Iterative Combinatorial Auctions with Linear Prices: Results of Numerical Experiments

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Abstract

Iterative Combinatorial Auctions (ICA) have been getting increasing attention in computer science and economics, as they provide an efficient solution to resource allocation problems with superadditive and subadditive valuations. Several auction designs have been proposed in the literature using different types of bidding languages and different types of ask prices. A few auction designs have focused on simple linear prices. Although, it can be shown that exact linear prices are often impossible to calculate, ICAs based on linear prices have performed very well in the laboratory with respect to allocative efficiency. In this paper we focus on three promising auction designs, the Combinatorial Clock Auction, the Resource Allocation Design (RAD) and a modified version of RAD and analyze their performance in discrete event simulations.

1 Introduction

Combinatorial auctions (CA) allow the design of auctions for complex markets, where the bidders can place bids on combinations of items, called ”packages” or ”bundles” rather than just on individual items. This allows the bidders to better express their valuations and ultimately increases the overall economic efficiency [7, 3]. CAs have already found application in various domains ranging from the transportation to the industrial procurement and the allocation of spectrum licenses for wireless communication services. In comparison to sealed-bid designs, iterative CAs (ICAs) have been selected in many practical applications, since they help the bidders to express their preferences by providing feedback, such as provisional pricing and allocation information in each round.

In this work we focus on the ICA designs with *linear prices*, i.e. where each item in the auction is assigned an individual price, and the price of a package of items is the sum of the item prices. One can use item prices to compute any bundle price even if no bids were submitted for it. In addition, they are easy to understand for the bidders in comparison to the non-linear prices, where the number of prices to communicate in each round is exponential in the number of items [4, 12, 6].

In this paper we use numerical simulations as a tool to benchmark selected auction designs and compare their outcome based on the allocative efficiency and the auctioneer revenue. Our goal is to study the economic behavior of ICAs based on linear prices and to quantify potential efficiency losses due to imprecise linear pricing.

The paper is organized as follows. Section 2 provides an overview of the studied auction designs. Section 3 describes the central aspects of our simulation framework, the model parameters and the performance measures. In Section 4 we discuss the numerical results. Section 5 concludes.

2 ICA Designs Based on Linear Prices

Due to the size restrictions of this paper we refer the reader to [9] for the general introduction to ICAs and ICA pricing schemes. *Resource Allocation Design (RAD)*, *Combinatorial Clock Auction (CC-auction)* and *Clock-Proxy Auction* have been proposed as ICA designs mainly based on linear prices. Newly we also proposed a number of improvements for the RAD design, which we now combine to a modified RAD auction and refer to it as *RADm*.

The Combinatorial Clock Auction (CC-auction) proposed in [10] utilizes anonymous linear prices called item clock prices. In each round bidders submit bids on the packages they would like to purchase at the current prices. As long as demand exceeds supply for at least one item the price clock ”ticks” upwards for those items (the item prices are increased by a fixed price increment), and the auction moves on to the next round. If there is no excess demand and no excess supply, the items are allocated corresponding to the last round bids and the auction terminates. If there

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is no excess demand but there is excess supply, the auctioneer solves the winner determination problem considering all bids submitted during the auction runtime end terminates. The advantages of the CC auction is it's cognitive, computational and communicative simplicity.

The Clock-Proxy Auction is a relatively new extension of the CC auction proposed in [3] and [2]. Since it is not a purely based on linear prices, it is out of scope of this paper.

Resource Allocation Design (RAD) auction proposed in [7] also uses the anonymous linear pricing scheme. However, instead of increasing the prices incrementally, the auction lets the bidders submit priced bids. It then computes a provisional allocation and a set of approximative linear ask prices for the next round based on the submitted bid prices. In the next round the losing bidders have to bid more then the ask prices plus a fixed minimum increment, and the winning bids remain valid. Since the prices may sometimes fall, the auction termination relies on additional eligibility rules defined as in the *Simultaneous Multiround Auction (SMR)* [5]. The strength of RAD lies in it's communicative simplicity and in the flexible price calculation mechanism, which can be tuned to lead the auction to the efficient allocation and to lower some undesirable effects like the well known threshold problem.

Resource Allocation Design (RADm). In [11] we analyzed some pitfalls in the RAD price calculation algorithm and the termination condition. So it is possible that the calculated prices are not minimal, not unique and the price deviations are not minimized. It is also possible that the auction prematurely terminates due to the looping phenomenon (endless switching between two specific inefficient allocations). In the same paper we proposed a number of ways to avoid these problems. Now we introduce a new design called *Modified RAD (RADm)*, which differs from RAD in the following points: a) we use the modified price calculation rule, b) bidder has to overbid her own own bids and c) the auction terminates when no more valid losing bids exist. In the following we will see that the RADm generally produced better results than RAD.

In the following we compare the CC auction, RAD and RADm auction designs using discrete event simulations and simple artificial bidders bidding on all possible bundles, which provide positive value to them.

3 Setup for Numerical Experiments

3.1 Simulation Framework

We developed a generic simulation framework for ICAs including three main components: *Value Model, Bidding Agents* and *Auction Processor.*

A ***value model*** generates valuations of all possible bundles for each bidder. We analyzed the following three value models: *Pairwise Synergy, Transportation* and *Matching.*

The *Pairwise Synergy* value model is a slightly modified implementation of the model described in [1]. It is defined by a set of valuations of individual items and a matrix of pairwise item synergies (complementarities). A synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the bundle valuation is twice as high as the sum of the individual item valuations. The most important parameters are the interval for the randomly generated item valuations and the interval for the randomly generated synergy values.

The *Transportation* value model is built following the idea of the *Paths in Space* model from the Combinatorial Auction Test Suite (CATS) [8] with necessary adjustments for the use in simulations of ICAs. It simulates a nearly planar transportation graph, where each bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are edges (routes) of the graph.

The *Matching* value model is an implementation of the *Matching* scenario in CATS with necessary modifications. It models the four large airports each having a predefined amount of starting and landing time slots. Each bidder is interested in obtaining one starting and one landing slot in two randomly selected airports. The bidder valuation is proportional to the distance between the airports, but can be reduced if the plane arrives later as planned or/and has to delay landing until the landing slot becomes available.

A ***bidding agent*** implements a bidding strategy adhering to the given value model and to the restrictions of the specific auction design. In this set of simulations we did not use agents that emulate real-world bidding behavior or myopic best-response bidders, but agents bidding the minimal allowed price on all possible bundles that provide positive payoff at the current prices.

An ***auction processor*** implements the auction logic, enforces the auction protocol rules, calculates the ask prices and provisional allocation for the current round.

3.2 Performance Measures

The most important characteristic of a (combinatorial) auction design is it's *allocative efficiency* (or simply *efficiency*). Intuitively it shows whether the auction allocates items to the bidders who value them most highly. We measure the efficiency following [7] as the ratio of the total valuation of the resulting allocation to the total valuation of an efficient (best possible) allocation.

Another important characteristic of an auction design is the *auctioneer normalized revenue* (or simply *revenue*), which shows the auctioneer's share in the overall economical gain, that could have been generated by the auction. Given the resulting allocation and the bid prices, the revenue is measured as the ratio of the auctioneers income to the total valuation of an efficient allocation.

While achieving maximum efficiency is usually desirable for any auction design, it can sometimes compete with the revenue maximization goal. It is also possible for two auction designs with similar efficiency to produce significantly different auctioneer revenues.

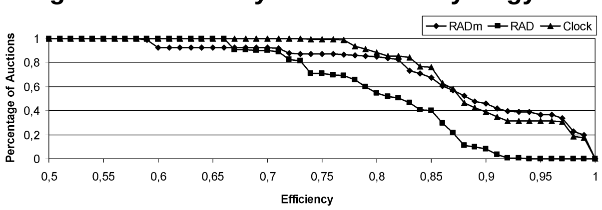
4 Simulation Results

4.1 Efficiency of Different ICA Designs

To compare performance of different ICA designs we have tested auctions of three selected designs against different value models.

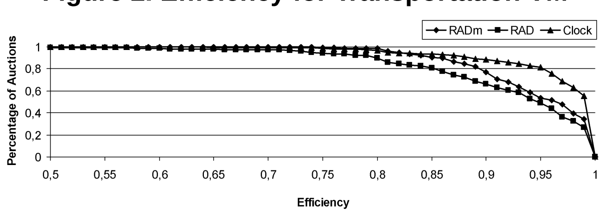
Within the Pairwise Synergy value model we have run 250 auctions with 5 items and synergy values ranging from 0 to 2.5. To create a competitive market with different but similar valuations, private value models for each bidder were set using different parameters, taken from the same narrow distribution, and bidders were restricted to a maximum bundle size of 3.

Figure 1. Efficiency for Pairwise Synergy VM



The Transportation value model was tested in 264 auctions with number of items varying from 5 to 26. There was no restriction on the bundle size in this case, and the amount of bidders was varying from 12 to 23 proportionally to the amount of traded items to achieve sufficient competition.

Figure 2. Efficiency for Transportation VM



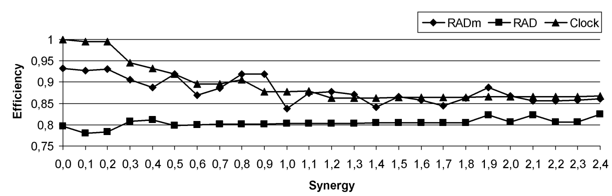
Another batch of 160 auctions was set up using the Matching value model with number of items varying from 40 to 100. Bidder settings were similar to the setup of the Transportation value model experiment.

The simulation results are presented in Fig. 1 and 2 (the matching case is essentially similar to the transportation and was omitted), which show which share of all auctions (y-axis) achieved at least the given efficiency (x-axis). They indicate generally a very good efficiency of the CC Auction. The RADm design outperforms RAD in most cases and is often very close to the CC design. Having analyzed several auction instances where the difference between RADm and RAD was especially large, we can say that RAD looses efficiency primarily for two reasons. First, it tends to terminate prematurely, when some bidders may still have valuations exceeding current prices. Second, unbalanced prices can prevent a bidder from further participating in the auction. The good performance of the CC design can be also explained by up to 30% higher number of rounds compared to the RAD designs with the same minimum increment. Furthermore, the CC auction forces bidders to provide more information to the auctioneer, see also Subsection 4.3.

4.2 Complementarity vs. Efficiency

In order to quantify potential efficiency losses in case of high synergies between the item valuations, we have used the Pairwise Synergy value model and measured the auction efficiency across different synergy values varying from 0 to 2.5 for different auction designs.

Figure 3. Impact of Synergies on Efficiency



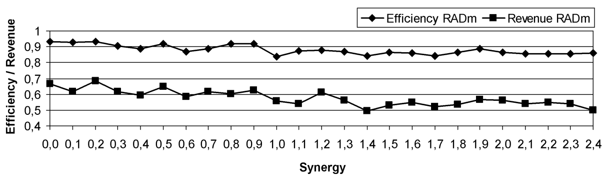
The simulation results are shown in the Figure 3. As expected, auction efficiency decreases as complementarity between items grows. The CC auction outperforms the RADm in the low synergy range; starting from a synergy value of 0.5 both auctions perform equally well. The efficiency of the RAD auction is limited because of other reasons (see Section 4.1) and lies slightly below both other designs.

Interestingly, auction efficiency remains relatively high for all auction designs even in case of high synergy values. This fact can be explained by the specifics of the Pairwise Synergy value model, where the valuations of a bidder for all bundles are relatively high compared to other value models, where some bundles might have a value of null. In addition, we have indicated that the simulation results can be interpreted as upper bounds of efficiency, which are less likely in laboratory settings.

4.3 Utility Distribution

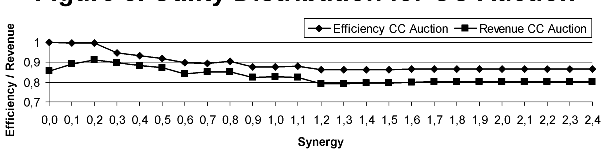
Another important characteristic is the auctioneer revenue. In a broader sense, it is particularly interesting which part of the overall utility (economic gains of the trade) goes to the auctioneer, and which part is distributed to the bidders. In cases where the auction is not 100% efficient, still part of the overall utility is simply lost.

Figure 4. Utility Distribution for RADm



Our simulations indicate different utility distributions in the three auction designs (essentially independent of the value model). Figures 4 and 5 show efficiency and revenue values for different auction designs based on the the Pairwise Synergy value model. For each synergy value on the x-axis, the distance under the *Revenue* curve is proportional to the auctioneer revenue, the distance between the *Revenue* and the *Efficiency* curves is proportional to the part of the overall utility which is distributed to the bidders, and the rest distance between the *Efficiency* curve and line $y = 1$ can be interpreted as the utility lost due to the inefficiency of the final allocation.

Figure 5. Utility Distribution for CC Auction



One important observation is that the CC design in all cases produced revenue nearly as high as efficiency, which means that the most part of the overall auction utility goes to the auctioneer. On the other hand, RAD and RADm auction designs result in a lower utility share for the auctioneer, as they attempt to construct minimal prices given the submitted bids and do not just increase prices as the CC design does. This will simplify the bidders strategy, since the bidders do not need to try to shade their bids.

5 Conclusion and Outlook

In this paper we used computer simulations to compare of three ICA designs based on different value models. We analyzed how complementarity among items affects its efficiency in case of linear ask prices. We have also looked into the distribution of utility between the auctioneer and the bidders. The results provide upper bounds on efficiency and a starting point for further experimental analysis.

In our future work we plan to extend our work with additional sensitivity analyzes using linear and non-linear price designs under different parameter regimes. In particular, we want to study the influence of minimum increment size, number of bidders, different bidding behavior, and budget constraints on the auction outcome.

References

[1] N. An, W. Elmaghraby, and P. Keskinocak. Bidding strategies and their impact on revenues in combinatorial auctions. *Journal of Revenue and Pricing Management*, 2005.

[2] L. Ausubel, P. Crampton, and P. Milgrom. The clock-proxy auction: A practical combinatorial auction design. Technical Report 04mit5, University of Maryland, Department of Economics, 2004. available at http://ideas.repec.org/p/pcc/pccumd/04mit5.html.

[3] L. Ausubel, P. Crampton, and P. Milgrom. The clock-proxy auction: A practical combinatorial auction design. In P. Cramton, Y. Shoham, and R. Steinberg, editors, *Combinatorial Auctions*. MIT Press, Cambridge, MA, 2005.

[4] S. Bikhchandani and J. M. Ostroy. From the assignment model to combinatorial auctions. In P. Cramton, Y. Shoham, and R. Steinberg, editors, *Combinatorial Auctions*. MIT Press, Cambridge, MA, 2005.

[5] P. Cramton. Ascending auctions. *European Economic Review*, 42(3-5):745–756, 1998.

[6] A. Drexl, K. Jornsten, and D. Knof. Non-linear anonymous pricing in combinatorial auctions. Working Paper October 2005, Christian-Albrechts-University, Kiel, Germany, 2005.

[7] T. Kwasnica, J. O. Ledyard, D. Porter, and C. DeMartini. A new and improved design for multi-objective iterative auctions. *Management Science*, 51(3):419–434, 2005.

[8] K. Leyton-Brown, M. Pearson, and Y. Shoham. Towards a universal test suite for combinatorial auction algorithms. In *ACM Conference on Electronic Commerce*, pages 66–76, 2000.

[9] D. C. Parkes. Iterative combinatorial auctions. In P. Cramton, Y. Shoham, and R. Steinberg, editors, *Combinatorial Auctions*. MIT Press, Cambridge, MA, 2005.

[10] D. Porter, S. Rassenti, A. Roopnarine, and V. Smith. Combinatorial auction design. *Proceedings of the National Academy of Sciences of the United States of America (PNAS)*, to appear, 2003.

[11] P. Shabalin, A. Pikovsky, and M. Bichler. An analysis of linear prices in iterative combinatorial auctions. Working paper, IBIS, October 2005. Available at http://ibis.in.tum.de/staff/pikovsky/index.htm.

[12] M. Xia, G. J. Koehler, and A. B. Whinston. Pricing combinatorial auctions. *European Journal of Operational Research*, 154(1):251–270, 2004.