On the Impact of Cognitive Limits in Combinatorial Auctions: An Experimental Study in the Context of Spectrum Auction Design

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Combinatorial auctions have been studied analytically for several years, but only limited experimental results are available, in particular for auctions with more than 10 items. We investigate the Hierarchical Package Bidding auction (HPB), the Combinatorial Clock auction (CC), and one pseudo-dual price auction (PDP) experimentally, as all these formats were used or suggested for high-stakes spectrum auctions. We want to understand the impact that different auction formats have on bidder behavior and allocative efficiency. Interestingly, we find that the main source of inefficiency in all formats is the bidders' preselection of packages, rather than their strategies or auction rules; bidders mostly preselect a small number of packages of interest early in the auction. CC achieves high efficiency and revenue in all experiments, but HPB yields similar results even in value models, where hierarchical pre-packaging is difficult. Due to their influence on the decision of the US Federal Communication Commission, we intentionally repeated a set of experiments conducted by Goeree and Holt (GH) [J.K. Goeree, C.A. Holt, Hierarchical package bidding: A paper & pencil combinatorial auction, Games Econ. Behav. (2008)]. We get similar aggregate results but in contrast to GH, we find that small bidders have more difficulties in coordinating their bids to outbid a national bidder in a threshold problem with HPB. In addition to the value models with global synergies in GH, we used value models where bidders have local synergies, which play a significant role in spectrum auctions in the field and lead to different results.

Key words: auctions, lab experiments, group behavior, individual behavior

1. Introduction

Designing *combinatorial auction* (CA) markets is a formidable task: Many theoretical results are negative in the sense that it seems quite unlikely to design fully efficient and practically applicable CAs with a strong game-theoretical solution concept (Cramton et al., 2006a). Experimental research has shown that *iterative combinatorial auction* (ICA) formats with linear prices achieve very high levels of efficiency (Porter et al., 2003; Kwasnica et al., 2005; Brunner et al., 2009; Scheffel et al., 2010). Most of these experiments are based on value models with only a few items of interest. While it is important to understand bidder behavior in small CAs, we need to know whether the promising results carry over to larger auctions, since applications of CAs can easily have more than 10 items. We ran laboratory experiments with those CA formats that have been used or analyzed for the sale of spectrum licenses, the most prominent and most thoroughly investigated application domain for CAs. However, the analysis is not restricted to spectrum sales and the results are also relevant to the design of auctions in other domains, such as procurement and transportation. We do not only observe which package bids bidders submit, but also which packages they evaluate. Interestingly, we find that auction design rules matter, but the cognitive barriers of bidders, i.e. the number of packages they evaluate, are the biggest barriers to full efficiency. These cognitive barriers have not been an issue in the experimental literature so far.

1.1. Spectrum auctions

There has been a long and ongoing discussion on appropriate auction mechanisms for the sale of spectrum rights in the USA (Porter and Smith, 2006). Since 1994, more than 70 spectrum auctions were run using the simultaneous multiround auction (***SMR***), an auction format which is based on work by Paul Milgrom, Robert Wilson, and Preston McAfee (Milgrom, 2000). While in the SMR auction several items are sold in a single auction, package bidding is not allowed. This leads to a number of strategic problems for bidders (Cramton, 2009b), such as limited substitution of spectrum due to the activity rules employed and the *exposure problem*. The latter refers to the phenomenon that a bidder is exposed to the possibility that he may end up winning a collection of licenses that he does not want at the prices he has bid, because the complementary licenses have become too expensive. CAs allow for bids on packages of items, avoiding exposure problems. The design of such auctions, however, led to a number of fundamental design problems, and many contributions during the past few years (Cramton et al., 2006b).

Since 2005 countries such as Trinidad and Tobago, the UK, the Netherlands, Denmark, and the USA have adopted CAs for selling spectrum rights (Cramton, 2009b). The USA used an auction format called Hierarchical Package Bidding (***HPB***) (Goeree and Holt, 2008), in which only restricted combinatorial bidding is allowed and bidders can only bid on hierarchically structured pre-defined packages. Other countries used a version of the Combinatorial Clock (***CC***) auction (Porter et al., 2003) or the Clock-Proxy auction (Ausubel et al., 2006), which extends the CC auction by a sealed-bid phase.[1] The pricing in the CC auction is rather simple, the price of over-demanded items raises by the specified increment and bidders can only place package bids at the current prices. A third type of CA formats, which has been analyzed for spectrum auctions in the US, uses pseudo-dual linear prices, i.e., they use an approximation of the dual variables of the winner determination problem. Different versions have been discussed in the literature (Brunner et al. (2009); Bichler et al. (2009)), which are all based on the Resource Allocation Design by Kwasnica et al. (2005) and earlier by Rassenti et al. (1982). We refer to these auction formats as pseudo-dual price (***PDP***) auctions in the following.

All auction formats which have been used for selling spectrum use linear (i.e., item-level) ask prices. This is an interesting observation, as there are negative theoretical results by Kelso and Crawford (1982), and Bikhchandani and Ostroy (2002) which show that only non-linear and personalized competitive equilibrium prices can always exist for general valuations. Disadvantages of non-linear price CAs have been studied by Scheffel et al. (2010), who show that these auction formats are impractical, due to the enormous communication complexity when selling more than only a few items. The linearity of ask prices, however, also seems to be an important requirement in other domains such as electricity markets, which allow for package bids (Meeus et al., 2009). There has so far been no game-theoretical equilibrium analysis on any of the linear-price auction formats discussed above.

In a Public Notice, the US Federal Communications Commission (FCC) justified their choice of HPB by stating that "the mechanism for calculating CPEs [current price estimates] is significantly simpler than other package bidding pricing mechanisms that adequately address coordination issues" (FCC, 2007, §221). While this argument is true when comparing HPB against PDP,

[1] A specific version of a two-stage clock auction with a supplementary bids phase has been implemented by Ofcom in the UK and other European countries. The clock phase allows for a single package bid per bidder in each round. Then, the supplementary bids phase allows for multiple package bids in a sealed bid phase. After this second stage the winners are determined, and bidder Pareto-optimal core prices are calculated (Day and Raghavan, 2007). This two-stage clock auction design in the UK is considerably different from single-stage formats discussed in this paper. There is also an active discussion on core-selecting auctions (Lamy, 2009; Goeree and Lien, 2010). We focus on the single-stage combinatorial clock auction, as it has been described by Porter et al. (2003), and do not discuss more complex multi- stage and core-selecting auction formats.

it does not necessarily hold when comparing HPB to the CC auction. Apart from the simplicity of the respective auction formats, the main arguments of the FCC were based on a set of lab experiments conducted by Brunner et al. (2009) and, more specifically, Goeree and Holt (2008). In their main value model, specific auction rules like purchase and activity limits apply and the result in a different value model that does not offer such an obvious hierarchical pre-packaging might be different.

Given the importance of spectrum auctions for the telecommunications sector and the significant costs that companies have to bear for licenses, these auction formats demand more empirical results.

1.2. Contribution of this Paper

Based on the influence of the experiments by Goeree and Holt (2008) on the FCCs decision, we intentionally decided to partly replicate their experiments but extended them with the CC auction. Since in their value model simple simulations indicate that even without package bids high levels of efficiency can be obtained, we extend our experiments further with a new value model that shows different characteristics and captures the local synergies of licenses, which can be well motivated from observations in the field (Ausubel et al. (1997); Moreton and Spiller (1998)).

From these extensions we derive new insights on bidder behavior in three important ICA formats, which have been used or proposed for selling spectrum licenses: CC, HPB, and PDP. The way ask prices are determined in these auction formats varies considerably, and there has been a long discussion on the pros and cons of each approach (FCC, 2007). While we find statistically significant differences in efficiency and revenue among the auction formats, these differences are small. We replicate some important findings of Goeree and Holt (2008) in their value model with global synergies, namely that HPB achieves higher efficiency and auctioneer revenue than PDP. Interestingly, HPB achieved similar results to PDP even in a second value model with local synergies, which is less suitable to a hierarchical pre-packaging. The fully combinatorial CC auction, however, realizes the highest median efficiency and revenue in both value models.

The reasons for inefficiencies in linear-price ICAs are not easy to explain, but our analysis reveals that the bidders' restricted package selection is the largest barrier to efficiency, while the differences in auction rules had much less impact. Bidders mostly pre-selected packages in the first round and then focused on this small subset of packages throughout the auction, even though they had enough time in each round and adequate monetary incentives to analyze more packages. They hardly evaluated new packages based on the price feedback throughout the auction.

Participants of spectrum auctions typically spend substantial time and money to prepare for the auction and one might challenge that results from the lab can be transferred to the field. However, even if they were able to evaluate more combinations, with sufficiently many items of interest in the auction the number of possible packages increases exponentially and the main phenomenon can probably be observed in all but small auctions.[2] It might be even more pronounced in other domains such as procurement or transportation, where bidders cannot invest the same amount of time to prepare for an auction. The result suggests that the positive experimental results on efficiency in small CAs do not necessarily carry over to larger instances. Issues such as pre-packaging and bidder decision support might play a much bigger role in the design of large CAs and deserve more attention in research and in practical auction design.

HPB has been used for selling spectrum by the US FCC and was therefore a main treatment in our experiments. Due to the limitation of package bids in HPB, bidders frequently took an exposure risk but still had more difficulties in coordinating and outbidding the big bidder in a threshold problem. In the fully combinatorial auction formats CC and PDP, small bidders were able to outbid the national bidders more frequently.

[2] For example, in the US Advanced Wireless Service auction in 2006, the FCC sold 1122 licenses, which leads to $2^{1122} - 1$ possible packages.

Apart from these main results, we report on a number of observations relevant to auctioneers and policy makers. For example, we find that opening bids on individual items have a significant positive effect on efficiency, while activity limits have a negative impact.

The paper is structured as follows: Section 2 describes the related literature and gives an overview of experimental studies on CAs so far. Section 3 describes the experimental environment, and the CA formats analyzed in this paper are introduced. In Section 4 we present our results both on an aggregate level and on the level of individual bidder behavior. A summary and conclusions follow in Section 5.

2. Related Literature

A number of experimental studies deals with CAs and their comparison to simultaneous or sequential auctions. Their main finding is that in the presence of complementarities between the items, package bidding in CAs leads to higher efficiencies (cf. Banks et al. (1989), Ledyard et al. (1997) and Banks et al. (2003)).

Brunner et al. (2009) compare the SMR auction to different kinds of CAs in values models with high and low complementarities. They find the CC auction to achieve the highest revenues among the CAs, which is consistent with the results of our experiments. In the high complementarity case, PDP and CC auctions significantly outperform SMR in terms of efficiency, while the opposite is the case with low complementarities. Their analysis of bidder behavior shows that bidders are more likely to bid on packages with high profit. We, however, also explain which factors determine the bidders' package selection, which helps understand inefficiencies in CAs.

Goeree and Holt (2008) introduce the HPB auction, in which bidders are allowed to place package bids only on predefined packages. They suggest defining predefined packages in a hierarchical manner, which allows solving the winner determination problem in linear time. They also suggest a pricing mechanism which is based on the hierarchical pre-packaging. In an experimental comparison of HPB, SMR, and PDP, they find HPB to achieve higher efficiency and revenue than SMR and PDP. They also analyze the threshold problem in PDP and HPB and find that smaller bidders can more effectively solve the threshold problem with HPB due to the pre-packaging. While they focus on the comparison between simultaneous auctions, restricted and fully combinatorial auctions in a value model with global synergies, we investigate only linear-price CAs and extend the analysis also to a value model with local synergies. We further analyze the bidder behavior in CC, in addition to HPB and PDP and provide insights into the reasons for inefficiencies in those CAs. In contrast to Goeree and Holt (2008), we find small bidders in CC and PDP to have less problems to coordinate in a threshold problem.

Finally, Chen and Takeuchi (2009) compare the Vickrey auction against an ascending version of the Vickrey auction in a small value model consisting of 16 packages. They find the ascending auction to generate higher bidder profit and efficiency, while the sealed bid auction generates a higher revenue. A large part of their study deals with the comparison of human bidders versus artificial bidding agents. In this context, they study the bidder behavior with different levels of information about the artificial agents strategies and find that bidders learn the bidding strategy from the agents which use straightforward bidding. In their analysis of the bidder behavior, they also find that it is more likely that bidders bid on packages with higher profits, which is consistent with the findings of Brunner et al. (2009). Whereas their analysis shows the bidder behavior in a Vickrey and an ascending version of the Vickrey auction, we investigate the bidder behavior in linear-price auctions.

3. Experimental Design

In the following, we provide an overview of the auction formats analyzed, the valuations of bidders, and the behavioral assumptions. Then we discuss the treatments used in our experiments, and the experimental procedures.

3.1. Market Mechanisms

The main difference between the CC, HPB, and the PDP auction is the calculation of the ask prices. All auction formats are multi-round auctions, meaning that the auction runs over several rounds, in which bidders are able to place new bids according to the current prices. At the end of each round, bidders are provided with information about their own provisionally winning and losing bids (referred to as standing or active bids in the CC auction), as well as with ask prices on items and packages for the new round. The bidders use an OR bidding language in our reported experiments. This means, they can submit and win more then one bid, with the restriction that each item can be included at most once in the winning allocation. This allows for comparability with the results in Goeree and Holt (2008).

Bidders in spectrum auctions are usually forced to be active and place bids early in the auction through an activity rule. We used a monotonicity rule that is based on eligibility points in all treatments. This means, the bidders were not able to increase the number of distinct items they are bidding on from round to round. Similar rules are used in spectrum auctions in the field (Cramton, 2009a).

3.1.1. The Combinatorial Clock Auction A core issue of running CAs is the computational complexity of the winner determination. Several iterative CA designs solve the winner determination problem after each round. In contrast, Porter et al. (2003) propose the CC auction, which alleviates some of the computational requirements; In the best case the auctioneer is not required to solve the winner determination problem at all. The price calculation in the CC auction is simple. Prices for all items are initially zero, and in each round of an auction, bidders bid on the packages they desire at the current prices. Jump bidding is not allowed. The auction moves to the next round as long as at least one item is demanded by two or more bidders. In this case the respective item prices are increased by a fixed increment. In a simple scenario, in which supply equals demand, the items are allocated to the corresponding bidders of the last round, and the auction terminates. If at some point there is excess supply for at least one item and no other item is over-demanded, the auctioneer solves the winner determination problem by considering all bids submitted so far. If all bidders who are active in the last round are included in the allocation, the auction terminates. Otherwise, the prices on those items not allocated to an active bidder are further increased and the auction continues. The feedback in the CC auction is restricted to the new price vector; additionally, our experimental software shows bidders if on any of their packages the price remains the same, which means that the respective bidder is the only one left, who demands these items at the current prices.

3.1.2. The Pseudo-Dual Price Auction The PDP auction uses a restricted dual formulation of the winner determination problem to derive ask prices after each auction round. Ask prices for individual items are determined via linear programming such that they sum up to the provisionally winning (package) bids and are lower than the losing bids after each round. Unfortunately, such item-level prices do not always exist in the presence of complementarities. In these cases, prices are set, such that violations of the constraints mentioned above are minimized. In the next round the losing bidders need to bid more than the sum of ask prices for a desired package plus a fixed minimum increment for each item in the package. All bids remain active throughout the auction, and bidders are allowed to submit jump bids as in a single-item English auction. Details of the ask price calculation can be found in (Bichler et al., 2009).

To avoid cycling the minimum increment has to be raised after a round in which new bids were submitted but auctioneer revenue did not increase. In our case the minimum increment is raised from 3 to 6 to 9 etc. until the auctioneer revenue increases at which point it is decreased to its initial value. It is possible that prices on an item also decrease during the course of an auction. If the bidders have placed a bid on the desired package in earlier rounds, they have to overbid their bid-price as well by the minimum increment, not only the current ask price. As soon as no bidder submits a bid in a round any more, the auction terminates and the winning bids become the final bids that determine the allocation and prices to be paid. The round feedback in the PDP auction includes information, whether a bid is provisionally winning, and the new individual-item prices as well as the minimum increment.

3.1.3. The Hierarchical Package Bidding Auction Goeree and Holt (2008) introduce HPB, an auction format which imposes a hierarchical structure of allowed package bids. This hierarchy reduces the winner determination problem to a computationally simple problem, which can be solved in linear time (Rothkopf et al., 1998). Goeree and Holt (2008) propose a simple and transparent pricing mechanism, which calculates item-level prices. They use a algorithm to determine new ask prices, which starts with the highest bid on every individual item as a lower bound, adding a surcharge if the next level package in the hierarchy received a bid, which is higher than the sum of the individual-item bids contained in the package. The difference is distributed uniformly across the respective item prices or bids on the lower level. This procedure is recursively applied to each level of the hierarchy and ends by evaluating the package(s) at the root level. In each round the bidders have to bid more than the sum of ask prices for a desired package plus a fixed minimum increment for each item in the package. Also, all bids remain active and can become winning throughout the auction. The auction terminates when bidding ends and provisionally winning bids become final bids that determine the allocation and prices to be paid. Similar to PDP, the feedback after each round in the HPB auction includes information, whether a bid is provisionally winning as well as the new individual-item prices.

3.2. Economic Environment

Ausubel et al. (1997, Page 499) define "*local synergies* as those gains in value that specifically arise from obtaining two or more geographically neighboring licenses [..., and] *global synergies* as those gains in value that accrue from obtaining increased numbers of licenses". Both Moreton and Spiller (1998) and Ausubel et al. (1997) find evidence for global and local synergies (complementarities) in spectrum auctions. While Goeree and Holt (2008) only analyze global synergies, we believe that due to the regional awarding of licenses in countries like the USA it is important to understand whether the results transfer to such value models with local synergies. We study these two different types of complementarities in our experiments. One of our value models uses global complementarities, so we refer to it here as global-synergy value model (*global-SVM*). Another value model uses local complementarities between items (*local-SVM*). In contrast to the global-SVM, the topology of items matters in the local-SVM.

3.2.1. The Global-Synergy Value Model The global-SVM is a replication of the first experiment in Goeree and Holt (2008), which was very influential on the FCC's decision for using HPB, and involves seven bidders and 18 items. Figure 1 represents the bidders' preferences. The six regional bidders (labeled 1 through 6) are each interested in four adjacent items of the national circle (consisting of items A through L) and two items of the regional circle (consisting of items M through R) while for the national bidder (labeled 7) the twelve items of the national circle are relevant. This information was common knowledge, but it was not known which bidders were interested in a particular item. For example experimental subjects did not know that besides bidder 7 also 3 and 4 were interested in license H.

The values for the individual items are randomly determined. For the national bidder the baseline draw distributions are uniform on the range $[0, 10]$ for items A-D and I-L and uniform on the range $[0, 20]$ for items E-H. For regional bidders the baseline draw distributions are uniform on the range $[0, 20]$ for items A-D, I-L and M-R and uniform on the range $[0, 40]$ for items E-H. These value distributions (not the actual draws) were common knowledge among the experimental subjects. For

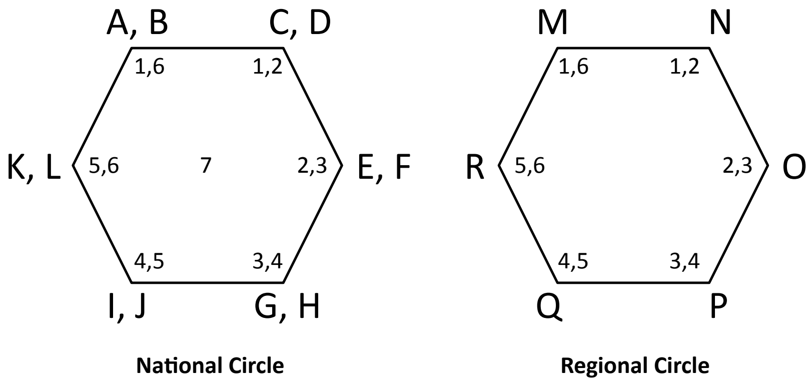


Figure 1 **Competition structure of the Global-SVM. Regional bidders 1-6 are interested in four items from the national circle and two items from the regional circle. National bidder 7 is interested in all twelve items from the national circle (Goeree and Holt, 2008).**

comparison, we use the same draws as in Goeree and Holt (2008), which the authors have kindly provided. For both bidder types the value of items in a package increases by 20% (with two items), 40% (with three items), 60% (with four items), etc. and by 220% for the package containing twelve items. For the calculation of the complementarities the identity of the items does not matter, e.g. a bidders' valuation of a package of items increases by the same percentage independent of the adjacency of the items; i.e, this is a value model in which global complementarities apply.

Activity and purchase limits are such that regional bidders can bid on and acquire at most four items in a single round, while the national bidder is able to acquire all his twelve items of interest. The allocation constraints such as the tight activity and purchase limits lead to additional difficulties, as we see in Section 4.4. As a consequence, we ran another set of experiments with another value model.

3.2.2. The Local-Synergy Value Model The local-SVM consists of 18 items arranged rectangularly and considers the scenario in which complementarities are gained from spatial proximity. In this value model items are placed on a rectangular map. The arrangement of items matters for the calculation of the complementarities, which only arise if the items are neighboring.

This model also contains two different bidder types: one national bidder, interested in all items, and five regional bidders. Each regional bidder is interested in a randomly determined preferred item, all horizontal and vertical adjacent items, and the items adjacent to those. This means that a regional bidder is interested in six to eleven items with local proximity to their preferred item. Examples are shown in Table 1, in which the preferred item of a regional bidder is Q or K, and all gray shaded items in the proximity of the preferred item have a positive valuation. For each bidder i we draw the valuation $v_i(k)$ for each item k in the proximity of the preferred item from a uniform distribution. Item valuations for the national bidder are in the range of $[3, 9]$ and for regional bidders in the range of $[3, 20]$. These value distributions (not the actual draws), the procedure to determine the preferred item and the number of bidders participating in the auction were common knowledge among the experimental subjects.

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q*	R

A	B	C	D	E	F
G	H	I	J	K*	L
M	N	O	P	Q	R

Table 1 **Local-SVM with the preferred items Q and K of two regional bidder. All their positive valued items are shaded.**

We assume that bidders experience only low complementarities on small packages, but complementarities increase heavily with a certain amount of adjacent items. We further assume that adding items to already large packages do not increase the complementarities anymore. The explanation for these assumptions is the lack of economies of scale with small packages and a saturation of this effect with larger packages. Therefore, complementarities arise based on a logistic function, which assigns a higher value to larger packages than to smaller ones:

$$v_i(S) = \sum_{C_h \in P} \left(\left(1 + \frac{a}{100\,(1+e^{b-|C_h|})} \right) * \sum_{k \in C_h} v_i(k) \right)$$

P is a partition of S containing maximally connected subpackages C_h, i.e. any item $k \in C_h \subseteq S$ has no vertical or horizontal adjacent item $l \in S \backslash C_h$. For our experiments we choose $a = 320$ and $b = 10$ for the national bidder and $a = 160$ and $b = 4$ for all regional bidders.

3.3. Behavioral Assumptions and Differences of the Value Models

Bidding in CAs is complex due to the auction rules (pricing and activity rules) and the large number of potential bids, which is exponential in the number of items. If bidders face this complicate situation, we conjecture that they rather select packages at the beginning of the auction and then concentrate on their bidding strategy.

Conjecture 1*Bidders select a limited number of packages at the beginning of the auction, which they focus on throughout the auction due to cognitive limits (Miller (1956)).*

However, at the beginning of the auction, bidders have only the information about their own valuations, i.e. their baseline valuations and the rules to calculate their complementarities. They do not have any information about the market prices and have only little information about the preferences of their competitors.

Conjecture 2*Bidders select packages according to their baseline valuations at the start of the auction.*

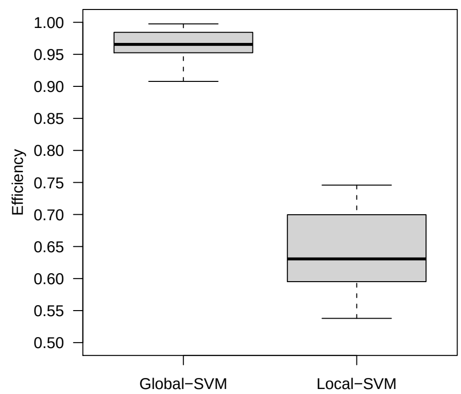


Figure 2 **Numerical experiments with the two different value models; all bidders are reporting their true valuations on individual items.**

Since complementarities among items in the global-SVM are the same for all bidders and all packages, eliminating the possibility to bid on packages has only a small effect on efficiency. Figure 2 describes the efficiency of numerical experiments with sealed-bid auctions and truth-revealing bidders, who only bid on individual items. The numerical experiments are based on all draws which are used in our experiments. Unlike in the global-SVM, truthful bidding on individual items in the local-SVM only results in an average efficiency of around 60%. This is due to the fact that complementarities occur only on adjacent items and the function is neither linear in the number of items nor the same for both types of bidders. In this case the availability of package bids is more important to achieve high levels of efficiency.

Conjecture 3*Efficiency in the global-SVM is higher as bidders know their market power through their baseline valuations. It is sufficient to stick (only) to the high valued items (packages) to achieve high levels of efficiency.*

	Global-SVM			Local-SVM		
	# items of interest	# packages		# items of interest	# packages	
		CC/PDP	HPB		CC/PDP	HPB
National bidder	12	4,095	16	18	262,143	28
Regional bidder	6	56	7	6 - 11	63 - 2,047	7-15

Table 2 Bidding space concerning value model, auction format and number of baseline draws.

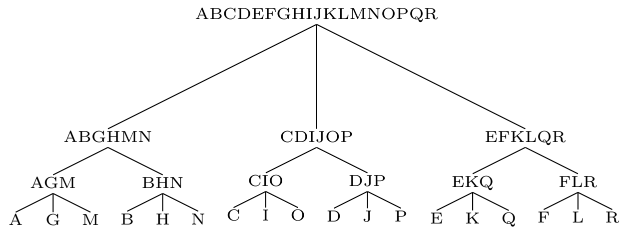
Table 2 shows the number of possible bids a specific type of bidder could submit in the different value models and auction formats. A large number of possible bids makes it more difficult in the CC and PDP auctions to select the "right" bids and coordinate with other bidders. This is particularly true, since no game-theoretical solution concept is known for any of the auction formats. The local-SVM provides even more alternatives for each bidder to choose from in each round. Miller (1956) show that the amount of information which people can process and remember is often limited.

Conjecture 4*Bidders evaluate and bid on the same amount of packages regardless of the number of possible package bids due to cognitive limits.*

A crucial factor to achieve high levels of efficiency in HPB is that the hierarchy meets the bidders' preferences; if this is the case, the auction is likely to achieve efficient outcomes, and reduces the strategic complexity for bidders as there are less exposure problems. In other words, it is easier for bidders to coordinate in a threshold problem if fewer overlapping packages are available. Otherwise HPB reduces to a set of parallel auctions as bidders cannot use the predefined packages. To be able to compare the results in HPB over the different value models, we use a similar pre-packaging in both value models. Bidders should be able to bid on all individual items and we tried that they are able to bid their preferences.

Since the competition structure is deterministic in the global-SVM it is simple to create a good hierarchy that fits the bidders' preferences. We take the structure from Goeree and Holt (2008) which yield higher efficiencies in their results, so the odd-numbered bidders were able to bid on their desired package of four items in the national circle directly. Hence, the hierarchical structure of possible package bids in HPB is the following: Bidders may bid on the package of items A-L on the top level, A-D, E-H and I-L on the middle level, and on all individual items on the bottom level.

Due to the random competition structure in the local-SVM, it is harder for the auctioneer to design a good hierarchy of allowed packages in HPB. With random competition structure we mean that we cannot tell a priori which bidders are interested in which items due to the randomly determined preferred item. We assume the auctioneer to have only a basic understanding of bidder valuations in the local-SVM and designed a hierarchy that allows bidders much flexibility with a four-level hierarchy:



Since the exposure problem is a well known phenomenon in the SMR auction, we expect this problem to occur in the HPB auctions as bidders are not able to express their preferences, due to the restriction of package bids.

Conjecture 5 *In HPB bidders have to deal with the exposure problem and take the risk of paying more for a package than it is worth to them.*

There are always two regional bidders and the national bidder competing for an item in the national circle, and two regional bidders competing on the items in the regional circle. In real applications, there are often hot spots with a lot of competition, while other areas exhibit very low competition. The threshold problem becomes more crucial in the local-SVM due to the stochastic competition structure. Goeree and Holt (2008) find that the threshold problem is more a coordination problem than a "free rider" problem and that HPB helps regional bidders in coordinating their bids to outbid the national bidder in the global-SVM. While this might be true when bidders are able to reveal their preferences through the predefined packages in HPB, we believe that as soon as they cannot reveal them through package bids, the exposure risk regional bidders have to take compensate the coordination advantage in HPB.

Conjecture 6 *The threshold problem is harder to overcome in the local-SVM due to the stochastic competition structure.*

3.4. Treatment Structure

Table 3 provides an overview of all six treatment combinations. For both the global-SVM and the local-SVM, we provided the bidders with a simple and easy to use tool to determine the value of a particular package. The user interface and the basic auction process was similar across all treatments[3]. The bid increment was three francs per item in our experiments.

A phenomenon that can occur in CAs is that items remain unsold, although bidders have positive valuations for these items. Since bidders are not restricted to bid on individual items only, their package bids might overlap so that the revenue maximizing allocation does not contain all items, which results in efficiency losses. Due to the restriction on package bids in HPB we find bidders are more likely to bid on individual items, which results often in complete sales. This is not the case in the fully combinatorial auctions, which often terminate with unsold items. In order to avoid unsold items in our comparison of the CAs and to guide bidders in solving their coordination problem, we introduce *opening (singleton) bids*; this means, in the first round bidders have to place individual-item bids on those items they want to be able to bid on in future rounds. Such a rule

[3] All experiments were conducted using the *MarketDesigner* software, which was developed at the TU München. It is a web-based software that was already applied for other lab experiments (Scheffel et al. (2010)). The software will be provided for replication of the experiments.

is regularly used in procurement, in which auctions are typically preceded by an initial request for quotes, i.e., a tender on individual items. To keep the experimental setup as close as possible to the experiments by Goeree and Holt (2008), we introduce opening bids only in the local-SVM. In treatment 3 we deviated from the OR bidding language for the regional bidders. Instead they used XOR bidding, i.e. each bidder can win at most one of his bids. This decision was based on the fact that regional bidders have only a moderate number (56) of bundles to choose from in each round and with XOR bidding they can directly express their purchase limits.

Treatment No.	Auction format	Bidding language	Opening bids	Value model
1	CC	OR	No	global-SVM
2	HPB	OR	No	global-SVM
3	PDP	OR/XOR	No	global-SVM
4	CC	OR	Yes	local-SVM
5	HPB	OR	No	local-SVM
6	PDP	OR	Yes	local-SVM

Table 3 Treatment structure.

In summary, we conducted 24 sessions with 156 participants. For each treatment combination we had four sessions. Each session consisted of two practice auctions and a series of six auctions in the global-SVM and four auctions with the local-SVM. In a session a single treatment combination was tested, but the value draws changed from one auction to the next in the same session. We used four different sets of value draws for each treatment combination, but the same set of draws for other treatments.

3.5. Procedures

All experiments were conducted from November 2008 to July 2009 with undergraduate students of the TU München. Each session started by giving subjects printed instructions which were read aloud. The subjects participated in two training auctions to ensure that they were familiar with the market design and user interface. Subjects took part in a short exam and were encouraged to ask questions to ascertain that they understood the economic environment as well as the market rules. The following four to six auctions were used to determine the earnings of each bidder. The number of auctions was announced in advance. Every auction started with period of five minutes in which bidders received their new valuations simultaneously and submitted their first round bids. In all subsequent rounds, bidders had at most three minutes for placing bids. Bidder roles were reassigned randomly after each auction in order to attenuate earning differences across national and regional bidders and to help bidders understand the strategic considerations faced by both bidder types. Earnings were calculated by converting the achieved gains to EUR by 2:1 in the auctions with the global-SVM and 1:1 in the auctions with the local-SVM. The earnings were distributed between the €10 show-up fee and a maximum of €80 per subject; average earnings were €43.55. The average duration of a session was 4 hours and 17 minutes.

4. Results

First we present our results on an aggregate level. In order to better understand and explain these results we analyze the bidders' package selection in a second step, and continue with the analysis of the threshold problem in our experiments. Finally, we provide more detailed insights about the individual bidder behavior.

4.1. Aggregated Measures

We use ***allocative efficiency*** (or simply ***efficiency***) as a primary aggregate measure. Given an allocation X and price set $\mathcal{P}_{pay}$, let $\pi_i(X, \mathcal{P}_{pay})$ denote the payoff of the bidder i for the allocation X and $\pi_{all}(X, \mathcal{P}_{pay}) := \sum_{i \in \mathcal{I}} \pi_i(X, \mathcal{P}_{pay})$ denote the ***total payoff of all bidders*** for an allocation at the prices $\mathcal{P}_{pay}$. Further, let $\Pi(X, \mathcal{P}_{pay})$ denote the ***auctioneer's revenue***. We measure efficiency as the ratio of the total valuation of the resulting allocation X to the total valuation of an efficient allocation X^* (Kwasnica et al., 2005):

$$E(X) := \frac{\Pi(X, \mathcal{P}_{pay}) + \pi_{all}(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, 1]$$

In addition, we measure ***revenue distribution***, which shows how the overall economic gain is distributed between the auctioneer and bidders. Given the resulting allocation X and prices $\mathcal{P}_{pay}$, the ***auctioneer's revenue share*** is measured as the ratio of the auctioneer's revenue to the total sum of valuations of an efficient allocation X^*:

$$R(X) := \frac{\Pi(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, E(X)] \subset [0, 1]$$

We also include the payoff or revenue share of different bidders in our analysis, which is measured by the proportion of the efficient solution that a single bidder gains. For the pairwise comparisons of selected metrics we use the *nonparametric Wilcoxon rank sum test* (Hollander and Wolfe, 1973). $\sim$ indicates an insignificant order, $\succ^*$ indicates significance at the 10% level, $\succ^{**}$ indicates significance at the 5% level, and $\succ^{***}$ indicates significance at the 1% level.

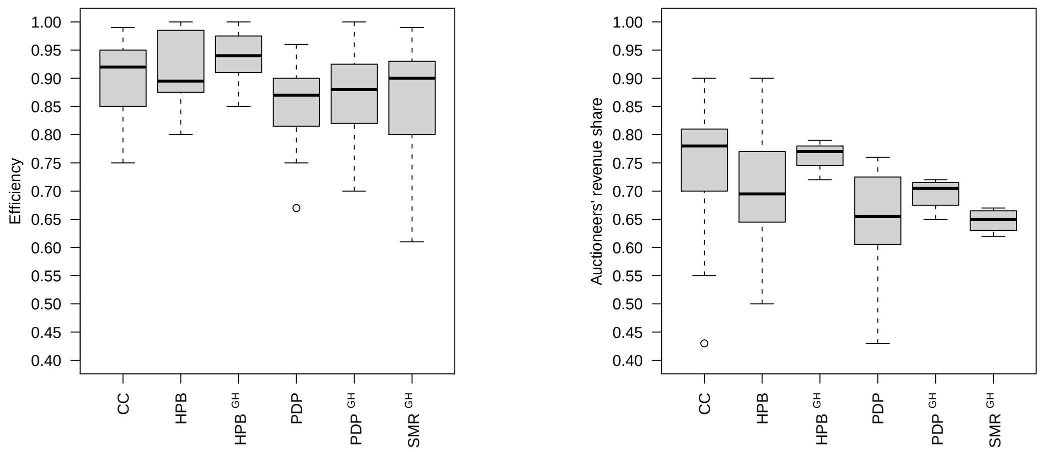
Auction Format	$E(X)$	$R(X)$	Payoff Nationals	Payoff Regionals	Number of Bids	Unsold Items	Rounds
CC	89.6%	74.6%	0.2%	2.5%	224.3	1.0	15.6
HPB	91.2%	70.5%	0.7%	3.3%	82.1	0.1	13.3
HPB^{GH}	94.0%	76.5%	1.6%	2.6%	n.a.	0.1	13.2
PDP	85.3%	64.8%	0.5%	3.3%	70.6	2.5	11.2
PDP^{GH}	89.7%	70.8%	2.3%	2.8%	n.a.	1.0	15.1
SMR^{GH}	85.1%	64.6%	-2.6%	3.0%	n.a.	2.1	17.8

Table 4 **Average aggregate measures of auction performance in the global-SVM. The results of Goeree and Holt (2008) are labeled with a superscript GH.**

4.1.1. Aggregate Performance Metrics of the global-SVM On an aggregated level, our analysis covers mainly the average numbers of efficiency, auctioneer's revenue, and the payoff of regional and national bidders. We further provide the average number of bids, the number of unsold items, and the number of auction rounds. In Table 4, we provide the results of these measures in the global-SVM. We show the distribution of the efficiency and the auctioneer's revenue in the box plots in Figure 3.

Result 1*(Efficiency and revenue distribution in the global-SVM). The CC auction and HPB achieve significantly higher efficiency than the PDP auction (HPB $\succ^{***}$ PDP, CC $\succ^{**}$ PDP, CC $\sim$ HPB). CC leads to the highest auctioneer's revenue share; auctioneer's revenues are ranked CC $\succ^{*}$ HPB $\succ^{**}$ PDP. The differences in the bidders' payoff is mainly insignificant; only the payoff of the regional bidders is significantly lower in CC compared to PDP (PDP $\succ^{**}$ CC).*

Support 1 *We performed Wilcoxon rank sum tests, and find significant differences in efficiencies between the PDP auction and CC ($p = 0.049$), and PDP and HPB ($p = 0.006$). We do not find a significant difference between CC and HPB ($p = 0.452$). In Figure 3, the box plot shows the highest median auctioneer's revenue share for the CC auction. A Wilcoxon rank sum test confirms that the CC auction leads in our experiments to the highest auctioneer's revenue. HPB results in a significantly lower revenue ($p = 0.078$). PDP is outperformed by both CC ($p = 0.001$) and HPB ($p = 0.047$). We find only small differences in the bidder payoffs (cf. Table 4); only the PDP auction has significantly higher regional bidders' payoffs compared to the CC auction ($p = 0.039$).* □



(a)Efficiency (b)Auctioneer's revenue

Figure 3 **Efficiency and auctioneer's revenue in the global-SVM. The results of Goeree and Holt (2008) are labeled with a superscript GH.**

The number of auction rounds is often used as a metric for speed of convergence. If an auction format requires too many rounds to close, it is impracticable in the field and also in the laboratory. While we find significant differences in the number of auction rounds (CC $\succ^{**}$ PDP, CC $\succ^{*}$ HPB), the average is less than 20 rounds in all sessions and never caused a problem. Only the auction duration in some of the CC auctions kept us from conducting all auctions in two of the CC sessions (on one occasion we conducted only four auctions, and in another session only five auctions).

4.1.2. Comparison to the Experiments by Goeree and Holt (2008) Since our global-SVM is a replication of the first experiment in Goeree and Holt (2008), we continue the analysis with the comparison to their results. The hierarchical pre-packaging in our experiments is the same as in the HPB_{odd} treatment in Goeree and Holt (2008), which resulted in the highest efficiency in their experiments.

Result 2 *(Global-SVM: Comparison to Goeree and Holt (2008) experiment one). We find significant differences between our experiments and those of Goeree and Holt (2008). Efficiencies are ranked $HPB^{GH} \succ^{***}$ HPB, but we replicated their main finding that a pre-packaged auction format (HPB) achieves higher efficiencies than a PDP auction (HPB $\succ^{***}$ PDP, $HPB^{GH} \succ^{*} PDP^{GH}$). The fully combinatorial CC auction achieves similar outcomes in efficiency (CC $\sim$ HPB) and achieves higher auctioneer revenue than HPB (CC $\succ^{*}$ HPB).*

Support 2 *The average numbers of both experiments are included in Table 4 and the box plots in Figure 3 show the distribution of the efficiencies and auctioneer's revenue. The Wilcoxon rank sum tests confirm the visual observation of the box plot, that HPB^{GH} achieves a significantly higher efficiency compared to HPB ($p = 0.004$) and auctioneer's revenue ($p = 0.006$). We find the CC auction to achieve similar efficiency as HPB ($p = 0.452$), but the auctioneer's revenue is higher in CC ($p = 0.078$).* □

It is noteworthy that the main finding in Goeree and Holt (2008), the difference between HPB and PDP, is replicated. Although bidders in the PDP treatment have additional complexity due to the bidding language, we find no significant differences in efficiency and auctioneer's revenue between PDP and PDP^{GH}, however, the number of unsold items is significantly higher in PDP compared to PDP^{GH}. This difference can be explained by the use of the XOR bidding language, which allows only one winning bid per bidder. XOR bidding increases the chance for unsold items due to the reduction of possible allocations compared to OR bidding.

The significant differences in the results could be due to a number of smaller differences in the laboratory procedures and software. In our experiments, bidders were unable to see the other bidders' IDs and bids. We have also used a maximum round duration of three minutes, while in GH bidders had only 40 seconds to submit their bids. There are also differences in the user interface that might cause the differences in the observed results.

4.1.3. Aggregate Performance Metrics of the Local-SVM In addition to the experiments with the global-SVM, we performed a series of experiments with the local-SVM to see if the results transfer to a different value model, which can be well motivated from observations in the field (Ausubel et al. (1997); Moreton and Spiller (1998)).

Auction Format	$E(X)$	$R(X)$	Payoff Nationals	Payoff Regionals	Number of Bids	Unsold Items	Rounds
CC	92.6%	85.2%	0.5%	1.3%	472.2	0.0	12.7
HPB	86.7%	80.8%	3.2%	0.5%	91.1	0.0	6.5
PDP	89.9%	82.4%	1.1%	1.3%	138.7	0.0	9.0

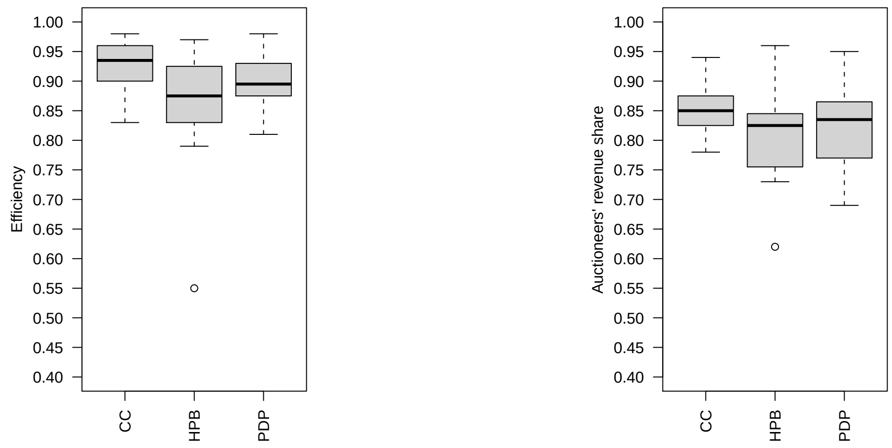
Table 5 **Average aggregate measures of auction performance in the local-SVM.**

In Table 5 we provide the average results of the auctions in the local-SVM and Figure 4 shows the distribution of efficiency and the auctioneer's revenue share in the local-SVM.

Result 3 *(Efficiencies and revenues in the local-SVM). CC leads to significantly higher efficiency (CC $\succ^{**}$ HPB) and auctioneer's revenue (CC $\succ^{*}$ HPB) than HPB . There was no significant difference between HPB and PDP in efficiency and auctioneer's revenue in the local-SVM.*

Support 3 *A Wilcoxon rank sum test on the efficiencies and auctioneer's revenue reveals that the CC auction outperforms the HPB auction in efficiency ($p = 0.026$) and auctioneer's revenue ($p = 0.054$). Further comparisons are insignificant.* □

4.1.4. Comparisons of the Auction Formats in the Different Value Models Although the comparison of efficiency and auctioneer's revenue share is difficult across two different value models, we observe that the three auction formats achieved similar levels of efficiency in the global-SVM and the local-SVM. This result confirms Conjecture 3 only partly. While in HPB, which is closer to our SMR simulation, on which the conjecture is based, the statement is true, it does not hold for the fully CAs CC and PDP. The comparison of the experiments in the two value models reveals that there are differences in the ranking of auction formats in terms of efficiency and auctioneer's revenue, but those differences are small.



(a)Efficiency (b)Auctioneer's revenue

Figure 4 Efficiency and auctioneer's revenue for each auction format in the local-SVM.

Result 4*(Comparison of auction formats in different value models). The results of the value model with global synergies do not fully carry over to a value model with local synergies. The ranking of efficiencies in the global-SVM (HPB $\succ^{***}$ PDP) and revenue (HPB $\succ^{***}$ PDP) cannot be replicated in the local-SVM, in which differences in efficiencies (HPB $\sim$ PDP) and revenues (HPB $\sim$ PDP) are insignificant. The CC auction achieves the highest median efficiency and auctioneer's revenue in both value models. Overall, the differences in these aggregate performance metrics between the different auction formats are small.*

Support 4*Results 1 and 3 show the different rankings of the auction formats in the different value models.* □

The ranking of HPB and PDP changes from the global-SVM to the local-SVM. There are several factors causing the different rankings. First, we find HPB to collect more bids in the global-SVM, in contrast to the local-SVM in which PDP collects more bids. Second, the number of unsold items is high in the PDP auction in the global-SVM. This number dropped from ∅ 2.46 in the global-SVM to ∅ 0.0 in the local-SVM (cf. Table 5). Opening bids in the local-SVM assures that all items are sold in the PDP as well. Third, the hierarchical structure of the predefined packages in HPB does not fit the bidders' preferences as well in the local-SVM as in the global-SVM. Finally, the tight activity limits of the regional bidders in the global-SVM have a considerable negative impact on PDP but not on HPB. We further elaborate on this in Section 4.4.

In both value models, the CC auctions achieve highest median efficiency and auctioneer's revenue share. One reason for this is the number of bids submitted. In both value models, bidders submit more bids in the CC auction than in any other auction format. We conjecture that the absence of temporary winners during the auction causes this phenomenon. Bidders are not aware of their current position in the auction. Since bidders in CC do not learn about the current allocation, they actively submit bids in each round. In HPB and PDP, if bidders provisionally win all the items they want in a round, there is no reason to submit additional bids and the bidders reveal less information about their preferences to the auctioneer. Another explanation is that jump bidding is not possible in the CC auction, which assures a high number of demand queries.

4.2. Package Selection and Package Evaluation

Efficiency is very high in the experiments but still there is a gap to full efficiency. There might be several reasons for the efficiency losses, as specific auction rules, the price calculation or the bidding strategies. Since we observe coordination problems among the regional bidders, we go on with a deeper analysis of the bidder behavior to find the reason for the efficiency losses. We identify the bidders' selection of packages to be a crucial factor.

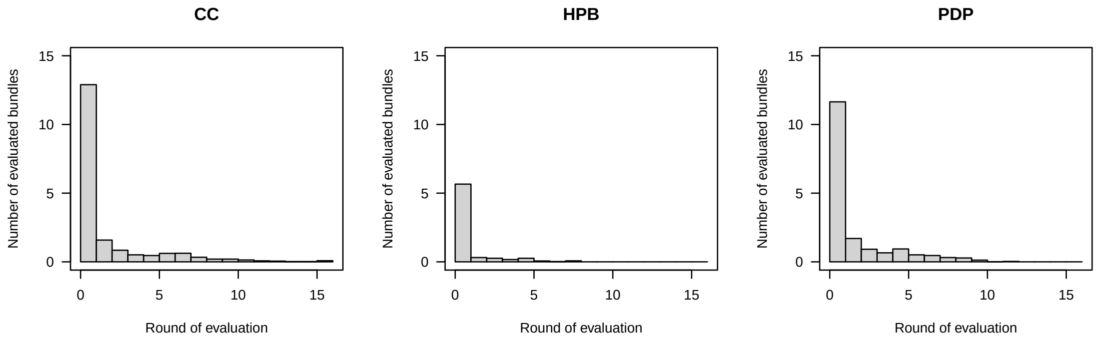


Figure 5 Number of packages evaluated by a bidder in the auctions on the local-SVM.

Local-SVM	CC	HPB	PDP
LAB	92.4%	86.7%	90.0%
EVALALL	93.8%	90.8%	93.2%
EVALEARLY	91.6%	90.1%	90.3%

Table 6 Efficiency in the laboratory experiments (LAB) compared to different simulations on packages selected by the bidders (EVALALL, EVALEARLY).

Both value models are challenging for bidders due to the large number of possible packages, especially in the local-SVM, in which bidders have to select from up to 262,143 different packages with positive valuation. Our experimental software provides a feature to calculate the value of a particular package, which gives us the opportunity to observe package evaluation. Since the calculation of package valuations in the global-SVM can easily be done without the tool, we concentrate on the local-SVM, where we can observe all evaluations. We conducted simulations with artificial bidding agents, who truthfully reveal their valuations on all individual items and packages that the bidders have evaluated in the actual lab auction (***EVALALL*** treatment). In another simulation treatment, ***EVALEARLY***, we consider the individual items and only those packages bidders have evaluated in the first and the second round of the auction.

Result 5 *(Preselection of packages). Bidders select and evaluate most packages in the very first round, which is consistent with Conjecture 1. New price information throughout the auction does not lead to the evaluation of many more packages later on. Further, package selection after the second round does not have a significant impact on the auction efficiency.*

Support 5 *We measure the number of evaluated packages in each round; the result is displayed in Figure 5. Bidders select over 80% of the packages in the first three rounds in CC and PDP and about 91% in the HPB auctions. To understand the impact of the late evaluated packages, we compare the simulations on the package selection, namely the treatments EVALALL and EVALEARLY. The results of EVALALL and EVALEARLY are not significantly different, which means that the late evaluated packages do not increase the possible efficiency (the average results are included in Table 6).* □

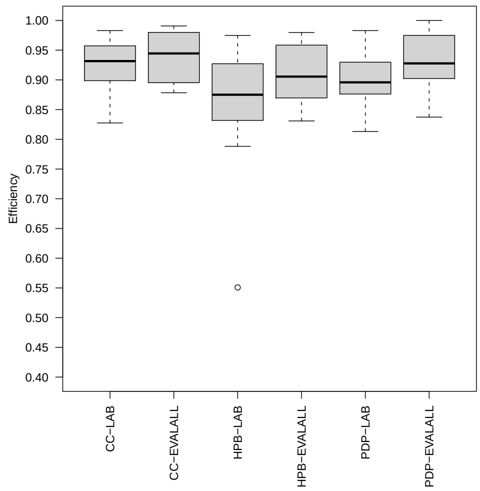


Figure 6 **Comparison of the efficiency in different auction formats in the laboratory (LAB) and simulation (EVALALL). Using a sealed bid auction in our simulations, we calculate the best allocation if bidders bid their true value on all packages they have evaluated during the actual lab auction.**

It is interesting to understand whether the efficiency loss is due to the package selection in a CA or whether it is mostly a result of the bidding strategies and auction rules. We compare the outcome in our laboratory experiments (***LAB***) to the results of the EVALALL treatment. While Table 6 contains all average values of our laboratory experiments and different numerical simulations, Figure 6 also contains the distribution of the values for the treatments LAB and EVALALL.

Result 6*(Efficiency loss due to package selection). The selection of packages (and not auction rules or bidding strategies) accounts for most of the efficiency losses in the local-SVM. Only in the PDP auction the bidders in the lab achieve a worse solution than in the simulations with a direct revelation mechanism and restricted package selection (PDP-EVALALL$\succ^{**}$PDP-LAB).*

Support 6*The results in Figure 6 and Table 6 show that much of the efficiency loss in the local-SVM is due to package selection of the bidders. In CC and HPB the results in our laboratory experiments (LAB) are not statistically different from the* EVALALL *treatments. This means that, in these two auction formats, the auction ends up in an allocation which is similar to the simulation EVALALL. Only in PDP do we find the simulation (EVALLALL) to achieve significantly higher efficiency than the LAB result ($p = 0.028$).* □

Result 6 implies that package selection accounts to a large extent for the efficiency loss in CAs. Therefore, we performed an analysis of the package selection in the local-SVM by using a logistic regression (Hastie and Pregibon, 1992), which describes the bidders' package selection. We only take the evaluations from regional bidders in CC and PDP auctions into account, since these auction formats are fully CA formats and are not biased by limitations of the auction mechanisms in terms of package selection.

To denote the bidders' selection of a package, we list all packages the bidder has a positive valuation for and assign a binary variable, which indicates 1 if the bidder has evaluated the value of the respective package and zero otherwise. We identify several independent variables which have an impact on the bidders' package selection. The variable ***MEANBASEVAL*** holds the package's mean baseline valuation divided by the bidder's highest baseline valuation. This means that, this variable shows whether the package contains items with high baseline valuations, relative to the bidders highest baseline valuation.

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-0.0482	0.1466	-0.33	0.7421
MEANBASEVAL	4.6504	0.1856	25.06	0.0000
CONN	-2.4844	0.0561	-44.32	0.0000
ITEMS7	-1.5378	0.1083	-14.20	0.0000
ITEMS8	-2.1002	0.0843	-24.92	0.0000
ITEMS9	-2.3324	0.0812	-28.72	0.0000
ITEMS10	-3.3378	0.0909	-36.71	0.0000
ITEMS11	-3.8165	0.0897	-42.57	0.0000
N	94992		$-2LogL_0$	25259
Freq. of 1:	2801		$-2LogL_{full}$	17547
Freq. of 0:	92191		AIC	17563

Table 7 **Logistic regression of the regional bidders' likelihood to evaluate a package. The dependent variable is the decision whether or not a bidder has evaluated a particular package.**

Another important dimension in the bidders' decision for evaluating a package is the number of not connected subpackages within the selected package. Since complementarities only arise for maximally connected subpackages, we count the number of them within the selected package in the variable ***CONN***, i.e. if all items are connected through neighboring items, this variable is 1, otherwise if for example the package can be divided in two sub-packages in which none of the items are horizontally or vertically neighboring, this variable is 2 for the respective package.

Consequently we investigate whether the number of packages a bidder is interested in has an effect on the number of evaluated packages. We model this attribute as a factor (***ITEMS6*** - ***ITEMS11***).

Result 7 *(Package selection by baseline valuations). Bidders select packages according to the relative hight of the baseline valuations. Additionally, they take into account that the whole package has to be maximally connected.*

Support 7 *Table 7 shows the results of the logistic regression among several potential factors which can explain the bidders' package selection. We find only the relative hight of the baseline valuations (MEANBASEVAL) to have a significant positive impact on the bidders' package selection, i.e. the higher the relative baseline valuations of the items are, the higher is the likelihood that the bidders evaluate the value of the package. The coefficient for the CONN variable is significantly negative, meaning it is more likely that bidders select packages with a low number of maximally connected sub-packages.* □

As Conjecture 2 states, bidders follow the two important dimensions in selecting the packages. They know the baseline valuations of the individual items and they know that the complementarities are higher if the package consists only of one maximally connected subpackage. A further significant factor is the number of items the bidder is interested in, as the number of packages increases exponentially in the number of items, bidders do not evaluate the same ratio of packages as the number of packages increases, according to the coefficients of the variables ITEMS7 - ITEMS11 in the logistic regression.

Therefore, we analyze the number of packages which bidders have evaluated in the auction, and compare those to the number of items they have a positive valuation for. Note that the regional bidders are interested in six to eleven items, while the national bidder is interested in 18 items. Figure 7 shows that the number of evaluated packages is independent of the number of items a

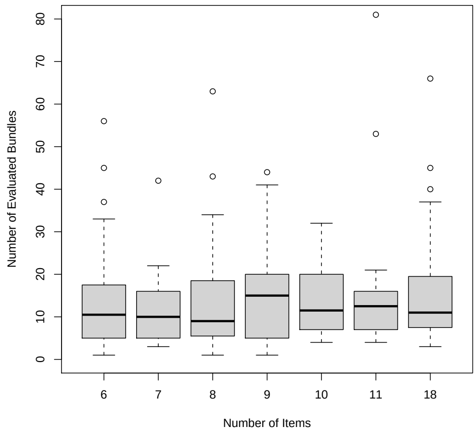


Figure 7 **Impact of number of items bidders are interested in on the number of packages evaluated by bidders in the local-SVM.**

bidder is interested in. We conjecture that this has to do with cognitive limits in the number of items people can concentrate on as have been analyzed in psychology for years (Miller, 1956).

Result 8*(Package selection: Impact of number of possible packages). Independent of the number of possible packages, bidders only evaluate approximately 14 packages including two or more items. The average number of packages evaluated does not increase with the number of items of interest, even though the number of possible packages increases exponentially with the number of items.*

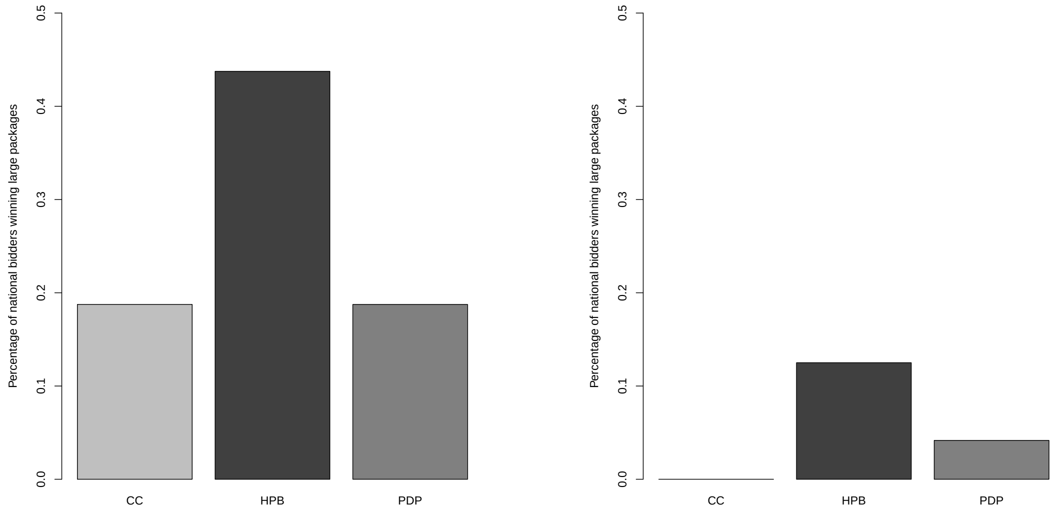
Support 8*In Figure 7 we see bidders select on average the same number of packages independent from the number of items they are interested in. For example, the national bidder has* $2^{18} - 1 = 262,143$ *packages with a positive valuation; however, bidders typically evaluate only around 14 packages including two or more items.* □

The observations about the bidders' package selection deserves a more in-depth discussion. First, one could argue that for the specific case of high-stakes spectrum auctions, bidders will spend the time to evaluate a much larger number of packages than in a lab experiment with students. In an auction with 20 items of interest bidders have more than one million possible packages. Teams in spectrum auctions have usually one or two hours per round and typically multiple people are involved in the decision. We do not expect that bidders are able to evaluate a few hundreds or even thousands of packages and bid on those.

Second, some auctioneers do not only reveal prices, but detailed information about other winning bids, and sometimes even all bids and the bidder identities. When a bidder assumes that another bidder is strong and there is a low probability to overbid this bidder on a particular package, this might in fact provide additional incentives to evaluate one or the other relevant packages during the auction. However, as the number of packages grows exponentially in the number of items, we consider it unlikely that even with different information feedback rules the observed phenomenon of the bidders' package selection vanishes. The development of additional bidder decision support and adequate information feedback is certainly important for future research.

4.3. Threshold, Coordination and Exposure Problem

We analyze the coordination problem by considering two different aspects. First, when some small bidders have to coordinate their bids to outbid a bidder interested in a package covering many items, they are confronted with the threshold problem (Rothkopf et al., 1998). In our value models, we have several regional bidders, interested only in a small set of items, competing against a national bidder. Second, since in our experiments every item is sold in the efficient solution, we can also evaluate the coordination problem in measuring the number of unsold items. We know that some bidders have a positive valuation for unsold items, which means ex-post they regret not having bid on these items at least a price of ϵ.



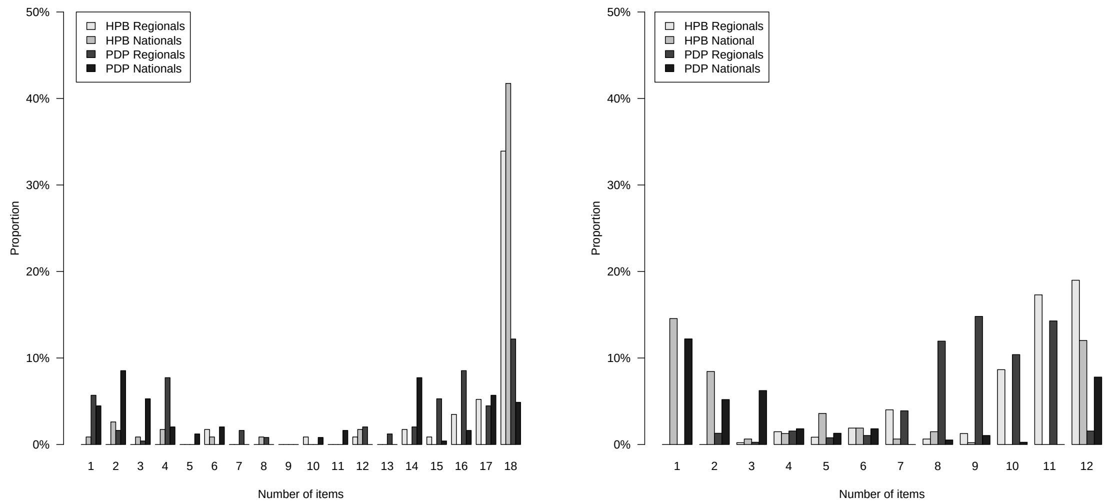
(a)Auctions in which the national bidder wins more than 16 items in the local-SVM (b)Auctions in which the national bidder wins all 12 items of the national circle in the global-SVM

Figure 8 Threshold problem.

Figure 8 shows the percentage of auctions in which the national bidder managed to win a huge share of items, even though based on their valuations the regional bidders were able to outbid him. In the global-SVM we had 24 auctions and in the local-SVM 16 auctions for each auction format. In HPB, regional bidders obviously have more difficulties coordinating among themselves in order to outbid the national bidder in a threshold problem. This is due to the fact that the package bids are restricted in HPB. The national bidder has an advantage, as there the package of all items is at the top hierarchy level, which fits his preferences. This is not the case for regional bidders. While in HPB pre-packaging guides bidders to solve their coordination problem by limiting the number of overlapping bids, bidders with a preference structure not matching the pre-packaging possibly face an exposure risk. In our data we find, contrary to Goeree and Holt (2008), evidence that the bidders' exposure risk dominates the coordination advantage in HPB in both value models.

Result 9*(Coordination: Strong threshold problem due to pre-packaging). In HPB the national bidder has an advantage over the regional bidders due to the limitation of package bids. In fully CAs regional bidders can often outbid the national bidder, although they do not manage to win all items.*

Support 9*In Figure 8 we find the national bidder to win considerably more often a huge package of items in HPB than in CC and PDP, in both value models. We extend our analysis to the provisional winners in each round (Figure 9) and find that the national bidder managed to win all items in nearly 45% of the rounds in HPB in the local-SVM. In the results of the global-SVM, we see that bidders in PDP are able to outbid the national bidder by winning not all items, which partially*



(a)All items of the local-SVM

(b)National circle of the global-SVM

Figure 9 Number of items regional and national bidders provisionally win in a round.

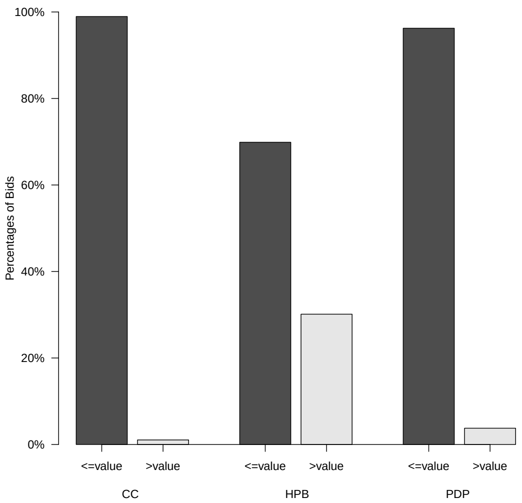
explains the observed high number of unsold items. In Table 4 we find only $\varnothing$ *0.1 of the items to remain unsold in the HPB auction, compared to* $\varnothing$ *1.0 in CC and* $\varnothing$ *2.5 in PDP (HPB* $\prec^{***}$ *CC* $\prec^{***}$ *PDP).* □

In the PDP auction in the global-SVM, a large number of items remain unsold and the efficiency is low. The combination of activity and purchase limits (cf. Section 4.4.2), the XOR bidding language for the regional bidders, and a fully CA format leads to such a coordination problem. The CC auction, also a fully CA format, better solves this coordination problem due to the large number of bids submitted by the bidders. However, also in the CC auctions in the global-SVM one item remains unsold on average. In contrast, HPB, with the pre-packaging, solves the coordination problem by limiting the number of overlapping bids, resulting in the lowest number of unsold items. With the introduction of opening bids in the local-SVM, all items are sold in CC and PDP, while in HPB pre-packaging ensures complete sales.

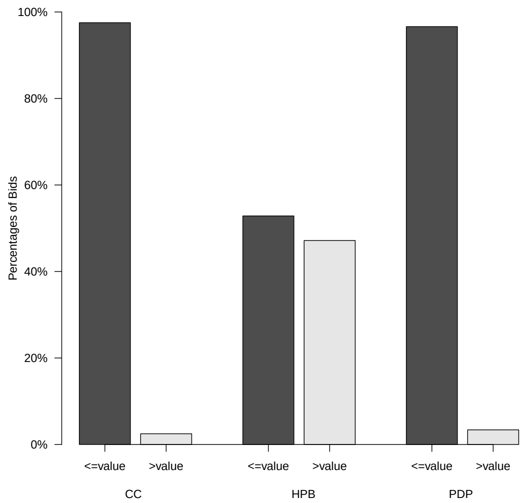
We find HPB to favor the national bidder. One reason for this is that the national bidder's preferences are perfectly matched by the biggest package available in HPB. Regional bidders, in contrast, usually find their preferences not matched, so they might have to overbid their valuation on a smaller package or an individual item to be able to win a package not included in the pre-packaging. The risk for them is to win only part of that package and paying too much for it, which is the exposure problem. Figure 10 describes the ratio of final bids that regional bidders submitted in different auction formats. We classified bids under or at, and over the bidder's valuation.

Result 10*(Exposure risk due to pre-packaging). Pre-packaging in HPB forces the regional bidders to overbid their valuations. In the fully CA formats CC and PDP, overbidding is rare.*

Support 10*Figure 10 reveals that* 47.2% *of the regional bidders' final bids in HPB in the global-SVM are above their respective valuation, as well as* 30.1% *in the local-SVM. In CC and PDP, the number of final bids above the valuation is below* 3.8%*.* □



(a)Final bids in the local-SVM

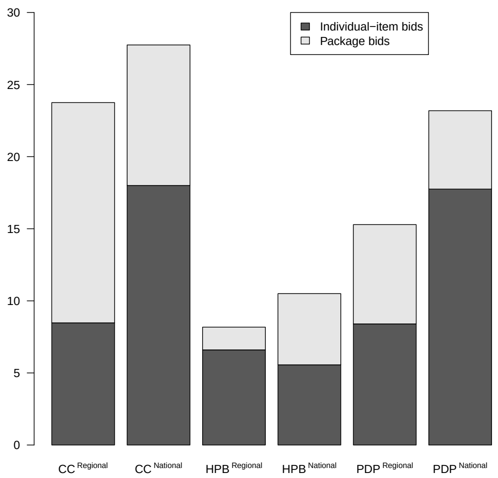


(b)Final bids in the global-SVM

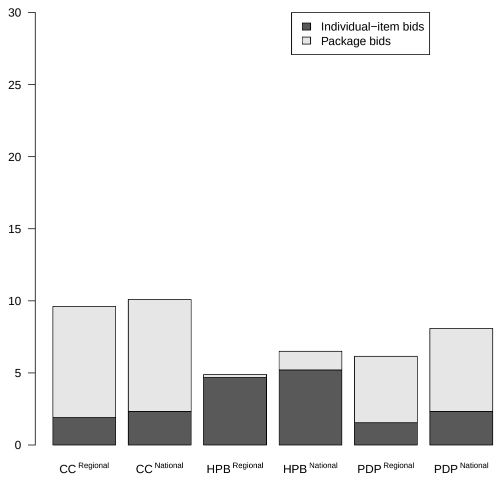
Figure 10 **Percentage of the regional bidders' final bids over their respective valuation compared to the share of bids on or below their valuation.**

4.4. Individual Bidder Behavior

Section 4.2 shows that bidders evaluate only a small number of packages. Since an equilibrium strategy is unknown in the three auction formats, the question is what strategy bidders follow and whether they bid on all evaluated packages.



(a)Distinct package bids in the local-SVM



(b)Distinct package bids in the global-SVM

Figure 11 **Number of distinct packages on which bidders submitted a bid.**

4.4.1. Analysis of Individual-Item and Package Bids Pre-packaging in HPB limits the number of packages a bidder is able to bid on. Figure 11 illustrates on how many distinct packages a bidder submits a bid in each auction format and value model.

Result 11*(Package bids vs. individual-item bids). Independent of the number of possible bids, bidders only submit bids on a small number of packages. Excluding the opening bids of the first round, bidders only submitted up to 15 distinct bids during an auction. Pre-packaging in HPB leads to small number of package bids submitted by regional bidders.*

Support 11*Figure 11 shows the number of individual-item bids vs. the number of package bids. In HPB, the regional bidders submit ∅ 0.2 distinct package bids during an auction in the global-SVM and ∅ 1.57 in the local-SVM. In contrast, regional bidders in CC submitted ∅ 7.7 package bids in the global-SVM and ∅ 15.2 in the local-SVM.* □

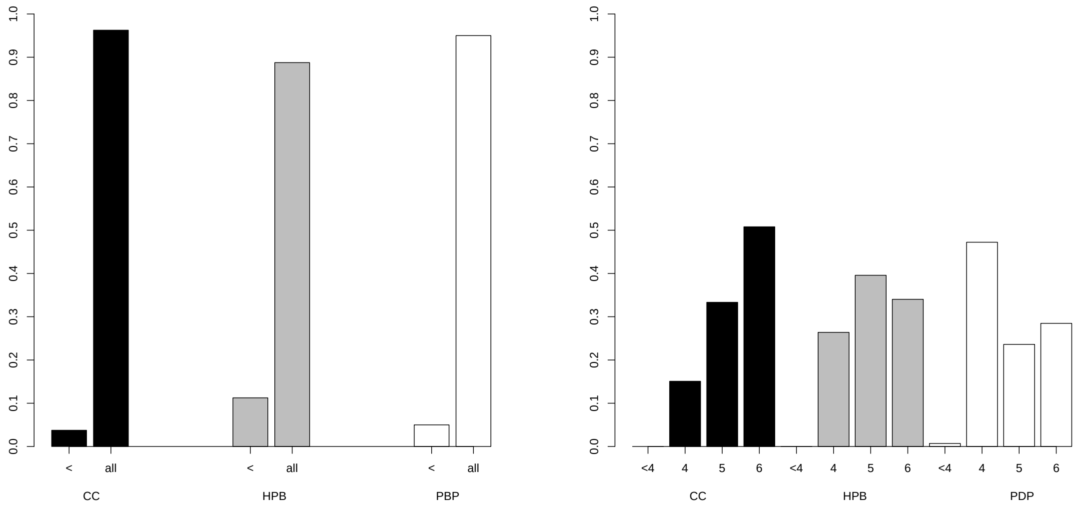
On average, bidders only bid on six to ten different packages throughout the auction, independent of auction format or bidder type (regional or national) in the global-SVM, although they had 56 or 4095 packages with positive valuations, respectively. Note that since HPB is not fully combinatorial, regional bidders have only up to seven and national bidders 16 different packages to bid on in the global-SVM, including the six and twelve individual items respectively. CC and PDP collected only a few individual-item bids (fewer than two per bidder and auction) in the global-SVM. In contrast, the regional bidders in HPB hardly made use of any package bid.

Due to a significantly larger number of possible bids in the local-SVM, we observed that bidders bid on more distinct packages on average compared to the global-SVM. However, the 15 to 27 distinct package bids throughout the auctions in the local-SVM still comprise only a small proportion of the possible bids (63-2,047 for regional bidders). Because of the restricted bid possibility in HPB, only eight to ten different packages were bid on. Unlike in the global-SVM, bidders bid on packages in HPB more frequently. Clearly, with the opening bids we forced bidders to more bid activity in the first round, which bias the average number of submitted bids in the local-SVM. Taking this into account, bidders did not bid on many more different packages on average than in the global-SVM, which suggests cognitive barriers in the number of packages a bidder can focus on independent of the number of possible bids.

4.4.2. Impact of Activity and Purchase Limits Regional bidders in the global-SVM have valuations for six distinct items, but have an activity and purchase limit of four items, i.e. they are only able to place bids on four different items in a round. Bidders in the local-SVM are not limited initially. We explore whether these limitations have an impact on bidder behavior by analyzing the distinct items the bidders placed a bid on. Figure 12 shows on how many distinct items a regional bidder placed a bid throughout the auction process, either individually or in a package.

Result 12*(Impact of tight activity limits). In the global-SVM more than half of the bidders in the PDP auction and HPB did not bid on all items with positive valuation, due to jump bids and activity limits. This phenomenon vanished in the local-SVM.*

Support 12*As shown in Figure 12, about* 50% *of the regional bidders in the PDP auction bid only on four items in the global-SVM. In HPB almost* 30% *of the regional bidders only bid on four items. This means that they never bid on two of the items for which they have positive valuations, neither individually nor in a package. Only very few bidders in the local-SVM did not bid on all items of interest (Figure 12).* □



(a)local-SVM (b)global-SVM

Figure 12 Proportion of regional bidders who bid on a particular number of items throughout the auction.

A reason for such behavior is jump bidding, which leads to the fact that many bidders win particular items for many rounds. If, due to a jump bid, they win a package of four items, they cannot submit bids on packages including any other items in these rounds in the PDP auction. Afterwards, the ask prices for other items often increase to a level such that a bid is not profitable. In the CC auction no jump bids are possible and no winning bids are announced in the auction process, so that bidders submit bids on many more items. In HPB, regional bidders face a similar problem as in the PDP auction; due to the activity limit and the announcement of the provisional winning bids, bidders are not always able to bid on all desired items. In the local-SVM, without activity limits and with opening bids, the situation is different. Most bidders submit bids on all items of interest.

5. Conclusions

The design of combinatorial auctions has been and remains a research challenge. Auction formats with strong game-theoretical solution concepts have shown to be impractical, or equilibrium strategies only hold for restricted valuations (Ausubel and Milgrom, 2006; Schneider et al., 2010). Linear-price combinatorial auctions have shown to be robust in simulations and lab experiments with small value models (Bichler et al., 2009; Kagel et al., 2009; Scheffel et al., 2010). The efficiency of combinatorial auctions in situations with more than 10 items of interest has not received much attention, although it is not clear whether the results of small combinatorial auctions also transfer to larger instances.

In this paper, we report on laboratory experiments comparing the Combinatorial Clock auction, an auction format with pseudo-dual prices, and the Hierarchical Package Bidding format in large value models with 18 items. All these auction formats have been used or suggested for selling spectrum, but they have not been compared in the same experimental setting. We use two different value models to analyze aggregate performance metrics such as efficiency and auctioneer's revenue share, as well as individual level bidder behavior. There are minor differences in efficiency and revenue across the auction formats, but the efficiency of all these linear-price formats is fairly high.

Interestingly, we find the limited number of packages that bidders evaluate to be the greatest barrier to efficiency, much more so than differences in the auction formats. On average, bidders evaluate about 14 packages besides the individual items, most of them in the first auction round, although they have a much larger number of profitable packages. They only submit six to 14 distinct package bids during an auction overall. We conjecture that this has to do with cognitive limits in terms of the number of items people can simultaneously concentrate on as have been analyzed in psychology for years (Miller, 1956). Practical auction formats need to take such cognitive barriers into account, either by limiting the number of items, by pre-packaging, or by advanced tooling, which allows a large number of packages to be easily explored. We also observe that changes in the auction rules matter. For example, activity rules can cause inefficiency in pseudo-dual price auctions with jump bids, while opening bids in the first round increase efficiency. While much fundamental theory in algorithmic mechanism design has been developed, the design of combinatorial auctions remains a formidable research challenge.

Appendix A: Auction Results

Auction Format	Auction Number	Wave	$E(X)$	$R(X)$	Payoff Nationals	Payoff Regionals	Number of Bids	Unsold Items	Rounds
CC	1	W1	0.96	0.90	2.00	0.80	330.00	1.00	16.00
CC	2	W1	0.93	0.85	0.00	1.30	235.00	0.00	14.00
CC	3	W1	0.96	0.81	0.00	2.60	290.00	0.00	17.00
CC	4	W1	0.91	0.82	0.00	1.60	304.00	2.00	14.00
CC	1	W2	0.82	0.63	0.50	3.20	197.00	0.00	8.00
CC	2	W2	0.91	0.70	0.00	3.50	176.00	1.00	18.00
CC	3	W2	0.83	0.65	0.00	3.00	197.00	2.00	15.00
CC	4	W2	0.98	0.77	0.00	3.40	198.00	0.00	23.00
CC	5	W2	0.74	0.43	0.00	5.20	70.00	3.00	6.00
CC	6	W2	0.85	0.62	0.00	3.70	114.00	3.00	12.00
CC	1	W3	0.86	0.73	0.50	2.10	313.00	0.00	21.00
CC	2	W3	0.92	0.76	0.00	2.70	254.00	1.00	13.00
CC	3	W3	0.83	0.55	0.00	4.60	183.00	1.00	18.00
CC	4	W3	0.93	0.84	0.00	1.40	243.00	0.00	12.00
CC	5	W3	0.95	0.86	0.40	1.50	277.00	0.00	20.00
CC	1	W4	0.91	0.79	6.80	0.70	236.00	0.00	14.00
CC	2	W4	0.95	0.81	-1.40	2.60	287.00	0.00	15.00
CC	3	W4	0.86	0.81	0.00	0.90	199.00	3.00	18.00
CC	4	W4	0.94	0.81	0.20	2.10	189.00	0.00	13.00
CC	5	W4	0.84	0.74	-4.40	2.30	161.00	3.00	14.00
CC	6	W4	0.94	0.78	0.00	2.70	257.00	0.00	26.00
HPB	1	W1	0.94	0.58	-0.10	6.00	62.00	0.00	9.00
HPB	2	W1	0.98	0.83	0.00	2.60	64.00	0.00	10.00
HPB	3	W1	0.98	0.74	0.00	4.10	80.00	0.00	17.00
HPB	4	W1	0.85	0.90	-7.40	0.50	97.00	0.00	20.00
HPB	5	W1	0.96	0.79	0.00	2.80	93.00	0.00	15.00
HPB	6	W1	0.87	0.74	1.20	2.10	69.00	0.00	7.00
HPB	1	W2	0.99	0.50	1.40	7.90	66.00	0.00	13.00
HPB	2	W2	0.80	0.70	-9.30	3.30	93.00	0.00	10.00
HPB	3	W2	0.89	0.63	1.10	4.10	96.00	0.00	15.00
HPB	4	W2	0.86	0.69	0.00	2.90	88.00	0.00	9.00
HPB	5	W2	0.92	0.62	0.00	4.90	83.00	0.00	19.00
HPB	6	W2	0.89	0.80	3.10	1.00	113.00	0.00	14.00
HPB	1	W3	0.86	0.69	0.50	2.70	68.00	0.00	11.00
HPB	2	W3	0.88	0.64	0.40	4.00	76.00	0.00	16.00
HPB	3	W3	0.98	0.57	0.00	6.80	66.00	0.00	8.00
HPB	4	W3	0.86	0.76	4.60	1.00	69.00	0.00	12.00
HPB	5	W3	0.88	0.73	0.00	2.40	80.00	0.00	14.00
HPB	6	W3	0.82	0.67	7.30	1.40	75.00	1.00	15.00
HPB	1	W4	0.99	0.78	14.90	0.90	96.00	0.00	11.00
HPB	2	W4	0.90	0.68	1.40	3.30	72.00	1.00	12.00
HPB	3	W4	0.89	0.82	-0.80	1.20	99.00	0.00	20.00
HPB	4	W4	0.91	0.68	-0.20	3.90	113.00	0.00	17.00
HPB	5	W4	0.99	0.73	0.00	4.30	75.00	0.00	7.00
HPB	6	W4	0.99	0.65	0.00	5.70	77.00	0.00	18.00
PDP	1	W1	0.67	0.58	0.00	1.40	57.00	6.00	17.00
PDP	2	W1	0.82	0.52	-1.80	5.40	60.00	3.00	10.00
PDP	3	W1	0.75	0.65	7.50	0.40	73.00	3.00	10.00
PDP	4	W1	0.94	0.64	1.80	4.60	51.00	0.00	8.00
PDP	5	W1	0.75	0.43	0.00	5.50	53.00	4.00	10.00
PDP	6	W1	0.84	0.50	0.00	5.70	43.00	2.00	11.00
PDP	1	W2	0.92	0.64	2.00	4.30	66.00	1.00	10.00
PDP	2	W2	0.81	0.58	0.30	3.70	63.00	2.00	10.00
PDP	3	W2	0.82	0.71	0.40	1.80	94.00	2.00	11.00
PDP	4	W2	0.85	0.66	0.20	3.10	72.00	3.00	13.00
PDP	5	W2	0.87	0.63	0.00	4.00	75.00	3.00	19.00
PDP	6	W2	0.86	0.55	0.00	5.20	54.00	5.00	9.00
PDP	1	W3	0.89	0.76	0.00	2.20	79.00	2.00	11.00
PDP	2	W3	0.96	0.66	0.90	4.90	107.00	1.00	9.00
PDP	3	W3	0.95	0.72	0.00	4.00	125.00	1.00	18.00
PDP	4	W3	0.89	0.74	0.40	2.50	93.00	1.00	14.00
PDP	5	W3	0.87	0.76	-3.40	2.40	133.00	0.00	14.00
PDP	6	W3	0.78	0.73	0.00	0.90	79.00	5.00	11.00
PDP	1	W4	0.87	0.74	0.00	2.10	67.00	3.00	7.00
PDP	2	W4	0.90	0.67	1.40	3.50	57.00	4.00	11.00
PDP	3	W4	0.87	0.63	2.80	3.60	56.00	3.00	9.00
PDP	4	W4	0.80	0.68	0.00	2.10	51.00	4.00	9.00
PDP	5	W4	0.94	0.73	0.40	3.40	46.00	0.00	7.00
PDP	6	W4	0.86	0.65	0.00	3.60	40.00	1.00	10.00

Table 8 Auction results in the global-SVM.

Auction Format	Auction Number	Wave	$E(X)$	$R(X)$	Payoff Nationals	Payoff Regionals	Number of Bids	Unsold Items	Rounds
CC	1	W2	0.83	0.83	0.20	0.00	375.00	0.00	16.00
CC	2	W2	0.95	0.90	0.40	0.90	322.00	0.00	13.00
CC	3	W2	0.93	0.83	0.00	1.80	362.00	0.00	14.00
CC	4	W2	0.86	0.84	2.00	0.00	417.00	0.00	12.00
CC	1	W3	0.97	0.80	1.30	3.00	279.00	0.00	11.00
CC	2	W3	0.92	0.85	0.00	1.40	466.00	0.00	11.00
CC	3	W3	0.87	0.78	0.90	1.50	484.00	0.00	11.00
CC	4	W3	0.98	0.94	0.00	0.90	422.00	0.00	14.00
CC	1	W4	0.98	0.87	1.70	1.80	581.00	0.00	16.00
CC	2	W4	0.88	0.88	0.20	0.00	647.00	0.00	13.00
CC	3	W4	0.92	0.85	0.20	1.50	526.00	0.00	12.00
CC	4	W4	0.94	0.82	0.00	2.30	596.00	0.00	11.00
CC	1	W5	0.92	0.81	0.00	2.10	442.00	0.00	12.00
CC	2	W5	0.98	0.91	0.00	1.40	353.00	0.00	13.00
CC	3	W5	0.94	0.87	0.00	1.40	605.00	0.00	12.00
CC	4	W5	0.94	0.86	1.40	1.20	678.00	0.00	12.00
HPB	1	W2	0.83	0.77	5.80	0.00	84.00	0.00	5.00
HPB	2	W2	0.88	0.74	14.00	0.00	60.00	0.00	5.00
HPB	3	W2	0.97	0.91	0.00	1.30	91.00	0.00	4.00
HPB	4	W2	0.86	0.78	8.30	0.00	70.00	0.00	6.00
HPB	1	W3	0.55	0.62	0.00	-1.40	96.00	0.00	6.00
HPB	2	W3	0.91	0.83	0.00	1.70	86.00	0.00	7.00
HPB	3	W3	0.79	0.73	5.90	0.00	109.00	0.00	8.00
HPB	4	W3	0.94	0.82	0.00	2.40	124.00	0.00	8.00
HPB	1	W4	0.86	0.83	2.30	0.00	79.00	0.00	6.00
HPB	2	W4	0.91	0.86	0.00	0.90	74.00	0.00	6.00
HPB	3	W4	0.83	0.74	8.60	0.00	79.00	0.00	7.00
HPB	4	W4	0.90	0.96	0.00	-1.20	95.00	0.00	7.00
HPB	1	W5	0.83	0.82	2.80	-0.30	109.00	0.00	7.00
HPB	2	W5	0.87	0.84	3.20	0.00	72.00	0.00	8.00
HPB	3	W5	0.97	0.83	0.00	2.80	120.00	0.00	8.00
HPB	4	W5	0.97	0.85	0.80	2.30	109.00	0.00	6.00
PDP	1	W2	0.81	0.91	-9.70	0.10	175.00	0.00	9.00
PDP	2	W2	0.87	0.95	0.00	-1.60	190.00	0.00	11.00
PDP	3	W2	0.90	0.81	0.50	1.60	192.00	0.00	11.00
PDP	4	W2	0.90	0.87	6.40	-0.60	115.00	0.00	11.00
PDP	1	W3	0.89	0.74	7.90	1.40	146.00	0.00	7.00
PDP	2	W3	0.89	0.85	0.00	0.90	145.00	0.00	8.00
PDP	3	W3	0.87	0.83	0.00	0.90	156.00	0.00	9.00
PDP	4	W3	0.98	0.93	0.00	1.10	146.00	0.00	6.00
PDP	1	W4	0.94	0.79	0.60	2.80	199.00	0.00	15.00
PDP	2	W4	0.83	0.77	4.70	0.20	100.00	0.00	9.00
PDP	3	W4	0.92	0.77	0.20	2.90	111.00	0.00	8.00
PDP	4	W4	0.91	0.69	0.00	4.40	97.00	0.00	8.00
PDP	1	W5	0.98	0.86	0.00	2.30	116.00	0.00	10.00
PDP	2	W5	0.88	0.86	1.60	0.10	123.00	0.00	8.00
PDP	3	W5	0.88	0.84	4.50	-0.10	111.00	0.00	7.00
PDP	4	W5	0.94	0.71	1.40	4.30	97.00	0.00	7.00

Table 9 Auction results in the local-SVM.

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