Yankee	11
C100.S	100	Computer	Prior distribution	1.0	Single object	11
C100.P	100	Computer	Prior distribution	0.7	Parallel	13
C100.Y	100	Computer	Prior distribution	0.7	Yankee	13
C100+.S	100	Computer	Prior & RNBNE bid fct.	1.0	Single object	10
C100+.P	100	Computer	Prior & RNBNE bid fct.	0.7	Parallel	11
C100+.Y	100	Computer	Prior & RNBNE bid fct.	0.7	Yankee	11

Table 2: Overview of treatment combinations in the experiments describing the number of auctions in which a bid function is used, the types of bidders against which a subject competed (human or computer bidder), the information available to bidders before an auction, the auction format (parallel or Yankee auction), and the number of bidders involved in auctions with this treatment combination.

We also ran additional computerized experiments of C100+ and C100 with a split parameter of $q = 0.9$ to make sure that the high predictive accuracy of the RNBNE function that we found for the split of $q = 0.7$ is robust against changes of the split parameter. This could be confirmed. We provide the results of the split of $q = 0.9$ only in Appendix 4, in order to limit the number of treatment combinations in the main part of the paper.

In addition, we performed experiments with single-lot auctions ($q = 1$) to understand how the results compare with traditional reverse auctions. This is necessary, because we are not aware of similar experiments with a reverse first-price sealed-bid auction. Overbidding on high-cost draws in sales auctions might just be different from underbidding for low-cost draws in reverse auctions, and these experiments provide us with a baseline.

3.3 Experimental Procedures

All experiments were conducted from November 2011 to November 2013 with students in computer science, mathematics, physics, and mechanical engineering. The subjects were recruited via e-mail lists and experiments were conducted in a computer lab at our university. At the beginning of the experiment, participants were randomly assigned to seats in the laboratory. The participants obtained written instructions. These instructions varied slightly depending on the treatment (see Appendix 2). All the instructions were read aloud and participants had to participate in a test about the economic environment and the auction rules. In addition, we conducted a test auction to make sure that the subjects were familiar with the auction design and the user interface. Bidders could take as much time as they wanted to write down their bid functions. The average auction duration for all sealed-bid formats was around 10 minutes. The repeated auctions in H took less time, as bidders usually finished their update after 5-10 minutes. The experiments C1, C100, and C100+ took 45 minutes overall on average, while experiment H took 2 hours and 30 minutes on average.

Each participant received a show up fee of 10 Euro for H, which took more time, and 5 Euro for the computerized experiments. Losses could reduce the fee. Payoffs from auctions were translated from Franc, the experimental currency, into Euro through a fixed exchange rate. If participants had colluded, they would have been excluded from the experiment without any payment. However, we did not find evidence of collusion. If subjects had made a loss, which was not covered by the show up and the payoff of all auctions, they would also have been excluded from the experiment and the session would have been cancelled. Actually, there were no losses which were not covered by the show up fee. Subjects only participated once in one session. Overall, 103 students participated in the experiments with split-award auctions where the split was $q = 0.7$, and 57 students in experiments with a split parameter of $q = 0.9$ (see Appendix 4). An additional 49 students participated in the experiments with single-lot reverse auctions.

From the 103 students participating in split-award auctions ($q = 0.7$), 22 were in C100+ (average payoff 13.38 €), 26 in C100 (average payoff 13.82 €), 23 in C1 (average payoff 8.27 €), and 32 in H (average payoff 46.76 €). From the 49 students in single-lot reverse auctions, 10 participated in C100+ (average payoff 13.15 €), 11 in C100 (average payoff 13.49 €), 12 in C1 (average payoff 7.25 €), and 16 in H (average payoff 53.45 €).

4. Experimental Results

We will now describe bidder behavior in the lab. We want to test the theoretical predictions and understand how well the RNBNE bid function explains the empirical observations in the different treatments.

4.1 Single-lot Procurement Auction

Before we look at split-award auctions, we analyze single-lot procurement auctions in which the entire quantity goes to one supplier. This will provide us with a baseline against which we can compare bidding behavior in split-award procurement auctions. We will discuss the four hypotheses from the previous section and organize them in three results because we can aggregate C100 and C100+. Then we provide statistics supporting the results.

Results on single-lot procurement auctions

S1: *There is underbidding in treatment H, but the level of underbidding decreased after a few rounds. We fail to reject Hypothesis 1.*

S2: *The RNBNE strategy describes the empirical data in C100 and C100+ in the single-lot auction well. This suggests that wrong expectations and the strategic complexity have little impact on the bidding behavior. We fail to reject Hypothesis 3 and 4.*

S3: *We found underbidding on low-cost draws in C1 and reject Hypothesis 2. The difference to C100 indicates that risk aversion has substantial impact on bidder behavior.*

Support: In order to analyze the empirical bid functions, we analyze the outcome of linear regression models in the different treatments and compare it with the linear RNBNE bid functions for the small and the large lot.[1] We use a fixed effects model with a dummy variable u_i to estimate the unobserved heterogeneity of bidders i.

$$y_{it} = \alpha + \beta c_{it} + \gamma r_{it} + \delta w_{it-1} + u_i + \varepsilon_{it}$$

The dependent variable y_{it} describes the bid t submitted by bidder i. The unit costs c_{it} are used as the main independent variable. The coefficients u_i for the bidder ID of all the bidders

[1] Seemingly unrelated regression (SUR) is one possibility to deal with these two sources of data. However, because each equation contains exactly the same set of regressors, the estimators of a SUR are numerically identical to ordinary least squares estimators, which follows from Kruskal's theorem (Davidson and MacKinnon, 1993).

control for bidder idiosyncrasies, but they are omitted from the table in order to focus on the main variables. Variable r_{it} controls the number of the auction (or round) and is only used in repeated experiments in treatment combination H. γ is the coefficient for the number of an auction in an experiment. Variable w_{it-1} describes whether a bidder won in the previous auction and δ describes the impact of winning in the last round. Table 3 summarizes the main parameters: the intercept α, the regression coefficient β for the unit costs c_{it}, and the multiple R^2 of the linear regression.

We also compute the mean squared error (MSE) of the RNBNE function to understand how well the model explains the data in the different treatment combinations. This metric is lowest in C100+, indicating that the variance around the RNBNE bid function is low. Plots of the empirical bid functions can be found at the end of Appendix 3. We compare the MSE of the linear RNBNE function against the MSE of a LOESS estimation of the data (Cleveland and Devlin 1988). LOESS is also known as locally weighted polynomial regression, which can be considered a best case model for the empirical data. At each point in the data set a low-degree polynomial is fitted to a subset of the data. The value of the regression function for the point is obtained by evaluating the local polynomial using the explanatory variable values for that data point.

	Single lot						
	α	β (unit cost)	Std. error β (*p*-value)	Mult. R^2	MSE RNBNE	MSE LOESS	# bids / bidders
RNBNE	2.53	0.748					
H.S	2.11	0.821	0.005 (0.000)	0.923	0.854	0.676	2560/16
H.S (# 1)	1.51	0.869	0.028 (0.000)	0.897	2.127	1.936	160/16
H.S (# 7)	1.87	0.793	0.016 (0.000)	0.952	0.587	0.387	160/16
H.S (# 16)	1.70	0.829	0.014 (0.000)	0.966	0.699	0.353	160/16
C1.S	1.92	0.767	0.039 (0.000)	0.862	4.828 (0.598)	4.740 (0.382)	120/12
C100.S	2.53	0.727	0.033 (0.000)	0.922	0.415	0.392	110/11
C100+.S	2.08	0.797	0.008 (0.000)	0.991	0.104	0.083	100/10

Table 3: Regression coefficients for the empirical bid functions (w/o bidder ID) of the single lot auction

The lines H.S (#1) to H.S (#16) in Table 3 describe the results of the regression for the empirical bid functions in individual auctions (numbers 1, 7 and 16) in H. The low intercept α together with a higher β compared to the RNBNE function indicates that there is underbidding on average on low-cost draws in H compared to the RNBNE bid function. In auction #7, for example, there is an underbidding of 18.7% at a unit cost of 1 compared to the RNBNE bid function, while there is underbidding of 2.75% for high-cost draws of 9 Francs.

The value of intercept α, which can be used as an estimator for underbidding on low value draws, is at a mean value of 1.74. The α decreased slightly in the last six rounds from a value of 1.81 to a value of 1.70. This can be explained by some bidders who became more aggressive on low-cost draws in order to become winners before the experiment was over. More aggressive bidding across rounds overall is also illustrated by a significantly negative, but low, coefficient γ = -0.04. We found a small but significantly negative impact of winning in the last round (δ=-0.06), which cannot be explained by regret. This small negative impact can also be observed in the different split-award auctions. Note that in this paper we want to analyze when the RNBNE can explain bidding behavior in first-price auctions. This allows us to rule out explanations such as strategic complexity as reasons for underbidding. The question, whether risk-aversion or rather regret determine the underbidding in our experiments may be a fruitful exercise to look at in the future.

The high MSE (RNBNE) in treatment combination C1.S is due to a single bidder who bid substantially above the RNBNE bid function. Without this bidder, the MSE is 0.589. The average underbidding at a unit cost of 1 is 18.02% below the RNBNE bid function. The Chow test allows testing whether the regression coefficients of two linear regressions are significantly different from the RNBNE. There is no significant difference between the RNBNE prediction and treatments C100+ (p = 0.999) and C100 (p = 0.991). This means that, even without information about the bid functions of computerized agents, the empirical bid functions in C100, and C100+ are very close to the RNBNE bid function, which is also illustrated by the low MSE. Note that the difference in the intercept α between C100 and C100+ is caused by an outlier (see Figure 16), but that the MSE is lower in C100+ as expected. We will also find no significant difference between C100+, C100, and the RNBNE bid function in the split-award auctions.

Most, but not all, empirical bid functions in the single-lot auction increased monotonously, but there are also some spikes. These small non-monotonicities which we found across all treatments might be due to errors that bidders make when typing in the data in spite of the graphical display of their bid function.

Discussion: As already discussed, overbidding on large valuations in experiments on first-price sealed-bid sales auctions is a consistent pattern. Kirchkamp and Reiss (2011) report median overbidding of up to 30% over the RNBNE on high-value draws, but even modest underbidding for low-value draws. Pezanis-Christou and Sadrieh (2003) report average relative overbidding over the RNBNE prediction of 34-37% for their experiments with symmetric bidders. These authors also use the strategy method. However, since they test sales auctions and the number of auctions and competitors is different, the level of overbidding in their experiments cannot easily be used as an estimate for underbidding in our reverse auctions. Kirchkamp and Reiss (2011), for example, used 12 iterations and two bidders in experiments, while we had four bidders in 16 rounds of a reverse auction. Underbidding for low unit costs of 1 Franc in our experiments is on average 18-19% below the RNBNE bid function in treatments C1 and H, as described above. As we will see, the level of underbidding in treatments C1 and H increases in split-award auctions, while C100 and C100+ are not significantly different from the RNBNE prediction as well.

4.2 Parallel Auction

We will now discuss the results of the parallel auction, which requires bidders to think about their bids for the small and the large lot. The results are organized similar to those of the single-lot auction.

Results on single-lot procurement auctions

P1: *There is underbidding in treatment H, but the level of underbidding decreased after a few rounds. We fail to reject Hypothesis 1.*

P2: *The RNBNE strategy describes the empirical data in C100 and C100+ in the parallel auction well. This suggests that wrong expectations and the strategic complexity of the parallel auction have little impact on the bidding behavior. We fail to reject Hypothesis 3 and 4.*

P3: *We found underbidding on low-cost draws in C1 and reject Hypothesis 2. The difference to C100 indicates that risk aversion has substantial impact on bidder behavior.*

Support: Table 4 follows the format of Table 3, but describes the regression coefficients for the large and the small lot. The regression line of C1 has a much lower intercept compared to the RNBNE bid function, which shows underbidding on low-cost draws. This can be explained by the impact of risk aversion, because the only difference from C100 is the number of auctions in which the bid function is used afterwards. The line H.P in Table 4 describes the relevant regression coefficients of all human subject experiments where we control for bidder idiosyncrasies and the number of auctions. The subsequent lines describe the results of the regression for the empirical bid functions in individual auctions (numbers 1, 7 and 16) in treatment combination H. The low intercept α together with a higher β compared to the RNBNE bid function indicates that there is underbidding on average on low-cost draws in H compared to the computerized treatments in C100 and C100+. However, there is even more underbidding in C1. Note that in the initial sealed-bid treatments with C100 and C100+, we have elicited the bid function for 20 unit costs from 0.5 to 10 Francs, while for the human subject experiments where students had to submit their bid function multiple times; we reduced this to 10 parameters. In test experiments with treatment combination C1, we did not find that this had any impact on the shape of the bid function in the experiments.

	Large lot				Small lot				
	α	β (unit cost)	Mult. R^2	Std. error β (p-value)	α	β (unit cost)	Mult. R^2	Std. error β (p-value)	# bids / bidders
RNBNE	3.59	0.643			3.37	0.67			
H.P	1.70	0.778	0.951	0.004 (0.000)	2.68	0.735	0.929	0.004 (0.000)	2560/16
H.P (# 1)	1.45	0.777	0.937	0.017 (0.000)	2.07	0.765	0.950	0.015 (0.000)	160/16
H.P (# 7)	1.74	0.772	0.961	0.013 (0.000)	2.79	0.722	0.940	0.016 (0.000)	160/16
H.P (# 16)	1.28	0.785	0.963	0.013 (0.000)	1.85	0.731	0.950	0.015 (0.000)	160/16
C1.P	1.48	0.854	0.938	0.022 (0.000)	1.81	0.805	0.947	0.019 (0.000)	120/12
C100.P	3.70	0.697	0.966	0.008 (0.000)	3.67	0.680	0.977	0.007 (0.000)	260/13
C100+.P	3.49	0.626	0.963	0.008 (0.000)	3.39	0.621	0.957	0.009 (0.000)	220/11

Table 4: Regression coefficients for the empirical bid functions of the parallel auction

In Table 5, we have provided the MSE of all three models for the parallel auction. The MSE is again lowest in C100+ and in C100 (marked in bold in Table 5). Actually, C100 has even a lower MSE than C100+ for the small lot. The MSE of C1 is also much higher than that of C100 and C100+ and higher than H, where some of the risk aversion might be eliminated due to the 16 repetitions of the experiment. We have added additional statistics to compare the RNBNE against the predictive power of a model with a constant profit margin and the RNBNE of a single-item first-price sealed-bid auction. This should help understand how sensitive the predictions are. The model assuming bidders had a constant profit margin had the worst MSE in all treatments. For this model we used the average markup of the RNBNE function across all draws as the profit margin. For the treatment combinations C1 and H the RNBNE of the single item auction has a lower MSE than the RNBNE of the split-award auction. This can easily be explained by the underbidding observed in these treatments. In a single-item auction with the same number of bidders the competition is higher, which brings down the bid prices in equilibrium below that of the RNBNE in the split-award auction. Again, the MSE for the treatment C1 is highest, which can be explained by risk aversion and the differences in how bidders respond to risk aversion. A few bidders deviated substantially from the RNBNE prediction, which led to a high MSE (see Figure 4).

	Large lot				Small lot			
	MSE LOESS	MSE RNBNE	MSE Single Item RNBNE	MSE Constant Factor	MSE LOESS	MSE RNBNE	MSE Single Item RNBNE	MSE Constant Factor
H.P	0.329	1.160	**0.449**	1.124	0.403	0.607	**0.484**	1.089
H.P (# 1)	0.578	1.107	**0.629**	1.165	0.568	0.708	**0.690**	1.059
H.P (# 7)	0.269	1.037	**0.386**	1.059	0.319	0.496	**0.424**	1.050
H.P(# 16)	0.319	1.340	**0.508**	1.223	0.453	0.750	**0.539**	1.234
C1.P	0.827	1.668	**0.948**	1.234	0.682	1.057	**0.730**	1.129
C100.P	0.269	**0.328**	0.406	1.037	0.187	**0.192**	0.443	1.055
C100+.P	0.141	**0.175**	0.464	1.337	0.206	**0.260**	0.542	1.441

Table 5: MSE of the RNBNE in the split-award auction, the RNBNE of a single item auction, and the MSE of a constant profit margin model. The MSE of the LOESS estimate serves as a baseline to compare against.

As an example, Figure 1 describes the bid functions in the first and seventh auction of H for the large lot. A thick solid line describes the average bid function based on a linear regression, while a thick dashed line shows the RNBNE bid function. In both plots, we can see the underbidding in the lower values compared to the RNBNE bid function. Figure 2 to Figure 8 in Appendix 3 show the bid functions for all other treatments in the parallel auction.

The intercepts of both C1.P and H.P are much lower than those of the RNBNE bid function for the large and the small lot. Underbidding below the RNBNE for low costs of 1 Franc on the large lot is on average 40.63% for treatment H (auction #7) and 44.79% for C1. On the small lot, we observed underbidding of 13.07% for treatment H (auction #7) and 35.27% for C1. By comparison, in the single-lot reverse auction, we observed around 18-19% for both treatments.

Risk aversion can serve as a natural explanation for the underbidding in C1.P. In H.P, the residual risk aversion in spite of the 16 repetitions, but also other conjectures such as regret, can be potential reasons for underbidding on low-cost draws. However, given our experimental design where bidders do not learn about their opponents, we conjecture that risk aversion serves as the most likely explanation for underbidding in treatment H.

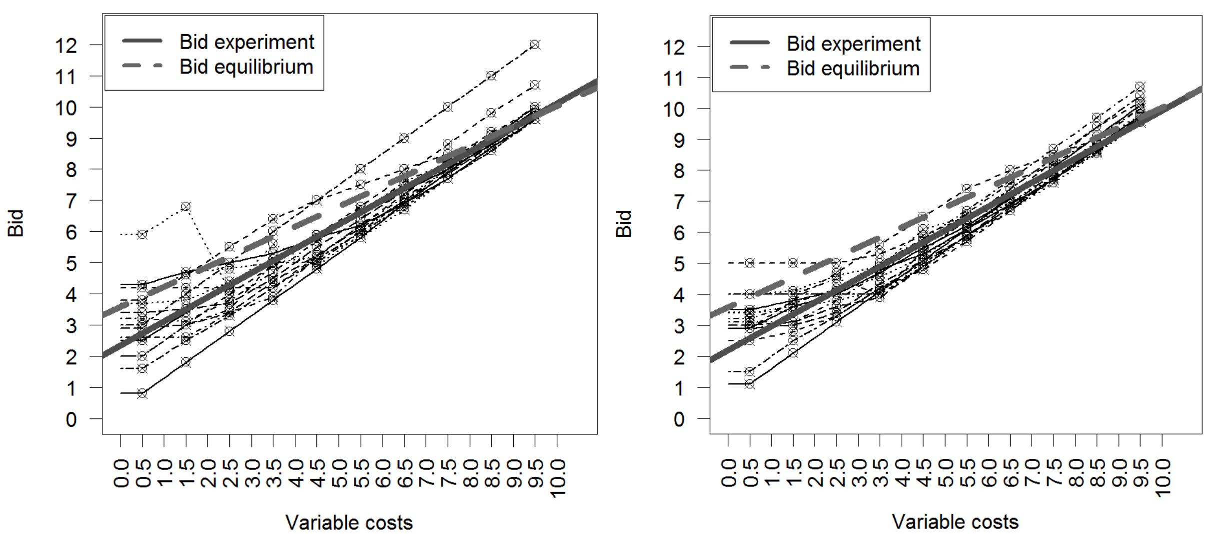

Figure 1: Scatter plot of bids and the optimal bid functions for H on the large lot for the first (left) and the 7th auction (right) for the parallel auction (q = 0.7) and a fixed cost of K=1.

The Chow test shows equivalence between the RNBNE bid function and those in C100+ for the large (p = 0.7399) and the small lot (p = 0.934). There is also no significant difference to the average bid functions in C100 for the large (p = 0.2698) and the small lot (p = 0.031). The test shows that the bid functions in C1 and H are both significantly different from the RNBNE bid function for both lots (p = 0.000).

P5: *The correlation between the markups of bidders in the large and in the small lot is high, and the markup on average in the small lot is significantly higher than in the large lot, as theory predicts.*

Support: We found the markups between the large and the small lot to be highly correlated (H (ρ = 0.981), C1 (ρ = 0.968), C100 (ρ = 0.977), and C100+ (ρ = 0.973)). In other words, bidders with a high markup on the large lot also have a high markup on the small lot. The differences in the markup between the large and the small lot in all treatments are significant throughout (paired t-test, α = 0.01). This suggests that bidders on average followed the same strategy in both the large and the small lot and on average they bid lower on the small lot where there is also less competition, which is in line with the equilibrium prediction.

Discussion: Overall, the results from the computerized experiments C100+.P and C100.P confirm rational bidding behavior according to the RNBNE model. Wrong expectations or strategic complexity do impact bidding behavior significantly. Underbidding in C1.P on low-cost draws can again be explained by risk aversion. This underbidding is higher than in the single-lot auction, in particular on the large lot. We conjecture that bidders tried to win the large lot with low prices, because it promised a higher total payoff with 70 units. Risk aversion can also serve as one of the reasons for underbidding in H.P. Due to the 16 repetitions risk aversion is mitigated, but not eliminated. Regret and spite might also play a role, but due to the experimental design, where bidders did not know their opponents, we assume these behavioral conjectures to be less important. This is supported by comments of participants after the experiments explaining their bidding strategy. Also in treatment H.P, bidders are aggressive on the large lot, and they started with a higher bid on the small lot in case of low cost draws. However, the bidding in the last rounds also became aggressive on the small lot with lower bids on the low-cost draws.

4.3 The Yankee Auction

In addition to the parallel auction, we also analyzed how well equilibrium bidding strategies explain bids in the different treatment combinations in the Yankee auction. The strategic complexity is higher than in the parallel auction, because bidders do not know if an aggressive bid will actually win the large lot, and if they do not win the large lot, they might win the small lot with a very low payoff. Again, we provide results in Table 6 and scatter plots (Appendix 3).

Results for the Yankee auction

Y1: *There is underbidding in treatment H, but the level of underbidding decreased after a few rounds. We fail to reject Hypothesis 1.*

Y2: *The RNBNE strategy describes the empirical data in C100 and C100+ in the Yankee auction well. This suggests that wrong expectations and the strategic complexity of the Yankee auction have little impact on the bidding behavior. We fail to reject Hypothesis 3 and 4.*

Y3: *We found underbidding on low-cost draws in C1 and reject Hypothesis 2. The difference to C100 indicates that risk aversion has substantial impact on bidder behavior.*

Support: We provide the same statistics as for the parallel auction in Table 6. The MSE values are comparable to the parallel auction. The lowest MSE values are again achieved for C100+ and C100. C1.Y has also a significantly lower intercept that can be attributed to risk aversion. In C1.Y, there is a clear outlier, a bidder who submitted very high bid functions leading to a high MSE RNBNE of 3.372. Without this bidder the MSE RNBNE for C1.Y is 1.024.

	α	β (unit cost)	Std. error β (p-value)	Mult. R^2	MSE LOESS	MSE RNBNE	MSE Single item RNBNE	MSE Constant factor	Number of bids / bidder
RNBNE	3.53	0.647							
H.Y	2.49	0.756	0.004 (0.000)	0.941	0.475	0.766	**0.532**	1.062	2550/16
H.Y (# 1)	2.68	0.724	0.018 (0.000)	0.930	0.907	**0.989**	1.297	1.623	160/16
H.Y (# 7)	2.19	0.743	0.012 (0.000)	0.963	0.317	0.573	**0.360**	0.941	160/16
H.Y(# 16)	1.92	0.800	0.011 (0.000)	0.975	0.355	0.914	**0.418**	0.892	160/16
C1.Y	2.05	0.783	0.028 (0.000)	0.860	3.191	**3.372**	3.531	3.647	110/11
C100.Y	3.21	0.684	0.007 (0.000)	0.973	0.192	**0.213**	0.396	1.039	260/13
C100+.Y	3.66	0.682	0.008 (0.000)	0.972	0.148	**0.166**	0.342	0.995	220/11

Table 6: Regression coefficients for the empirical bid functions (w/o bidder ID) of the Yankee auction with q = 0.7. The numbers in brackets for C1.Y describe MSE values w/o one outlier.

The line H.Y in Table 6 describes the regression coefficients of all the bid functions with the number of the auction as an additional covariate. As in the parallel auction, we find a low intercept α together with a higher β compared to the RNBNE function in the analysis of auctions

1, 7 and 16 in H. This means that, also in the Yankee auction, bidders in H underbid on low-cost draws compared to the RNBNE function, which we also observe in C100 and C100+.

We have included the MSE for the single-item RNBNE and that of a constant profit factor model in Table 6. In the Yankee auction, the split-award RNBNE model had the lowest MSE in treatments C100+.Y, and C100.Y. For H.Y and C.1 (w/o the outlier) the single-item RNBNE had a lower MSE, which can again be explained by the fact that the single-item RNBNE model leads to higher competition with the same number of bidders and lower equilibrium bid price, which better fits the average behavior of risk-averse bidders.

As an example, Figure 9 describes the bid functions in the first and seventh auctions of H. We provide the bid functions for other treatment combinations in the Yankee auction in Figure 9Figure 10 to Figure 16 in Appendix 3.

The Chow test shows that there is no significant difference between the RNBNE bid functions in C100+ (p = 0.847) and C100 (p = 0.907), but there is a difference to H and C1 (p = 0.000). Underbidding below the RNBNE for low-cost draws of 1 Franc is on average 29.78% for treatment H (auction #7) and 32.18% for C1. This is less than in the parallel auction.

Discussion: Bidders in the Yankee auction do not know a priori if they will win the large lot or the small lot with their single-bid price. The high predictive accuracy of the RNBNE function in C100 and C100+ is, therefore, an interesting result. In particular, there is no significant difference between C100 and C100+ and no significant underbidding, indicating that strategic complexity had little impact even in the Yankee auction. In line with what we see in the single-lot and in the parallel auction, we find significant underbidding below the RNBNE bid function in C1, which can be explained by risk aversion. In the treatment combination H, the level of underbidding on low-cost draws increases slightly across the 16 auctions in a session, which can be explained by losing bidders in the initial rounds who bid more aggressively in later rounds.

4.4 Predictive Accuracy across Auction Formats

Let us now summarize the results across all auction formats, the single-lot, the parallel, and the Yankee auction.

Results for the all three auctions

A1: *There is underbidding in treatment H. We fail to reject Hypothesis 1.*

A2: *The RNBNE strategy describes the empirical data in C100 and C100+ in the single-lot auction well. We fail to reject Hypothesis 3 and 4.*

A3: *We found underbidding on low-cost draws in C1 and reject Hypothesis 2. The difference to C100 indicates that risk aversion has substantial impact on bidder behavior.*

Support: First, the results S1-3, P1-3, and Y1-3 are in line. In addition, in order to analyze the predictive accuracy of the RNBNE model across auction formats and split parameters, we have pooled all observations (Parallel and Yankee auction) within each of the four treatment combinations, as well as the observations for different split parameters (0.7 and 0.9). We have then used the RNBNE prediction as right-hand side variable for the bids in a regression. A coefficient on the RNBNE prediction close to 1 is strong evidence for the RNBNE model, and it demonstrates that subjects understand the strategic differences across the auction sessions. Again, for C100+ and C100 we find evidence for the RNBNE model, while the results in Table 7 indicate underbidding in the treatments C1 and H. Apart from the standard t-test for β=0, we have also tested the null hypothesis of β=1. The difference of the coefficient β to a value of 1 is significant in all cases, although the β is already very close to 1 for C100 and C100+.

Treatment	α	β (RNBNE)	Std. Error (β)	p-value (β=0)	p-value (β=1)
H	-1.867	1.188	0.005	< 2e-16	0.000
C1	-2.151	1.268	0.052	< 2e-16	0.000
C100	-0.460	1.060	0.009	< 2e-16	0.000
C100+	0.139	0.980	0.006	< 2e-16	0.004

Table 7: Regression coefficients of the RNBNE prediction across auction formats.

4.5 Procurement Cost Comparisons

The final result of our theoretical analysis in section 2.3 is that the expected costs of the parallel and the Yankee auction are the same. In this subsection, we report on allocative efficiency and a cost ratio which normalizes the actual procurement costs by the costs of the bidders in the optimal solution. This allows for comparison across different cost draws in the auctions, because average procurement costs can differ significantly due to the cost draws of the bidders in individual auctions.

Allocative efficiency is computed as $E = (c_*^1 + c_*^2 + 2K)/(c_i^1 + c_j^2 + 2K)$, where c_*^1 and c_*^2 are the variable costs in the efficient allocation for the large and the small lot, and c_i^1 and c_j^2 are the costs of those bidders i and j who won the auction. Cost ratio is defined as $C = (c_*^1 + c_*^2 + 2K)/(b_i^1 + b_j^2)$, where b_i^1 and b_j^2 describe the winning bids by bidders i and j on the large and the small lot, respectively. If bid prices in the winning allocation decrease, this ratio increases, i.e. a higher number is better for the buyer.

Result on procurement costs across auctions

C1: *The parallel auction and the Yankee auction exhibit no significant differences in efficiency and cost ratio within the same treatment, as predicted by Proposition 5.*

Support: Overall, efficiency is high in all experimental treatments. We did not find a significant difference in efficiency *E* or cost ratio *C* within the same treatment combination between the parallel and the Yankee auction using a Wilcoxon rank sum test (α=0.01).

	Efficiency *E*	Cost ratio *C*
H.P	96.17%	83.63%
C1.P	96.62%	71.32%
C100.P	99.01%	51.91%
C100+.P	99.39%	51.63%
H.Y	96.49%	79.62%
C1.Y	99.24%	67.48%
C100.Y	98.50%	51.95%
C100+.Y	98.78%	51.60%

Table 8: Efficiency and auctioneer's cost ratio.

There are significant differences between the treatment combination H and C100+ for both split parameters (α=0.01). H has lower efficiency and a higher cost ratio, which means a lower cost, in both the parallel and the Yankee auction. The lower cost in H can be attributed to the underbidding that we described earlier in human subject experiments. So, in spite of differences in the bid function and the underbidding in the lab the outcomes of the auction are in line with the theoretical prediction.

5. Conclusions

In this paper, we analyze two sealed-bid split-award auctions which are regularly used in procurement practice. We obtain closed-form symmetric Bayes Nash equilibrium bidding strategies and several interesting model implications. We prove that if suppliers are risk-neutral and the production cost information is private, then the two auction mechanisms are equivalent, in the sense that they yield the same expected costs to the buyer. Experimental work on the first-price sealed-bid auction has shown a consistent pattern of underbidding. This can be due to risk aversion, regret and wrong expectations, but also due to the complexity of deriving the RNBNE bid function. It is not obvious that RNBNE strategies could be a good predictor for split-award procurement auctions, where bidders are exposed to increased strategic complexity compared to single-lot reverse auctions.

We provide the results of lab experiments with different levels of control. The experiments against computerized bidders are meant to limit the influence of risk aversion and regret as far as possible. Interestingly, there is no significant difference between the average bid function of bidders in the lab and the RNBNE bid function in our computerized experiments where bid functions are reused in 100 auctions. This is different to earlier experiments of first-price sealed-bid auctions, and we attribute the result to our experimental design. The ability to control risk aversion in the experiments allows us to analyze whether bidders are able to cope with the strategic complexity in these markets. The results provide evidence that bidders are well able to mimic their RNBNE bid function even in strategically complex Yankee split-award auctions. Overall, there is no evidence that the strategic complexity or wrong expectations of these auctions explain underbidding in human subjects experiments. However, the results show that risk aversion has substantial impact on the bidder behavior and we conjecture it is a major driver for the underbidding that we see in human subject experiments. We find underbidding for low-cost draws compared to the RNBNE bid function, which typically increases in the latter auction rounds.

Regret and risk aversion are two possible extensions of our models and also the experimental work in this area. It would also be interesting to analyze markets with very high fixed costs in the lab or explore more complex cost functions. However, we firstly wanted to find evidence that human subjects are able to mimic their equilibrium bid functions even without this added complexity.

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Appendices (For Reviewing Purposes)

We provide an appendix which includes the detailed derivations of the equilibrium strategies (Appendix 1), instructions for bidders in the lab (Appendix 2), scatter plots of the empirical bid functions (Appendix 3), and results of the experiments with a split parameter of 0.9 (Appendix 4).

Appendix 1

Proof of Proposition 1

Let $(\phi_1(b), \phi_2(b))$ denote the inverses of the equilibrium bidding strategies. Given the symmetry of the bidders we examine the decision problem faced by a typical bidder, say bidder 1. Let $C_{1:N\text{-}1}$ and $C_{2:N\text{-}1}$ denote the lowest and the second lowest order statistic among the $N-1$ private production costs of the rival bidders. Since private costs are independent and identically distributed the joint density of $(C_{1:N-1}, C_{2:N-1})$ is

(A.1)

$$g_{12}(C_{1:N-1}, C_{2:N-1}) = (N-1)(N-2)\, f(C_{1:N-1})\, f(C_{2:N-1})\left[1 - F(C_{2:N-1})\right]^{N-3}.$$

Bidder 1's expected payoff as a function of his bids (b_1, b_2) and his cost c can be written as[2]

(A.2)

$$\begin{aligned}\pi(b_1, b_2, c) = {} & \Pr(C_{1:N\text{-}1} \geq \phi_1(b_1))\left[(b_1 - c)q - K\right] \\ & + \Pr(\phi_1(b_1) > C_{1:N\text{-}1}, \phi_2(b_2) \leq C_{2:N\text{-}1})\left[(b_2 - c)(1-q) - K\right]\end{aligned}$$

The probability expressions in the first and second terms are given by

(A.3)

$$\Pr(C_{1:N\text{-}1} \geq \phi_1(b_1)) = \left[1 - F(\phi_1(b_1))\right]^{N-1}$$

and

[2] See derivation below.

(A.4)

$$\Pr(\phi_1(b_1) > C_{1:N-1}, \phi_2(b_2) \le C_{2:N-1})$$

$$= \begin{cases} \int_{\phi_2(b_2)}^{\bar{c}} \int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} & \text{if } \phi_1(b_1) \le \phi_2(b_2) \\ \int_{\phi_1(b_1)}^{\bar{c}} \int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} & \\ \quad + \int_{\phi_2(b_2)}^{\phi_1(b_1)} \int_{\underline{c}}^{c_{2:N-1}} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} & \text{if } \phi_1(b_1) > \phi_2(b_2) \end{cases}$$

Therefore, the expected payoff function becomes

(A.5)

$$\pi(b_1, b_2, c) =$$

$$= \begin{cases} \left[1 - F(\phi_1(b_1))\right]^{N-1} \left[(b_1 - c)q - K\right] + & \\ \left[(b_2 - c)(1-q) - K\right] \int_{\phi_2(b_2)}^{\bar{c}} \int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} & \text{if } \phi_1(b_1) \le \phi_2(b_2) \\ \left[1 - F(\phi_1(b_1))\right]^{N-1} \left[(b_1 - c)q - K\right] + & \\ \left[(b_2 - c)(1-q) - K\right] \left\{ \begin{aligned} & \int_{\phi_1(b_1)}^{\bar{c}} \int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} \\ & + \int_{\phi_2(b_2)}^{\phi_1(b_1)} \int_{\underline{c}}^{C_{2:N-1}} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1} \end{aligned} \right\} & \text{if } \phi_1(b_1) > \phi_2(b_2) \end{cases}$$

Taking the derivatives with respect to the decision variables (b_1, b_2) and imposing the equilibrium condition $c = \phi_1(b_1) = \phi_2(b_2)$, we get the following system of equilibrium differential equations:[3]

(A.6)

$$(1-q)\int_{c}^{\bar{c}} \int_{\underline{c}}^{c} g_{12}(C_{1:N-1}, C_{2:N-1}) dC_{1:N-1} dC_{2:N-1}$$

$$-\int_{\underline{c}}^{c} g_{12}(C_{1:N-1}, c) dC_{1:N-1} * \frac{1}{\beta_2'(c)} * \left[\left(\beta_2(c) - c\right)(1-q) - K\right] = 0$$

is the first-order condition for b_2 and the first-order condition for b_1 is

[3]See derivation below.

(A.7)

$$-\frac{(N-1)f(c)}{\beta_1^{'}(c)}\left[1-F(c)\right]^{N-2}*\left[\left(\beta_1(c)-c\right)q-K\right]$$
$$+\left[1-F(c)\right]^{N-1}q+\int_{c}^{\bar{c}}\frac{g_{12}(c,C_{2:N-1})}{\beta_1^{'}(c)}dC_{2:N-1}*\left[\left(\beta_2(c)-c\right)(1-q)-K\right]=0.$$

First-order condition (A.6) can be written as

(A.8)

$$\frac{\partial(1-q)\beta_2(c)\int_{c}^{\bar{c}}f(C_{2:N-1})\left[1-F(C_{2:N-1})\right]^{N-3}dC_{2:N-1}}{\partial c}=-(c(1-q)+K)f(c)\left[1-F(c)\right]^{N-3}.$$

Integrate both sides of equation (A.8) gives us

(A.9)

$$(1-q)\beta_2(c)\int_{y}^{\bar{c}}f(x)\left[1-F(x)\right]^{N-3}dx\,|_{c}^{\bar{c}}$$
$$=-\int_{c}^{\bar{c}}(x(1-q)+K)f(x)\left[1-F(x)\right]^{N-3}dx+C$$
$$\Leftrightarrow$$
$$(1-q)\beta_2(c)\int_{c}^{\bar{c}}f(x)\left[1-F(x)\right]^{N-3}dx=\int_{c}^{\bar{c}}(x(1-q)+K)f(x)\left[1-F(x)\right]^{N-3}dx+C$$
$$\Leftrightarrow$$
$$\beta_2(c)=\frac{K}{1-q}+\frac{\int_{c}^{\bar{c}}xf(x)\left[1-F(x)\right]^{N-3}dx}{\int_{c}^{\bar{c}}f(x)\left[1-F(x)\right]^{N-3}dx}+\frac{C}{(1-q)\int_{c}^{\bar{c}}f(x)\left[1-F(x)\right]^{N-3}dx}$$
$$=\frac{K}{1-q}+\frac{\frac{1}{N-2}c\left[1-F(c)\right]^{N-2}+\frac{1}{N-2}\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-2}dx}{\frac{1}{N-2}\left[1-F(c)\right]^{N-2}}$$
$$=\frac{K}{1-q}+c+\frac{\int_{c_i}^{\bar{c}}\left[1-F(x)\right]^{N-2}dx}{\left[1-F(c)\right]^{N-2}}\text{ with the boundary condition }\beta_2(\bar{c})=\frac{K}{1-q}+\bar{c}$$

First-order condition (A.7) can be reformulated as

(A.10)

$$\frac{\partial \beta_1(c)\left[1-F(c)\right]^{N-1} q}{\partial c} = -\int_{c_i}^{\bar{c}} g_{12}(c, C_{2:N-1}) dC_{2:N-1} * \left[\left(\beta_2(c)-c\right)(1-q)-K\right]$$

$$-(cq+K)(N-1)f(c)\left[1-F(c)\right]^{N-2} \text{ (using equation (A.9))}$$

$$= -(N-1)(N-2)f(c)\int_{c}^{\bar{c}} f(x)\left[1-F(x)\right]^{N-3} dx * \left\{\frac{(1-q)\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-2} dx}{\left[1-F(c)\right]^{N-2}}\right\} \cdot$$

$$-(cq+K)(N-1)f(c)\left[1-F(c)\right]^{N-2}$$

Integrate both sides of equation (A.10) gives us

(A.11)

$$\beta_1(x)\left[1-F(x)\right]^{N-1} q \,|_{c}^{\bar{c}} = -\int_{c_i}^{\bar{c}} (N-1)(N-2)f(y)\int_{y}^{\bar{c}} f(x)\left[1-F(x)\right]^{N-3} dx * \left\{\frac{(1-q)\int_{y}^{\bar{c}}\left[1-F(x)\right]^{N-2} dx}{\left[1-F(y)\right]^{N-2}}\right\} dy$$

$$-\int_{c}^{\bar{c}} (xq+K)(N-1)f(x)\left[1-F(x)\right]^{N-2} dx$$

$$\Leftrightarrow$$

$$-\beta_1(c)\left[1-F(c)\right]^{N-1} q = -(1-q)(N-1)\int_{c}^{\bar{c}} f(y)\int_{y}^{\bar{c}}\left[1-F(x)\right]^{N-2} dxdy$$

$$-(qc+K)\left[1-F(c)\right]^{N-1} - q\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-1} dx + C$$

$\Leftrightarrow$ (using boundary condition $\beta_1(\bar{c}) = \bar{c} + \dfrac{K}{q}$)

$$\beta_1(c) = c + \frac{K}{q} + \frac{\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-1} dx}{\left[1-F(c)\right]^{N-1}} + \frac{(1-q)(N-1)\int_{c}^{\bar{c}} f(y)\int_{y}^{\bar{c}}\left[1-F(x)\right]^{N-2} dxdy}{\left[1-F(c)\right]^{N-1} q}$$

$$= c + \frac{K}{q} + \frac{\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-1} dx}{\left[1-F(c)\right]^{N-1}} + \frac{(1-q)(N-1)\left[\int_{c}^{\bar{c}} F(x)\left[1-F(x)\right]^{N-2} dx - F(c)\int_{c}^{\bar{c}}\left[1-F(x)\right]^{N-2} dx\right]}{\left[1-F(c)\right]^{N-1} q}$$

This completes the proof for bidding strategies. The monotone increasing property of bidding strategies can be easily checked by the first derivatives of the bidding strategies. With the standard assumption of log-concavity on the cumulative distribution function $F(\cdot)$ such an equilibrium is unique and it is characterized as the solution to the system of differential equations corresponding to the first-order conditions (Lebrun, 2006). Actually, the result only requires log-concavity at the highest lower extremity of the supports.

Now we turn to prove the additional claim in Proposition 1.

Claim: $\beta_1(c)q > \beta_2(c)(1-q)$.

Proof:

$$\beta_1(c)q - \beta_2(c)(1-q) = cq + K$$

$$+\frac{1}{[1-F(c)]}\int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\{q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]\}dx$$

$$-c(1-q) - K - (1-q)\int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}dx$$

$$= c(2q-1) + \int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\left[\frac{q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]}{1-F(c)} - (1-q)\right]dx$$

$$= c(2q-1) + \int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\left[\frac{(2q-1)[1-F(x)]+(1-q)(N-2)[F(x)-F(c)]}{1-F(c)}\right]dx$$

$$> 0$$

since $q > 0.5$ and $1 > F(x) > F(c)$ for $c < x < \bar{c}$.

Claim: $\beta_2(c) > \beta_1(c)$

Proof:

$$\beta_2(c) - \beta_1(c) = -c - \frac{K}{q}$$

$$-\frac{1}{q[1-F(c)]}\int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\{q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]\}dx$$

$$+c + \frac{K}{1-q} + \int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}dx$$

$$= \frac{(2q-1)K}{q(1-q)} + \int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\left[1 - \frac{q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]}{q\left[1-F(c)\right]}\right]dx$$

$$= \frac{(2q-1)K}{q(1-q)} + \int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\left[\frac{\left[Nq-(N-1)\right][F(x)-F(c)]}{q\left[1-F(c)\right]}\right]dx$$

$$> 0 \text{ if } K > K_0$$

$$\textit{where } K_0 := -\frac{(1-q)}{(2q-1)}\int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\left[\frac{\left[Nq-(N-1)\right][F(x)-F(c)]}{q\left[1-F(c)\right]}\right]dx$$

.

Derivation of (A.2)

$$
\begin{aligned}
&\pi(b_1,b_2,c) = \text{Pr(bidder } i \text{ wins both Lots)}\left[(b_1-c)q-K\right]\\
&+\text{Pr(bidder } i \text{ wins Lot 1 and loses Lot 2)}\left[(b_1-c)q-K\right]\\
&+\text{Pr(bidder } i \text{ loses Lot 1 and wins Lot 2)}\left[(b_2-c)(1-q)-K\right]\\
&=\Pr(b_1 \le b_j^1\ \forall j\neq i \text{ and } b_2 \le b_j^2\ \forall j\neq i)\left[(b_1-c)q-K\right]\\
&+\Pr(b_1 \le b_j^1\ \forall j\neq i \text{ and } b_2 > b_j^2 \neq i \text{ for some } j)\left[(b_1-c)q-K\right]\\
&+\Pr(b_1 > b_j^1 \text{ for some } j\neq i \text{ and } b_2 \le b_j^2\ \forall j\neq i)\left[(b_2-c)(1-q)-K\right]\\
&=\Pr\left(b_1 \le b_j^1\ \forall j\neq i\right)\left[(b_1-c)q-K\right]\\
&+\Pr\left(\phi_1(b_1) > C_{1:N-1} \text{ and } \phi_2(b_2)\le C_{2:N-1}\right)\left[(b_2-c)(1-q)-K\right]\\
&=\Pr\left(\phi_1(b_1)\le C_{1:N-1}\right)\left[(b_1-c)q-K\right]\\
&+\left[\Pr\left(\phi_1(b_1)>C_{1:N-1}\right)-\Pr\left(\phi_1(b_1)>C_{1:N-1} \text{ and } \phi_2(b_2)>C_{2:N-1}\right)\right]\left[(b_2-c)(1-q)-K\right]
\end{aligned}
$$

Derivation of (A.6) and (A.7)

If $\phi_1(b_1) \le \phi_2(b_2)$, the first order condition with respect to b_2 gives

$$
\text{(A.12).}\quad
\begin{aligned}
&(1-q)\int_{\phi_1(b_1)}^{\bar{c}}\int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1}\\
&-\int_{\underline{c}}^{\phi_1(b_1)} g_{12}(C_{1:N-1},\phi_2(b_2))dC_{1:N-1}\frac{1}{\beta_2'(\phi_2(b_2))}\left[(b_2-c_i)(1-q)-K\right]=0
\end{aligned}
$$

and the first-order condition with respect to b_1 gives

$$
\text{(A.13).}\quad
\begin{aligned}
&-\frac{(N-1)f(\phi_1(b_1))}{\beta_1'(\phi_1(b_1))}\left[1-F(\phi_1(b_1))\right]^{N-2}\left[(b_1-c)q-K\right]+\left[1-F(\beta_1^{-1}(b_1))\right]^{N-1}q\\
&+\int_{\phi_1(b_1)}^{\bar{c}}\frac{g_{12}(\phi_1(b_1),C_{2:N-1})}{\beta_1'(\phi_1(b_1))}dC_{2:N-1}\left[(b_2-c)(1-q)-K\right]=0
\end{aligned}
$$

With the equilibrium condition $c=\phi_1(b_1)=\phi_2(b_2)$, hence, $\beta_1(c)=b_1$ and $\beta_2(c)=b_2$, , (A.12) becomes

$$
\text{(A.14).}\quad
\begin{aligned}
&(1-q)\int_{c}^{\bar{c}}\int_{\underline{c}}^{c} g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1}\\
&-\int_{\underline{c}}^{c} g_{12}(C_{1:N-1},c)dC_{1:N-1}\frac{1}{\beta_2'(c)}\left[(\beta_2(c)-c)(1-q)-K\right]=0
\end{aligned}
$$

and (A.13) becomes

(A.15)

$$-\frac{(N-1)f(c)}{\beta_1^{'}(c)}\left[1-F(c)\right]^{N-2}\left[\left(\beta_1(c)-c\right)q-K\right]$$
$$+\left[1-F(c)\right]^{N-1}q+\int_c^{\bar{c}}\frac{g_{12}(c,C_{2:N-1})}{\beta_1^{'}(c)}dC_{2:N-1}\left[\left(\beta_2(c)-c\right)(1-q)-K\right]=0 .$$

If $\phi_1(b_1)>\phi_2(b_2)$, the first order condition with respect to b_2 gives

(A.16)

$$(1-q)\left[\begin{array}{l}\int_{\phi_1(b_1)}^{\bar{c}}\int_{\underline{c}}^{\phi_1(b_1)}g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1}+\\ \int_{\phi_2(b_2)}^{\phi_1(b_1)}\int_{\underline{c}}^{C_{2:N-1}}g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1}\end{array}\right]$$
$$-\int_{\underline{c}}^{\phi_2(b_2)}g_{12}(C_{1:N-1},\phi_2(b_2))dC_{1:N-1}\frac{1}{\beta_2^{'}(\phi_2(b_2))}\left[\left(b_2-c_i\right)(1-q)-K\right]=0$$

and the first-order condition with respect to b_1 gives

(A.17)

$$q\left[1-F(\phi_1(b_1))\right]^{N-1}$$
$$-(N-1)\left[1-F(\phi_1(b_1))\right]^{N-2}f(\phi_1(b_1))\frac{1}{\beta'(\phi_1(b_1))}\left[\left(b_1-c\right)q-K\right] .$$
$$\left[\begin{array}{l}-\int_{\underline{c}}^{\phi_1(b_1)}g_{12}(C_{1:N-1},\phi_1(b_1))dC_{1:N-1}\\ +\int_{\phi_1(b_1)}^{\bar{c}}g_{12}(\phi_1(b_1),C_{1:N-1})dC_{2:N-1}\\ +\int_{\underline{c}}^{\phi_1(b_1)}g_{12}(C_{1:N-1},\phi_1(b_1))dC_{1:N-1}\end{array}\right]\frac{1}{\beta'(\phi_1(b_1))}\left[\left(b_2-c\right)(1-q)-K\right]=0$$

With the equilibrium condition $c=\phi_1(b_1)=\phi_2(b_2),$ hence, $\beta_1(c)=b_1$ and $\beta_2(c)=b_2$, , (A.16) becomes

(A.18)

$$(1-q)\int_c^{\bar{c}}\int_{\underline{c}}^{c}g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1}$$
$$-\int_{\underline{c}}^{c}g_{12}(C_{1:N-1},c)dC_{1:N-1}\frac{1}{\beta_2^{'}(c)}\left[\left(\beta_2(c)-c\right)(1-q)-K\right]=0$$

and (A.17) becomes

(A.19)

$$
\begin{aligned}
&(1-q)\int_{c}^{\bar{c}}\int_{\underline{c}}^{c} g_{12}(C_{1:N-1},C_{2:N-1})dC_{1:N-1}dC_{2:N-1} \\
&-\int_{\underline{c}}^{c} g_{12}(C_{1:N-1},c)dC_{1:N-1}\,\frac{1}{\beta_2^{'}(c)}\Big[\big(\beta_2(c)-c\big)(1-q)-K\Big]=0 .
\end{aligned}
$$

Thus, in equilibrium, the first order conditions for both bids are the same in the two scenarios.

Proof of Proposition 2

$$\frac{\partial \beta_1(c)}{\partial N} = \frac{1}{q[1-F(c)]}\int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)}\right)^{N-2} T(x,c,N,q)dx$$

where $T(x,c,N,q_2) := \left(q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]\right)\log\left(\frac{1-F(x)}{1-F(c)}\right)+(1-q)[F(x)-F(c)]$

We show that $T(x,c,N,q)<0$ for all values of the arguments satisfying

$c\in[\underline{c},\bar{c}],\ x\in[c,\bar{c}],\ N\geq 2$ and $1/2<q\leq 1.$

Since $\log\left(\frac{1-F(x)}{1-F(c)}\right)<\frac{F(c)-F(x)}{1-F(c)}$

$$T(x,c,N,q)<\left(q[1-F(x)]+(1-q)(N-1)[F(x)-F(c)]\right)\left(\frac{F(c)-F(x)}{1-F(c)}\right)+(1-q)[F(x)-F(c)]=:W(x,c,N,q)$$

Thus if we show that $W(x,c,N,q)<0,$ then it follows that $T(x,c,N,q)<0.$

One can rearrange the expression for W to get

$$W(x,c,N,q)=\left(\frac{F(x)-F(c)}{1-F(c)}\right)\left\{F(x)-1+(1-q)[2-N\ F(x)+(N-2)F(c)]\right\}$$

The sign is determined by the expression in braces

$Y(x,c,N,q):=F(x)-1+(1-q)[2-N\ F(x)+(N-2)F(c)].$

Note that $Y(c,c,N,q)=2[q-(1/2)][F(c)-1]<0$ since $q>1/2$ and $F(c)<1.$

Note also that $Y(\bar{c},c,N,q)=(1-q)(2-N)[1-F(c)]<0$ for all $N>2.$

To establish the sign of Y for $c\leq x\leq\bar{c},$ we note that

$Y_x(x,c,N,q)=f(x)(1-N(1-q)).$

Case (i) : If $q<(N-1)/N$ then $Y_x(x,c,N,q)<0$ for all $x\geq c.$ Since $Y(c,c,N,q)<0$ we get

$Y(x,c,N,q)<0$ for all $x\geq c.$

Case (ii) : If $q>(N-1)/N$ then $Y_x(x,c,N,q)>0$ for all $x\geq c.$ Since $Y(\bar{c},c,N,q)<0$ we get

$Y(x,c,N,q)<0$ for all $x\in[c,\bar{c}].$

The case for $\frac{\partial\beta_2(c)}{\partial N}<0$ is easily established since the integrand is negative for all values of the arguments:

$$\frac{\partial\beta_2(c)}{\partial N}=\int_c^{\bar{c}}\left(\frac{1-F(x)}{1-F(c)}\right)^{N-2}\log\left(\frac{1-F(x)}{1-F(c)}\right)dx.$$

Finally, the partial derivatives of β_1 and β_2 with respect to q are:

$$\frac{\partial \beta_1(c)}{\partial q} = \frac{-1}{q^2}\left(K + (N-1)\int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} \frac{F(x)-F(c)}{1-F(c)} dx \right) < 0,$$

$$\frac{\partial \beta_2(c)}{\partial q} = \frac{K}{(1-q)^2} > 0.$$

Proof of Proposition 3

The expected payoff of a generic bidder i when his cost is c and he bids b and all his rivals use a monotone equilibrium bidding strategy $\beta(c)$ with inverse $\phi(b)$ is[4]

(A.20)

$$\pi(b,c) = \left[1-F(\phi(b))\right]^{N-1}\left[\left(b-c\right)q - K\right]$$
$$+(N-1)F(\phi(b))\left[1-F(\phi(b))\right]^{N-2}\left[\left(b-c\right)(1-q) - K\right]$$

The first order condition with respect to b gives

(A.21)

$$\beta^{'}(c)\left\{\left[1-F(c)\right]^{N-1} q + (N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q)\right\}$$
$$+\begin{bmatrix} -(N-1)\left[1-F(c)\right]^{N-2} f(c)q + (N-1)f(c)\left[1-F(c)\right]^{N-2}(1-q) \\ -(N-1)F(c)(N-2)\left[1-F(c)\right]^{N-3} f(c)(1-q) \end{bmatrix} b_i$$
$$= c\begin{bmatrix} -(N-1)\left[1-F(c)\right]^{N-2} f(c)q + (N-1)f(c)\left[1-F(c)\right]^{N-2}(1-q) - \\ (N-1)F(c)(N-2)\left[1-F(c)\right]^{N-3} f(c)(1-q) \end{bmatrix}$$
$$+K\begin{bmatrix} -(N-1)\left[1-F(c)\right]^{N-2} f(c)q + (N-1)f(c)\left[1-F(c)\right]^{N-2}(1-q) \\ -(N-1)F(c)(N-2)\left[1-F(c)\right]^{N-3} f(c)(1-q) \end{bmatrix}$$

or,

[4] See derivation below.

(A.22)

$$\frac{d}{dc}\left\{\beta(c)\left\{\left[1-F(c)\right]^{N-1}q+(N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q)\right\}\right\}$$

$$=c\begin{bmatrix}-(N-1)\left[1-F(c)\right]^{N-2}f(c)q+(N-1)f(c)\left[1-F(c)\right]^{N-2}(1-q)-\\(N-1)F(c)(N-2)\left[1-F(c)\right]^{N-3}f(c)(1-q)\end{bmatrix}$$

$$+K\begin{bmatrix}-(N-1)\left[1-F(c)\right]^{N-2}f(c)q+(N-1)f(c)\left[1-F(c)\right]^{N-2}(1-q)\\-(N-1)F(c)(N-2)\left[1-F(c)\right]^{N-3}f(c)(1-q)\end{bmatrix}$$

Integrating both sides of (A.22) yields (A.23).

$$-\beta(c)\left\{\left[1-F(c)\right]^{N-1}q+(N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q)\right\}$$

$$=\int_c^{\bar{c}}xd\left\{\left[1-F(x)\right]^{N-1}q+(N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}$$

$$+K\int_c^{\bar{c}}d\left\{\left[1-F(x)\right]^{N-1}+(N-1)F(x)\left[1-F(x)\right]^{N-2}\right\}+C$$

$$=x\left\{\left[1-F(x)\right]^{N-1}q+(N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}\Big|_{c_i}^{\bar{c}}$$

$$-\int_c^{\bar{c}}\left\{\left[1-F(x)\right]^{N-1}q+(N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}dx$$

$$+K\left\{\left[1-F(x)\right]^{N-1}+(N-1)F(x)\left[1-F(x)\right]^{N-2}\right\}\Big|_c^{\bar{c}}+C$$

$$=-c\left\{\left[1-F(c)\right]^{N-1}q+(N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q)\right\}$$

$$-\int_c^{\bar{c}}\left\{\left[1-F(x)\right]^{N-1}q+(N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}dx$$

$$-K\left\{\left[1-F(c)\right]^{N-1}+(N-1)F(c)\left[1-F(c_i)\right]^{N-2}\right\}+C$$

Therefore,

(A.24)

$$\beta(c)=c+\frac{\int_c^{\bar{c}}\left\{\left[1-F(x)\right]^{N-1}q+(N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}dx}{\left\{\left[1-F(c)\right]^{N-1}q+(N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q)\right\}}$$

$$+K\frac{\left\{1+(N-2)F(c)\right\}}{\left\{\left[1-F(c_i)\right]q+(N-1)F(c)(1-q)\right\}}+C$$

With boundary condition $\beta(\bar{c})=\bar{c}+\dfrac{K}{1-q}$, the equilibrium bidding strategy is

(A.25)

$$\beta(c_i) = c + \frac{\int_c^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c)\right]^{N-1} q + (N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q) \right\}}$$

$$+K \frac{\left\{1+(N-2)F(c)\right\}}{\left\{\left[1-F(c)\right]q + (N-1)F(c)(1-q)\right\}}.$$

This completes the proof. Again, the monotone increasing property of the bidding strategy can be checked by calculating the first derivative of the equilibrium bidding strategy.

Derivation of (A.20)

$$\pi(b,c) = \text{Pr(bidder } i \text{ wins Lot 1)}\left[\left(b-c\right)q-K\right] + \text{Pr(bidder } i \text{ loses Lot 1 and wins Lot 2)} * \left[\left(b-c\right)(1-q)-K\right]$$

$$= \text{Pr(}b\text{ is the lowest bid)}\left[\left(b-c\right)q-K\right] + \left[\text{Pr(}b\text{ is the second lowest bid)}\right] * \left[\left(b-c\right)(1-q)-K\right]$$

$$= \Pr(\phi(b) \le C_{1:(N-1)})\left[\left(b-c\right)q_1-K\right] + \Pr(C_{1:(N-1)} < \phi(b) \le C_{2:(N-1)})\left[\left(b-c\right)(1-q)-K\right]$$

$$= \left[1-G_1(\phi(b))\right]\left[\left(b-c\right)q-K\right]$$

$$+\left[\Pr(C_{1:(N-1)} < \phi(b) - \Pr(C_{1:(N-1)} < \phi(b) \text{ and } \mathrm{C}_{2:(N-1)} < \phi(b))\right]\left[\left(b-c\right)(1-q)-K\right]$$

$$= \left[1-G_1(\phi(b))\right]\left[\left(b-c\right)q-K\right] + \left[G_1(\phi(b)) - G_{12}(\phi(b)\ ,\phi(b)\ )\right]\left[\left(b-c\right)(1-q)-K\right]$$

$$= \left[1-F(\phi(b))\right]^{N-1}\left[\left(b-c\right)q-K\right]$$

$$+\left\{1-\left[1-F(\phi(b))\right]^{N-1} - \sum_{j=2}^{N-1}\frac{(N-1)!}{j!(N-1-j)!}F(\phi(b))^j\left[1-F(\phi(b))\right]^{N-1-j}\right\}\left[\left(b-c\right)(1-q)-K\right]$$

$$= \left[1-F(\phi(b))\right]^{N-1}\left[\left(b-c\right)q-K\right]$$

$$+\left\{1-\left[1-F(\phi(b))\right]^{N-1} - 1 + \left[1-F(\phi(b))\right]^{N-1} + (N-1)F(\phi(b))\left[1-F(\phi(b))\right]^{N-2}\right\}\left[\left(b-c\right)(1-q)-K\right]$$

$$= \left[1-F(\phi(b))\right]^{N-1}\left[\left(b-c\right)q-K\right] + (N-1)F(\phi(b))\left[1-F(\phi(b))\right]^{N-2}\left[\left(b-c\right)(1-q)-K\right]$$

Proof of Corollary 1

For the winner of the large lot, ex post, his profit is

$$\text{Profit}_{\text{large}} = \beta(c_i)q - c_i q - K =$$

$$c_i q + \frac{\int_{c_i}^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c_i)\right]^{N-1} q + (N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}(1-q) \right\}} q$$

$$+Kq \frac{\left\{1+(N-2)F(c_i)\right\}}{\left\{\left[1-F(c_i)\right]q + (N-1)F(c_i)(1-q)\right\}} - c_i q - K \qquad .$$

$$= \frac{\int_{c_i}^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c_i)\right]^{N-1} q + (N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}(1-q) \right\}} q$$

$$+ \frac{K(N-1)F(c_i)(2q-1)}{\left\{\left[1-F(c_i)\right]q + (N-1)F(c_i)(1-q)\right\}}$$

Since $q > 0.5$ by assumption, $\text{Profit}_{\text{large}} > 0$.

Similarly, for the winner of the small lot, ex post, his profit is

$$\text{Profit}_{\text{small}} = \beta(c_i)(1-q) - c_i(1-q) - K =$$

$$c_i(1-q) + \frac{\int_{c_i}^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c_i)\right]^{N-1} q + (N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}(1-q) \right\}} (1-q)$$

$$+K(1-q) \frac{\left\{1+(N-2)F(c_i)\right\}}{\left\{\left[1-F(c_i)\right]q + (N-1)F(c_i)(1-q)\right\}} - c_i(1-q) - K$$

$$= \frac{\int_{c_i}^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c_i)\right]^{N-1} q + (N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}(1-q) \right\}} (1-q)$$

$$+ \frac{K\left[1-F(c_i)\right](1-2q)}{\left\{\left[1-F(c_i)\right]q + (N-1)F(c_i)(1-q)\right\}}$$

Since $q > 0.5$ by assumption, the second term in the last equality is negative and depends on K and the first term is positive and independent of K. Therefore, the profit for the winner of the small lot can be either be positive or negative depending on $K,\ c_i,\ F(\cdot),\ N$ and q.

Proof of Proposition 4

We first examine how equilibrium bidding strategy changes when the number of bidders increases. Denote

$$\beta_N(c) = c + \frac{\int_c^{\bar{c}} \left\{ \left[1-F(x)\right]^{N-1} q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q) \right\} dx}{\left\{ \left[1-F(c)\right]^{N-1} q + (N-1)F(c)\left[1-F(c)\right]^{N-2}(1-q) \right\}}$$
$$+K \frac{\left\{1+(N-2)F(c)\right\}}{\left\{\left[1-F(c)\right]q+(N-1)F(c)(1-q)\right\}}$$

as the equilibrium bidding strategy when there are N bidders and

$$\beta_{N+1}(c) = c + \frac{\int_c^{\bar{c}} \left\{ \left[1-F(x)\right]^{N} q + NF(x)\left[1-F(x)\right]^{N-1}(1-q) \right\} dx}{\left\{ \left[1-F(c)\right]^{N} q + NF(c_i)\left[1-F(c)\right]^{N-1}(1-q) \right\}}$$
$$+K \frac{\left\{1+(N-1)F(c)\right\}}{\left\{\left[1-F(c)\right]q+NF(c)(1-q)\right\}}$$

as the equilibrium bidding strategy when there are $N+1$ bidders, then

$$\beta_{N+1}(c) - \beta_N(c)$$
$$= \frac{\int_c^{\bar{c}} \{A+B+D\}\, dx}{\left[1-F(c)\right]^{N-1} E} + K \frac{\left[1-F(c)\right]F(c)\left[2q-1\right]}{E}$$

where

$$A = \left[F(c)-F(x)\right]\left[1-F(x)\right]^{N-1}\left[1-F(c)\right]q^2$$
$$B = \left[F(c)-F(x)\right]\left[1-F(x)\right]^{N-2} F(x)F(c)(1-q)^2$$
$$D = \left[F(c)-F(x)\right]\left[NF(c)-2(N-1)F(c)F(x)+NF(x)-1\right]\left[1-F(x)\right]^{N-2} q(1-q)$$
$$E = \left\{\left[1-F(c)\right]q+(N-1)F(c)(1-q)\right\}\left\{\left[1-F(c)\right]q+NF(c)(1-q)\right\}$$

Since $q > 0.5$, the second term is always positive. In the first term, since we have $F(c_i) < F(x)$, $A < 0$, B<0 but D can be positive or negative depending on *F(c)* and *N*. Therefore, the sign of the first term is uncertain. As a result, there exists a threshold condition on the fixed cost *K* as function of $N, F(\cdot)$ and c:

$$\beta_{N+1}(c) - \beta_N(c) > 0 (< 0) \Leftrightarrow K > (<) K_1(N,c,F)$$

where .

$$K_1(N,c,F) := -\frac{\int_c^{\bar{c}} \{A+B+D\}\, dx}{[1-F(c)]^N F(c)(2q-1)}$$

We then examine how equilibrium bidding strategy changes when the size of the first lot increases.

$$\frac{\partial \beta(c)}{\partial q} = \frac{1}{\{[1-NF(c)]q + (N-1)F(c)\}^2} \times$$
$$\left\{ K[1+(N-2)F(c)][NF(c)-1] + (N-1)\int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} [F(c)-F(x)]dx \right\}$$

There are several interesting features of the way equilibrium bidding strategy varies with the size of the first lot. First, note that the sign is independent of lot size. Second, the rate of change is not uniform in per unit cost. The integral in the second term is always negative, but the first term can be positive or negative depending on *F(c)* and *N*. If the unit cost is less than the $(1/N)th$ quantile of the type distribution, $F^{-1}(1/N)$, then both terms are negative, i.e., bidding becomes more aggressive as the size of the first lot increases. For larger values of the per unit variable cost, however, an increase in the size of the first lot may lead to less aggressive bidding in equilibrium depending on the values of $K, N, F(\cdot)$ and c. The precise condition can be expressed as a threshold condition on the fixed cost K as function of $N, F(\cdot)$ and c:

$$\frac{\partial \beta(c)}{\partial q} > 0 (< 0) \Leftrightarrow K > (<) K_2(N,c,F)$$

where

$$K_2(N,c,F) := \frac{1}{1+(N-2)F(c)} \frac{N-1}{NF(c)-1} \int_c^{\bar{c}} \left(\frac{1-F(x)}{1-F(c)} \right)^{N-2} [F(x)-F(c)]dx$$

The threshold $K_2(N,c,F)$ is always positive for $c > F^{-1}(1/N)$. It is easily seen that $K_2(N,c,F)$ decreases with *N* since it is the product of positive terms each decreasing in *N*. By a similar argument one can easily establish that the threshold $K_2(N,c,F)$ is decreasing in *c*. Thus, in situations characterized by high fixed costs, many bidders and high per unit cost, an increase in the size of the first lot leads to less aggressive bidding. On the other hand, low fixed costs, few bidders and low per unit cost make bidding negatively related to the size of the first lot - i.e. bidding becomes more aggressive when the first lot becomes larger.

Proof of Proposition 5:

In the parallel auction model, since the equilibrium bidding strategies for both lots are increasing, in equilibrium, the bidder with the lowest marginal production cost will submit the lowest bids on both lots and hence be awarded the larger lot. On the other hand, the bidder with the second lowest marginal cost will win the small lot. Therefore, the expected payment to a supplier with private production cost c_i from the buyer is

$$EP_{parallel}(c_i) = \beta_1(c_i)q\Pr(c_i \text{ is the lowest cost}) + \beta_2(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost})$$

Because of symmetry, the ex-ante buyer's expected cost in this model becomes

$$EC_{buyer} = N\int_{\underline{c}}^{\bar{c}}\left[\beta_1(c_i)q\Pr(c_i \text{ is the lowest cost}) + \beta_2(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost})\right]f(c_i)dc_i$$

In the Yankee model, every bidder submits only one bid. The bidder with the lowest bid wins the larger lot and the bidder with the second lowest bid wins the small lot. Therefore, the expected payment to a supplier with private production cost c_i from the buyer is

$$EP_{Yankee}(c_i) = \beta(c_i)q\Pr(c_i \text{ is the lowest cost}) + \beta(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost}).$$

Because of symmetry, the ex-ante buyer's expected cost in this model becomes

$$EC_{buyer} = N\int_{\underline{c}}^{\bar{c}}\left[\beta(c_i)q\Pr(c_i \text{ is the lowest cost}) + \beta(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost})\right]f(c_i)dc_i$$

(A.26)

$$
\begin{aligned}
\Delta &= EP_{parallel} - EP_{Yankee} \\
&= \beta_1(c_i) q \Pr(c_i \text{ is the lowest cost}) + \beta_2(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost}), \\
&-\beta(c_i) q \Pr(c_i \text{ is the lowest cost}) - \beta(c_i)(1-q)\Pr(c_i \text{ is the second lowest cost})
\end{aligned}
$$

where $\Pr(c_i \text{ is the lowest cost}) = \Pr(c_i < C_{1:N-1}) = \left[1-F(c_i)\right]^{N-1}$ and

$$
\begin{aligned}
&\Pr(c_i \text{ is the second lowest cost}) = \Pr(c_i > C_{1:N-1}, c_i \le C_{2:N-1}) \\
&= \Pr(c_i > C_{1:N-1}) - \Pr(c_i > C_{1:N-1}, c_i > C_{2:N-1}) \\
&= (N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}
\end{aligned}
$$

Then, together with the equilibrium bidding strategies defined in Proposition 1 and 3, we have

$$
\begin{aligned}
\Delta &= qc_i\left[1-F(c_i)\right]^{N-1} + K\left[1-F(c_i)\right]^{N-1} + q\int_{c_i}^{\bar{c}}\left[1-F(x)\right]^{N-1}dx \\
&+(1-q)(N-1)\left[\int_{c_i}^{\bar{c}} F(x)\left[1-F(x)\right]^{N-2}dx - F(c_i)\int_{c_i}^{\bar{c}}\left[1-F(x)\right]^{N-2}dx\right] \\
&+K(N-1)F(c_i)\left[1-F(c_i)\right]^{N-2} + c_i(1-q)(N-1)F(c_i)\left[1-F(c_i)\right]^{N-2} \\
&+\int_{c_i}^{\bar{c}}\left[1-F(x)\right]^{N-2}dx(1-q)(N-1)F(c_i) \\
&-c_i\left[1-F(c_i)\right]^{N-1}q - c_i(N-1)F(c_i)\left[1-F(c_i)\right]^{N-2}(1-q) \\
&-\int_{c_i}^{\bar{c}}\left\{\left[1-F(x)\right]^{N-1}q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\}dx \\
&-K\left\{1+(N-2)F(c_i)\right\}\left\{\left[1-F(x)\right]^{N-1}q + (N-1)F(x)\left[1-F(x)\right]^{N-2}(1-q)\right\} \\
&= K\left[1-F(c_i)\right]^{N-1} + K(N-1)F(c_i)\left[1-F(c_i)\right]^{N-2} \\
&-K\left\{1+(N-2)F(c_i)\right\}\left[1-F(c_i)\right]^{N-2} \\
&= K\left[1-F(c_i)\right]^{N-1} + K(N-1)F(c_i)\left[1-F(c_i)\right]^{N-2} \\
&-K\left[1-F(c_i)\right]^{N-2} - K(N-2)F(c_i)\left[1-F(c_i)\right]^{N-2} \\
&= K\left[1-F(c_i)\right]^{N-1} + KF(c_i)\left[1-F(c_i)\right]^{N-2} - K\left[1-F(c_i)\right]^{N-2} \\
&= K\left[1-F(c_i)\right]^{N-2}\left[1-F(c_i)+F(c_i)-1\right] \\
&= 0
\end{aligned}
$$

Therefore, $EC_{parallel} - EC_{Yankee} = \int_{\underline{c}}^{\bar{c}}\Delta f(c_i)dc_i = \int_{\underline{c}}^{\bar{c}} 0 f(c_i)dc_i = 0$

This completes the proof of expected cost equivalence of the two auction models.

Appendix 2

In the following, we provide examples of the bidder instructions. The text is identical across the various treatments, but some numbers differed according to the specific treatment combination. For example, in C1 the bid function is only used in a single computerized auction, whereas in C100 and C100+ it is used in 100 computerized auctions.

Computerized experiments

Instructions for the experiment for the parallel auction:

This is an experiment on decision making. The following instructions are simple, and if you follow them carefully and make good decisions, you may earn a considerable amount of money. During the experiment your payoff will be in experimental Francs that will be converted into Euro at the end of the experiment at the following rate:

16 Experimental Francs = 1 Euro

Payments will be made privately at the end of the experiment. If you have a question at any time, please raise your hand and the monitor will answer it. We ask that you not talk with one another for the duration of the experiment. On your desks you should have a checkout form, a pen, and a copy of the consent form.

Your Experimental Task

In each round of today's session you will be bidding against three computerized competitors. The computerized competitors have been programmed to bid in a way that would maximize their expected earnings when they bid against likewise programmed competitors. Their bidding strategies are only based on the cost draw, the distribution of costs, the fixed costs and the number of competitors, which are public to all bidders. The bids of the computerized bidders have also been determined, and they cannot be affected by your decisions today.

At the beginning of the experiment, you will receive a sheet of paper on which you will see a list of 20 numbers. Each number is between 0 and 10 Experimental Francs (randomly drawn with equal probability) and has been rounded to one decimal plac**e**. Each number represents a possible unit cost that you may have to produce a fictitious commodity. The process of selecting possible unit costs is exactly the same for everyone.

You can either win a contract on a small lot with 30 units of the commodity or on a large lot for 70 units of the commodity. You can bid on both lots. The unit costs determine the total cost for the small and large lot, which you can also find on the sheet. For each of your 20 possible cost values, you should write down a bid for the small lot and a bid for the large lot in the space provided on the sheet of paper for the small and the large lot. Then, bids are connected with a line to determine bids for all possible variable costs. We will call this your bid function. After all of the participants have chosen their bids for each of the 20 possible cost values, the lists will be collected.

The bids function will then be used in **100 auctions,** where you compete against the three computerized agents in each round. The strategy of the computerized agents does not change across the different auctions. However, in each of these auctions, your variable costs and also those of your competitors will be randomly drawn from a uniform distribution between 0 and 10. The bidder in each auction with the lowest bid for each lot wins this lot and pays the exact amount of his or her bid. If a single bidder wins both lots, he will get the large lot and the second best bidder on the small lot will win this lot. In the case of a tie, the winner will be determined randomly by the software. Winners in an auction will earn the difference between their bid and their true costs. If you are not a winner, you will not earn any money. You will be paid an average of your winnings in the 100 auctions. After the auction you will participate in a brief survey.

Before you submit the bid sheet, you should think about your bid strategy for

- high and low cost draws, and
- the small and the large lot.
- Would your strategy change, if a certain bid was only valid for 1 and not for 100 auction?

Results of both sessions will be e-mailed to you together with information about how much money you have won. You will not learn information about other bids in the auction, just whether you won or you lost. Are there any questions?

Instructions for the experiments for the Yankee auction:

This is an experiment on decision making. The following instructions are simple, and if you follow them carefully and make good decisions, you may earn a considerable amount of money.

During the experiment your payoff will be in experimental Francs that will be converted into Euro at the end of the experiment at the following rate:

16 Experimental Francs = 1 Euro

Payments will be made privately at the end of the experiment. If you have a question at any time, please raise your hand and the monitor will answer it. We ask that you not talk with one another for the duration of the experiment. On your desks you should have a checkout form, a pen, and a copy of the consent form.

Your Experimental Task

In each round of today's session you will be bidding against three computerized competitors. The computerized competitors have been programmed to bid in a way that would maximize their expected earnings when they bid against likewise programmed competitors. Their bidding strategies are only based on the cost draw, the distribution of costs, the fixed costs and the number of competitors, which are public to all bidders. The bids of the computerized bidders have also been determined, and they cannot be affected by your decisions today.

At the beginning of the experiment, you will receive a sheet of paper on which you will see a list of 20 numbers. Each number is between 0 and 10 Experimental Francs (randomly drawn with equal probability) and has been rounded to one decimal plac**e**. Each number represents a possible unit cost that you may have to produce a fictitious commodity. The process of selecting possible unit costs is exactly the same for everyone.

You can either win a contract on a small lot with 30 units of the commodity or on a large lot for 70 units of the commodity. You can submit a bid per unit that is valid for both lots. The unit costs determine the total cost for the small and large lot, which you can also find on the sheet. For each of your 20 possible cost values, you should write down a bid in the space provided on the sheet of paper that is valid for the small and the large lot. Then, bids are connected with a line to determine bids for all possible variable costs. We will call this your bid function. After all of the participants have chosen their bids for each of the possible cost values, the lists will be collected.

The bids function will then be used in **100 auctions,** where you compete against the three computerized agents in each round. The strategy of the computerized agents does not change across the different auctions. However, in each of these auctions, your variable costs and also those of your competitors will be randomly drawn from a uniform distribution between 0 and 10. The bidder in each auction with the lowest bid wins the large lot, the second best bidder the small lot and pays the exact amount of his or her bid. In the case of a tie, the winner will be determined randomly by the software. Winners in an auction will earn the difference between their bid and their true costs. If you are not a winner, you will not earn any money. You will be paid an average of your winnings in the 100 auctions. After the auction you will participate in a brief survey.

Before you submit the bid sheet, you should think about your bid strategy for

- high and low cost draws,
- Would your strategy change, if a certain bid was only valid for 1 and not for 100 auction?

Results will be e-mailed to you together with information about how much money you have won. You will not learn information about other bids in the auction, just whether you won or you lost. Are there any questions?

Instructions for the experiments for the single lot auction:

This is an experiment on decision making. The following instructions are simple, and if you follow them carefully and make good decisions, you may earn a considerable amount of money.

During the experiment your payoff will be in experimental Francs that will be converted into Euro at the end of the experiment at the following rate:

16 Experimental Francs = 1 Euro

Payments will be made privately at the end of the experiment. If you have a question at any time, please raise your hand and the monitor will answer it. We ask that you not talk with one another for the duration of the experiment. On your desks you should have a checkout form, a pen, and a copy of the consent form.

Your Experimental Task

In each round of today's session you will be bidding against three computerized competitors. The computerized competitors have been programmed to bid in a way that would maximize their expected earnings when they bid against likewise programmed competitors. Their bidding strategies are only based on the cost draw, the distribution of costs, the fixed costs and the number of competitors, which are public to all bidders. The bids of the computerized bidders have also been determined, and they cannot be affected by your decisions today.

At the beginning of the experiment, you will receive a sheet of paper on which you will see a list of 10 numbers. Each number is between 0 and 10 Experimental Francs (randomly drawn with equal probability) and has been rounded to one decimal plac**e**. Each number represents a possible unit cost that you may have to produce a fictitious commodity. The process of selecting possible unit costs is exactly the same for everyone.

You can either win the full amount of the commodity or nothing. You can submit only a bid per unit that is valid for the full amount. The unit costs determine the total cost fort, which you can also find on the sheet. For each of your 10 possible cost values, you should write down a bid in the space provided on the sheet. Then, bids are connected with a line to determine bids for all possible variable costs. We will call this your bid function. After all of the participants have chosen their bids for each of the possible cost values, the lists will be collected.

The bids function will then be used in **100 auctions,** where you compete against the three computerized agents in each round. The strategy of the computerized agents does not change across the different auctions. However, in each of these auctions, your variable costs and also those of your competitors will be randomly drawn from a uniform distribution between 0 and 10. The bidder in each auction with the lowest bid wins the full amount, and pays the exact amount of his or her bid. In the case of a tie, the winner will be determined randomly by the software. Winners in an auction will earn the difference between their bid and their true costs. If you are not a winner, you will not earn any money. You will be paid an average of your winnings in the 100 auctions. After the auction you will participate in a brief survey.

Before you submit the bid sheet, you should think about your bid strategy for

- high and low cost draws,
- Would your strategy change, if a certain bid was only valid for 1 and not for 100 auction?

Results will be e-mailed to you together with information about how much money you have won. You will not learn information about other bids in the auction, just whether you won or you lost.
Are there any questions?

Human subject experiments

For the human subject experiments, only the instructions for the parallel auction are presented. For the Yankee and the single lot auction, both, the auction and bidding rules, and the winner determination are the same as within the computerized experiments.

Instructions for the experiment for the parallel auction:

This is an experiment on decision making. The following instructions are simple, and if you follow them carefully and make good decisions, you may earn a considerable amount of money. During the experiment your payoff will be in experimental Francs that will be converted into Euro at the end of the experiment at the following rate:

16 Experimental Francs = 1 Euro

Payments will be made privately at the end of the experiment. If you have a question at any time, please raise your hand and the monitor will answer it. We ask that you not talk with one another for the duration of the experiment. On your desks you should have a checkout form, a pen, and a copy of the consent form.

Your Experimental Task

In each round of today's session you will be bidding against three other human bidders. These bidders will be determined randomly in each of 16 rounds from the pool of participants.

At the beginning of the experiment, you will receive a sheet of paper on which you will see a list of 10 numbers. Each number is between 0 and 10 Experimental Francs (randomly drawn with equal probability) and has been rounded to one decimal place. Each number represents a possible unit cost that you may have to produce a fictitious commodity. The process of selecting possible unit costs is exactly the same for everyone.

You can either win a contract on a small lot with 30 units of the commodity or on a large lot for 70 units of the commodity. You can bid on both lots. The unit costs determine the total cost for the small and large lot, which you can also find on the sheet. For each of your 10 possible cost values, you should write down a bid for the small lot and a bid for the large lot in the space provided on the sheet of paper for the small and the large lot. Then, bids are connected with a line to determine bids for all possible variable costs. We will call this your bid function. After all of the participants have chosen their bids for each of the 10 possible cost values, you can upload the sheet on a server. In the auctions, your variable costs and also those of your competitors will be randomly drawn from a uniform distribution between 0 and 10, and the computer determines the bids based on the randomly drawn costs. Next you will be randomly matched to three of the other players in the room and the bids of all bidders will enter the auction. The bidder in each auction with the lowest bid for each lot wins this lot and pays the exact amount of his or her bid. If a single bidder wins both lots, he will get the large lot and the second best bidder on the small lot will win this lot. In the case of a tie, the winner will be determined randomly by the software. Winners in an auction will earn the difference between their bid and their true costs. If you are not a winner, you will not earn any money. After each round you get the following feedback:

- Your own cost draw,
- Your bid for the small and the large lot according to your bid function,
- The bids of your competitors of both lots, and
- If you have won or not.

Additionally, the mean and all winning bids of each round will be shown aggregated on the webpage. Please consider both the feedback of the previous round and the aggregated winning bids.

Before you submit the bid sheet, you should think about your bid strategy for

- high and low cost draws, and
- the small and the large lot.

Results of the whole session will be e-mailed to you together with information about how much money you have won.

Are there any questions?

Appendix 3

Scatter plots of the parallel auction

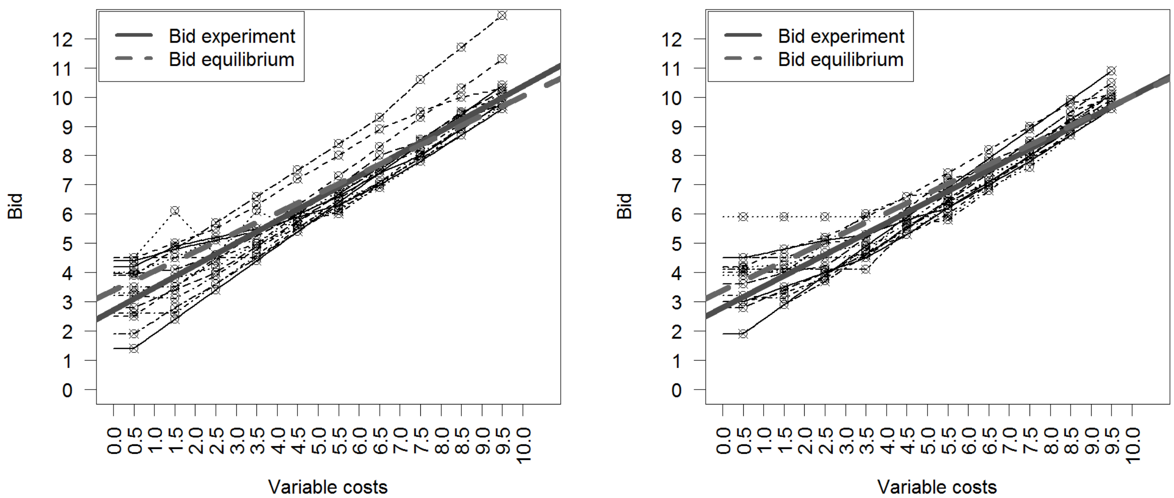

Figure 2: Scatter plot of bids and the equilibrium bid functions for **H** on the *small* lot for the first (left) and the 7th auction (right) for the parallel auction (*q=0.7*).

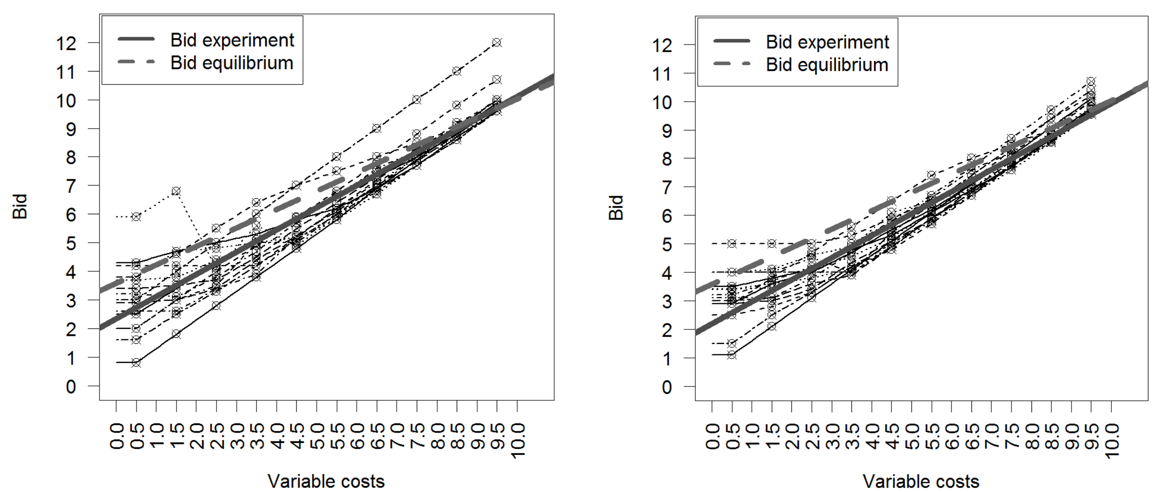

Figure 3: Scatter plot of bids and the equilibrium bid functions for **H** on the *large* lot for the first (left) and the 7th auction (right) for the parallel auction (*q=0.7*).

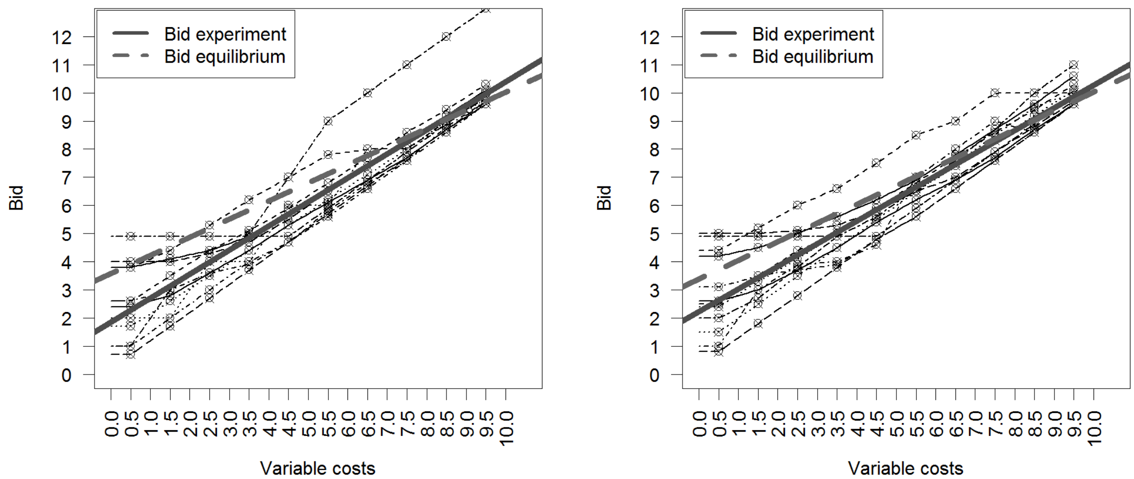

Figure 4: Scatter plot of bids and the equilibrium bid functions for **C1** on the large lot (left) and for the small lot (right) for the parallel auction (*q=0.7*).

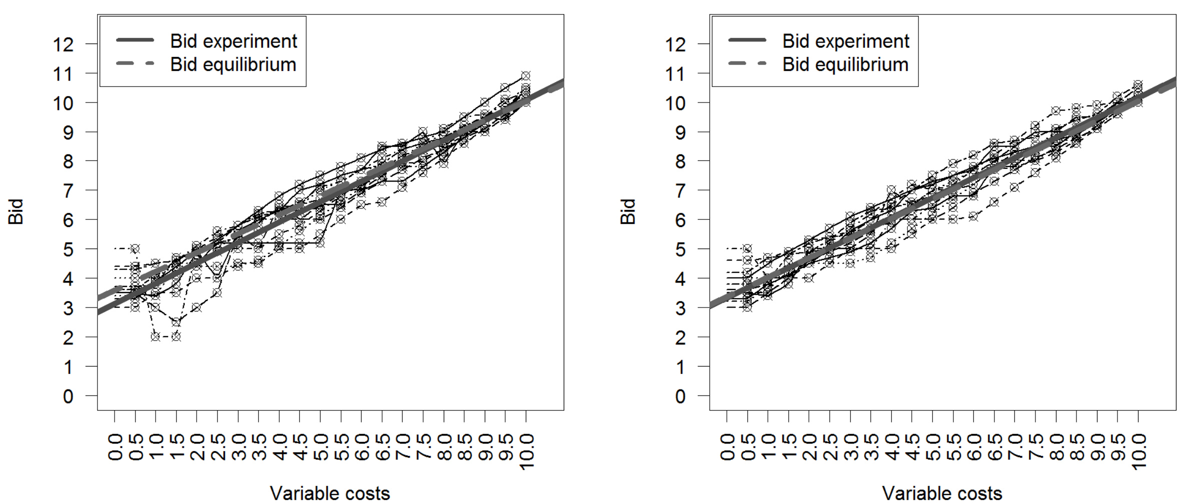

Figure 5: Scatter plot of bids and the equilibrium bid functions for **C100** on the large lot (left) and for the small lot (right) for the parallel auction (*q*=0.7).

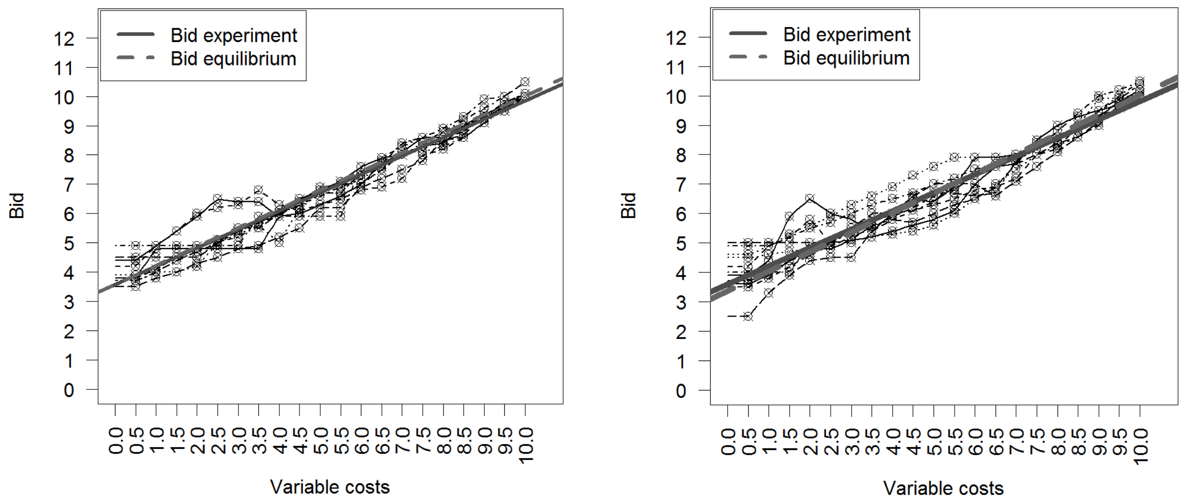

Figure 6: Scatter plot of bids and the equilibrium bid functions for **C100+** on the large lot (left) and for the small lot (right) for the parallel auction (*q=0.7*).

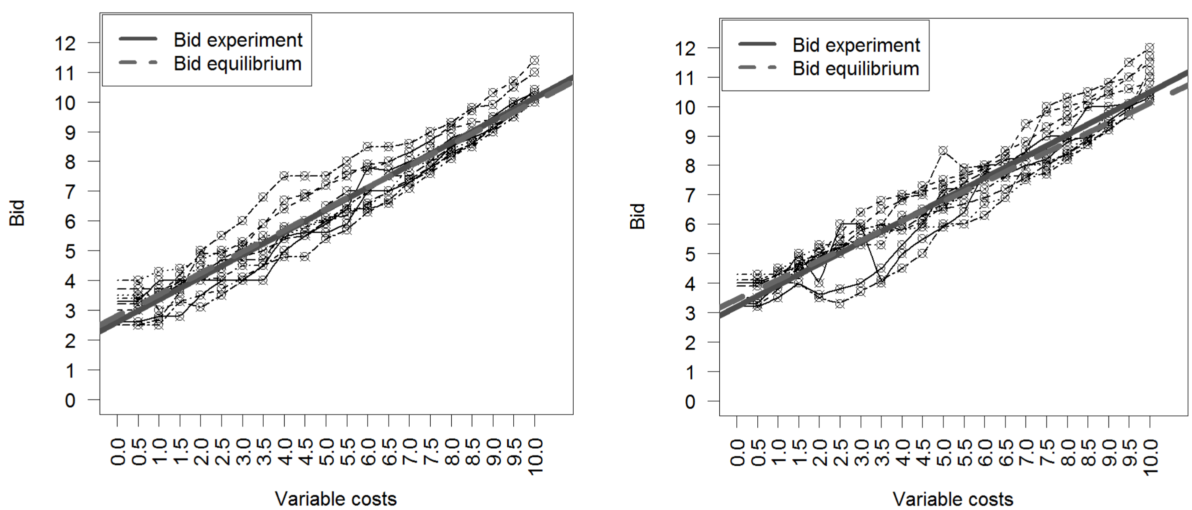

Figure 7: Scatter plot of bids and the equilibrium bid functions for **C100** on the large lot (left) and for the small lot (right) for the parallel auction (*q=0.9*).

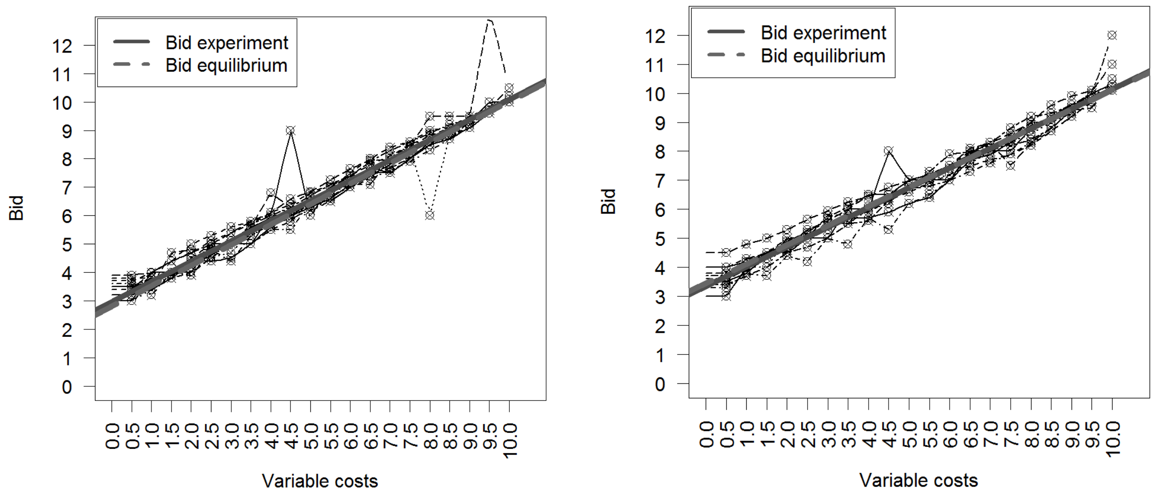

Figure 8: Scatter plot of bids and the equilibrium bid functions for **C100+** on the large lot (left) and for the small lot (right) for the parallel auction (*q=0.9*).

Scatter plots of the Yankee auction

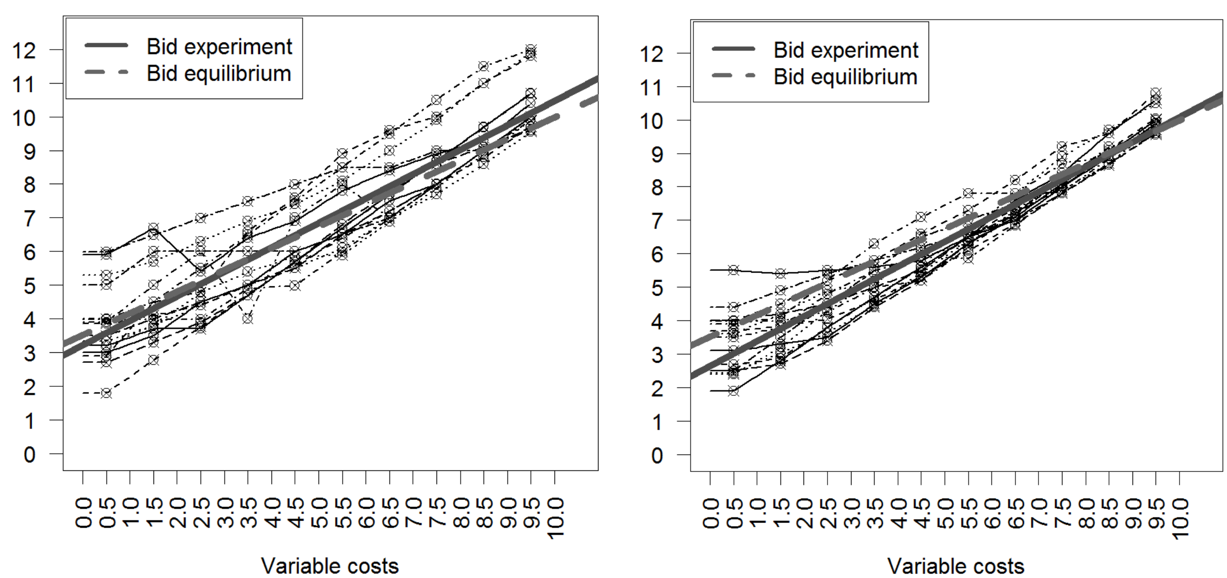

Figure 9: Scatter plot of bids and the optimal bid functions in H auction 1 (left) auction 7 (right) for different variable costs in the Yankee auction.

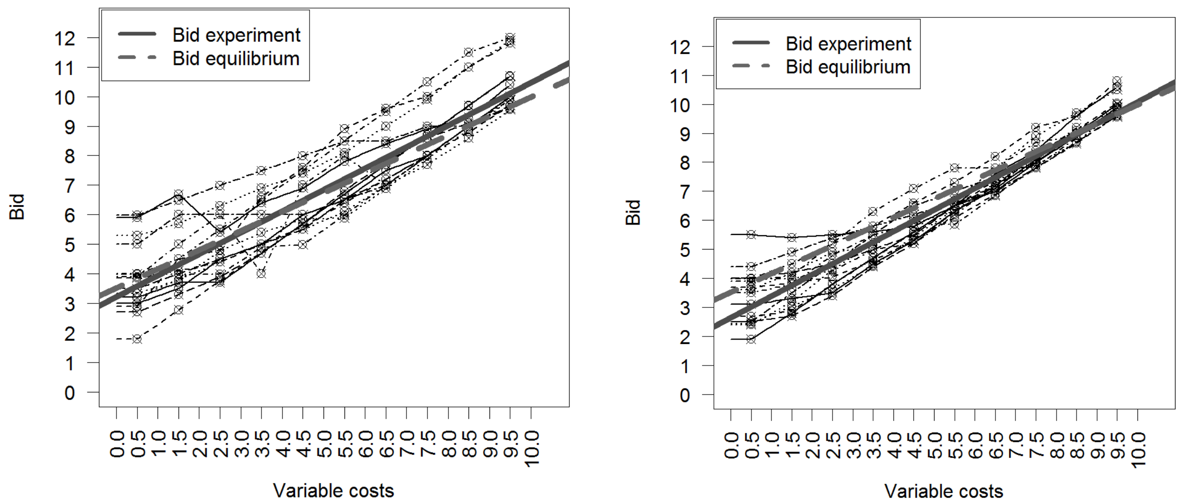

Figure 10: Scatter plot of bids and the equilibrium bid functions in **H** for the first (left) and the 7th auction (right) for Yankee auction.

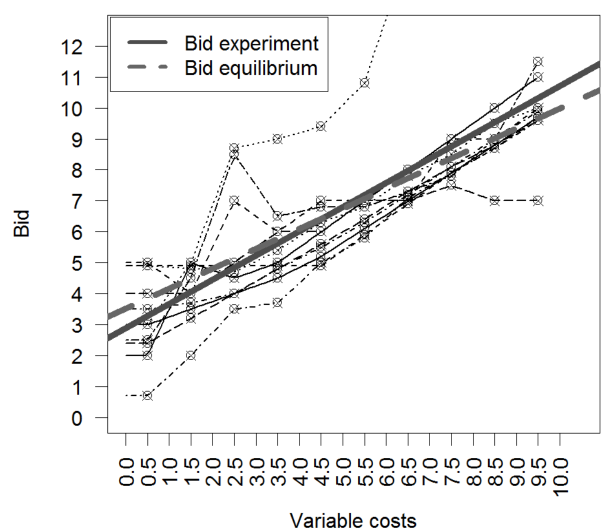

Figure 11: Scatter plot of bids and the equilibrium bid functions for **C1** for the Yankee auction.

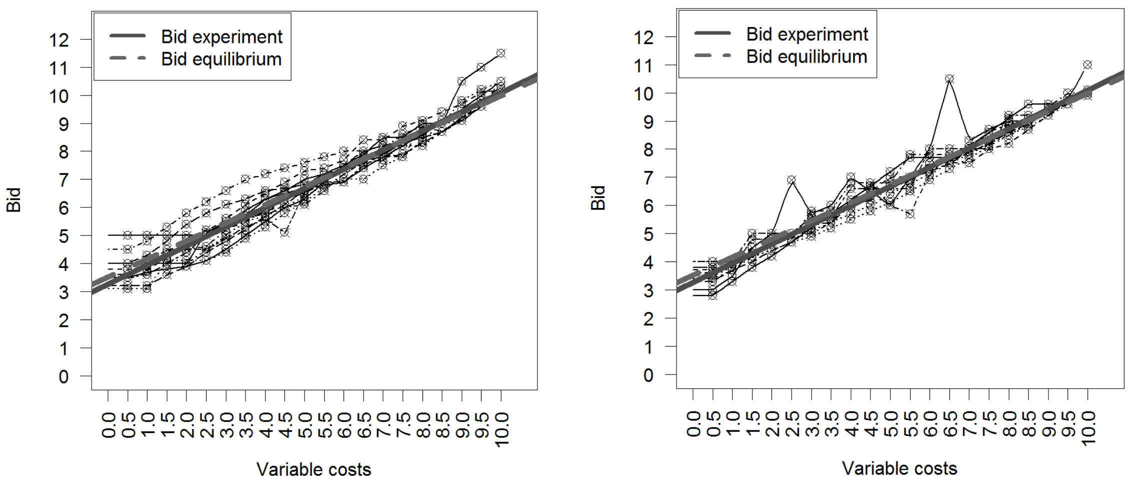

Figure 12: Scatter plot of bids and the equilibrium bid functions for *q=0.7* with **C100** (left) and **C100+** (right) for the Yankee auction.

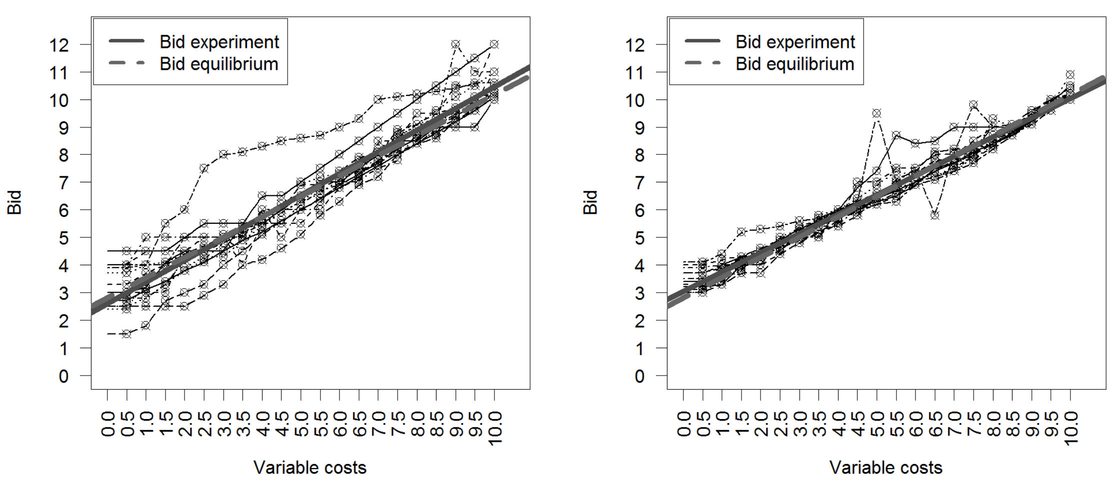

Figure 13: Scatter plot of bids and the equilibrium bid functions for q=0.9 with **C100** (left) and **C100+** (right) for the Yankee auction

Scatter plots of the single lot auction

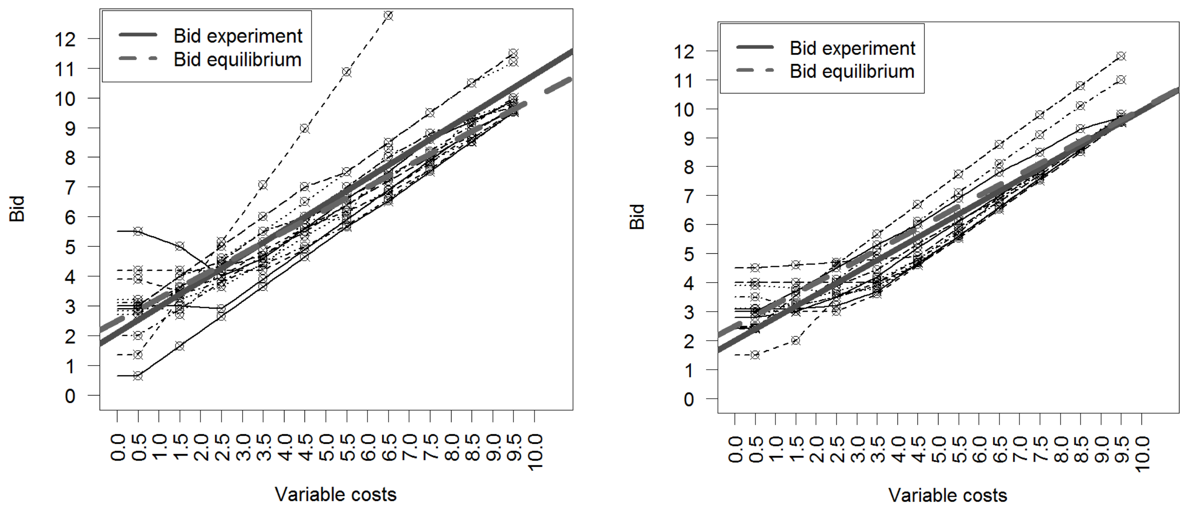

Figure 14: Scatter plot of bids and the equilibrium bid functions in **H** for the first (left) and the 7th auction (right) for the single lot auction.

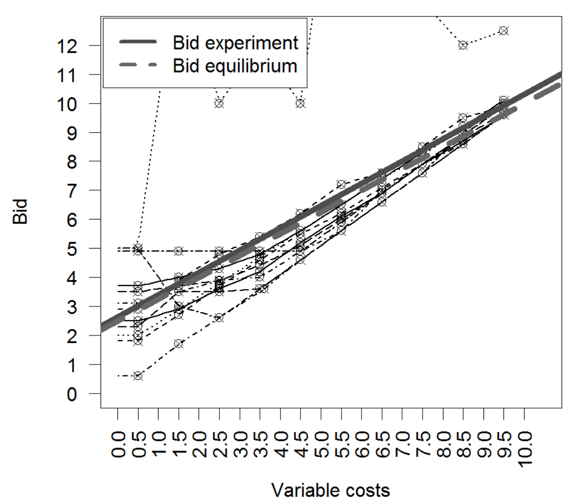

Figure 15: Scatter plot of bids and the equilibrium bid functions for **C1** for the single lot auction.

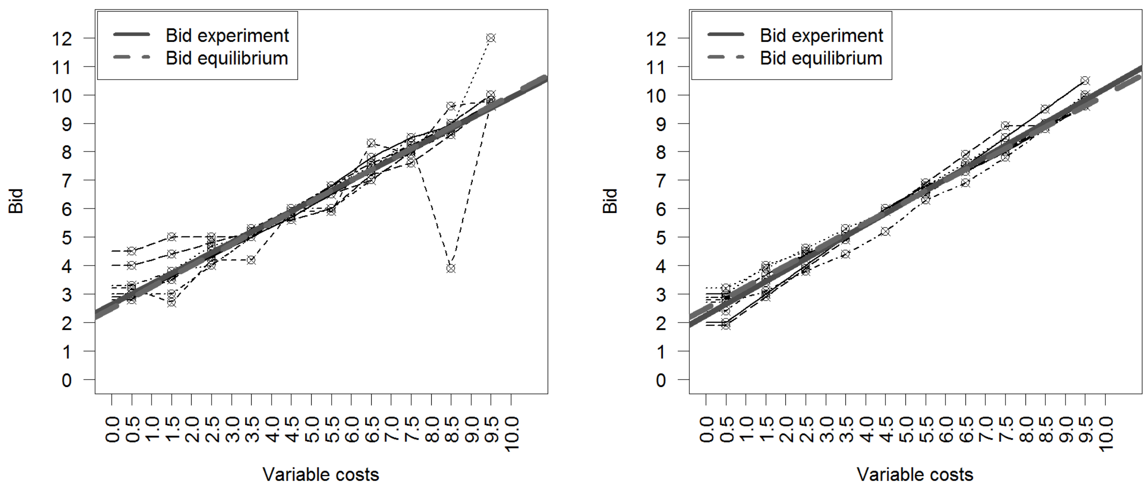

Figure 16: Scatter plot of bids and the equilibrium bid functions with **C100** (left) and **C100+** (right) for the single lot auction.

Appendix 4

Experiments with a Split Parameter of 0.9

In addition to our experiments with q=0.7 we tested in the laboratory the Yankee and the parallel auction also with q=0.9 for C100 and C100+ (see Table 9) to make sure that the high predictive accuracy of the RNBNE function found for a split of q=0.7 is robust against changes of the split parameter. We also wanted to test Propositions 2 and 4.

27 students took part in C100 and they earned an average payoff of 13.98€ (i.e., altogether 53 students played in C100 for q=0.7 and q=0.9 with an average payoff 13.90€). In C100+ 30 students were tested with q=0.9 with an average payoff of 15.18€, which results in 52 students in C100+ for both split points with an average payoff of 14.39€.

Treatment	Bid fct. reused	Subjects	Information	Split	Auction format	No. of Bidders
C100.P9	100	Computer	Prior distribution	0.9	Parallel	12
C100.Y9	100	Computer	Prior distribution	0.9	Yankee	15
C100+.P9	100	Computer	Prior & RNBNE bid fct.	0.9	Parallel	14
C100+.Y9	100	Computer	Prior & RNBNE bid fct.	0.9	Yankee	16

Table 9: Overview of treatment combinations in the experiments with a split of q=0.9 rather than 0.7 in the main part of the paper.

Result 1: *In the parallel auction as q increases, bidding becomes more (less) aggressive for the large lot (small lot), as theory predicts (Proposition 2). Only for the small lot, where the differences between the equilibrium bid function are small, the theoretical prediction does not hold.*

Support: For the *large lot* the intercept in q=0.9 is lower for C100 and C100+ and both regression lines are close to the equilibrium bid function (see Table 11). The equilibrium bid function for q=0.9 is lower than for q=0.7 until high unit costs between 8 and 9 monetary units. This is reflected in the regression lines of C100+, where both regressions intersect at a unit cost of 8. In C100, the regression line for q=0.7 is always above q=0.9, i.e., bidders bid more aggressive in q=0.9 throughout.

For the *small lot* the equilibrium bid function for q=0.9 is always below q=0.7 by a small number of 0.08. Also in C100+ the regression line for q=0.9 is slightly below q=0.7, in C100 it is the opposite. The differences between the C100 and the C100+ regression line are less than one monetary unit, however.

	Small lot				Large lot				
	α	β (unit cost)	Std. error (p-value)	Mult. R^2	α	β (unit cost) (p-value)	Std. error (p-value)	Mult. R^2	# bids / bidders
RNBNE	2.80	0.721			3.44	0.67			
C100	2.16	0.753	0.009 (0.000)	0.972	2.90	0.729	0.011 (0.000)	0.952	240/12
C100+	2.85	0.711	0.008 (0.000)	0.971	3.42	0.679	0.005 (0.000)	0.985	280/14

Table 10: Regression coefficients for the empirical bid functions (w/o bidder ID) of the parallel auction with q=0.9

		Large lot			Small lot			
	MSE LOESS	MSE RNBNE	MSE Single Item RNBNE	MSE Constant Factor	MSE LOESS	MSE RNBNE	MSE Single Item RNBNE	MSE Constant Factor
C100	0.312	0.327	0.330	0.864	0.387	0.448	0.736	1.090
C100+	0.163	0.235	0.244	0.860	0.095	0.102	0.350	0.981

Result 2: *As q increases, bidding becomes more aggressive for our experimental environment with fixed costs of 1 and uniformly distributed unit costs in the Yankee auction, which is in line with Proposition 4.*

Support: The intercept in *q*=0.9 is below that of *q*=0.7 (see Table 6 and Table 11). The equilibrium bid function of *q*=0.7 is higher than *q*=0.9 up to a unit cost of 9. The difference between both regression lines in C100 is small. In C100+ the regression line of *q*=0.9 is always below that of *q*=0.7. Overall, the data from the lab confirms the theoretical prediction in Proposition 4.

	α	β (unit cost)	Std. error β (p-value)	Multiple R^2	MSE LOESS	MSE RNBNE	MSE Single item RNBNE	MSE constant profit factor	Number of bids / bidders
RNBNE	2.82	0.735							
C100	2.42	0.786	0.010 (0.000)	0.961	0.572	0.6122	0.671	0.966	300/15
C100+	2.95	0.699	0.007 (0.000)	0.970	0.140	0.160	0.236	0.896	320/16

Table 11: Regression coefficients for the empirical bid functions (w/o bidder ID) of the Yankee auction with q=0.9

Corollary 1 states that if bidders submitted their RNBNE bid function in a Yankee auction, then they always make a positive payoff if they win the large lot, but they could also make a loss in the small lot. Losses are more likely with high fixed costs. In our experiments fixed costs were low, and the lowest payoff that we encountered was zero. Possible losses due to high fixed costs are another phenomenon to be analyzed in the future, but losses in experiments are difficult as in most experiments it is considered unethical to have bidders pay a loss and external validity of the experiments becomes an issue.

Result 3: *As q increases, the procurement costs decrease, but the efficiency remains high.*

	Efficiency E	Cost ratio C
C100+, parallel, q=0.9	98.83%	52.76%
C100, parallel, q=0.9	97.87%	51.71%
C100+, Yankee, q=0.9	99.50%	52.19%
C100, Yankee, q=0.9	97.98%	51.97%

Table 12: Efficiency and auctioneer's costs for C100 and C100+ with $q=0.9$

Support: If you compare Table 8 and Table 12, you will see that the efficiency is hardly changing. We did not find a significant difference in efficiency E and cost ratio C within the same treatment combination between the parallel and the Yankee auction using a Wilcoxon rank sum test (α=0.01) (see Table 13). However, since the supplier with a cost advantage wins a larger amount, when q is increasing, the procurement costs are reduced.

	C100+, Yankee, q=0.9	C100, Yankee, q=0.9	C100+, Single item	C100, Single item
C100, parallel, q=0.9		E: 0.1959 C: 0.9805		E: 0.0042 C: 0.0246
C100+, parallel, q=0.9	E: 0.0112 C: 0.4175		E: 0.0037 C: 0.0177	

Table 13: Significance tests (Wilcoxon rank sum tests) for a difference on all pairs of auction formats and treatments concerning efficiency E and cost ratio C.